THEORY OF THE RESISTANCE MINIMUM IN DILUTE PARAMAGNETIC ALLOYS

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The presence of a minimum in the electrical resistivity as a function of temperature, at low temperatures, appears to be a general property of dilute paramagnetic alloys of a transition metal in a noble metal.[1] Recent experiments[2] on Cu-Fe alloys indicate that the resistance minimum is a true bulk property depending on the presence of paramagnetic ions in random solid solution.

It has been shown that the scattering of conduction electrons by paramagnetic ions yields a temperature-independent contribution to the resistivity.[3] However, if the Zeeman levels of the ions are split by a magnetic field (for example), this contribution becomes temperature dependent. The anomalous resistivity of paramagnetic alloys suggests the presence of scattering centers with closely spaced energy states. Such centers occur when the interaction between nearest-neighbor pairs of paramagnetic ions is considered. If the spin of each ion is S, then states of the ion-pair are characterized by the total spin I, which has integer values between 0 and $2S$. The energy separation of these states of different I depends on an exchange integral W. We have calculated the resistance arising from scattering by such ion-pairs, and find that if the interaction of a pair is ferromagnetic ($W>0$), a resistance minimum should occur. Since this mechanism involves pairs of ions, the size of the minimum should be proportional to the square of the concentration of the transition metal. This quadratic dependence appears to be qualitatively confirmed for the most dilute Cu-Co alloys investigated by Jacobs and Schmitt.[4] [One expects this dependence to obtain only for extreme dilution, where the $(2I+1)$-fold degeneracy of each level is not split by other interactions.]

A mechanism for a temperature-dependent resistivity is described in the heuristic model of Schmitt.[5] The resistivity caused by elastic scattering will be temperature dependent since the occupancy of the different energy states, having different scattering cross sections, varies with temperature. Inelastic scattering is temperature dependent for the same reason and also because the available final states for the scattered electron depend on temperature.

We shall treat the s-d exchange scattering of conduction electrons by ferromagnetically coupled pairs. The model Hamiltonian is the sum of two terms:

$$H_0 = -W\vec{S}_1\cdot\vec{S}_2, \tag{1}$$

and

$$H_1 = V(\vec{r}) + V(\vec{r}-\vec{R}) - 2J(\vec{r})\vec{s}\cdot\vec{S}_1 - 2J(\vec{r}-\vec{R})\vec{s}\cdot\vec{S}_2. \tag{2}$$

$\vec{S}_1$ and $\vec{S}_2$ are the spin operators for the ions, $\vec{s}$ the spin operator for an electron in the conduction band of the solvent metal, and $\vec{R}$ is the nearest-neighbor distance. $V(\vec{r})$ and $J(\vec{r})$ denote the spin-independent and spin-dependent coefficients of the interaction between a conduction electron and an ion. The eigenvalues of H_0 are given by

$$E_I = \tfrac{1}{2}W[I(I+1) - 2S(S+1)],$$

each level being $(2I+1)$-fold degenerate.

We find that the average elastic scattering cross section for <u>each</u> of the $2I+1$ states of energy E_I is proportional to $I(I+1)$. If $W>0$, states of lower energy have larger elastic scattering cross section, so that this contribution to the resistance increases as temperature decreases. Therefore a resistance minimum would appear to be easily explained. However, one must take account of inelastic scattering, which becomes frozen out at low temperatures and yields a decreasing contribution to the resistance as temperature decreases. The net temperature dependence depends on the close competition of elastic and inelastic processes, so one must explore the model in detail to show that the elastic contribution predominates.

The scattering arising from the V terms of (2) is temperature independent and need not be considered further. All cross terms between V and J terms may be shown to vanish when square matrix elements are averaged over conduction electron spin states. The J terms yield the following elastic and inelastic contributions to the resistivity:

$$\rho_{\text{el}} = \alpha f_+ B_+, \tag{3}$$

$$\rho_{\text{inel}} = \alpha f_- B_-, \tag{4}$$

where

$$\alpha = m^2 N_p / 2\pi n e^2 \hbar^2 K^3,$$

N_p being the number of pairs per cc, n the electron concentration, and K the Fermi wave number. The factors $B_\pm$ are the following integrals of the Fourier transform $J(q)$ of $J(\vec{r})$:

$$B_\pm = \int_0^{2K} |J(q)|^2 [1 \pm (\sin qR/qR)] q^3 dq. \quad (5)$$

The second term in the square brackets of (5) represents the interference of the scattered waves from the two atoms of a pair (averaged over random directions of $\vec{R}$). One should observe that the interference is constructive for elastic scattering, whereas it is destructive for inelastic scattering. It is precisely this feature which allows the elastic scattering to predominate over the inelastic. The remaining factors, $f_\pm$, of (3) and (4) contain the temperature dependence of the resistivities and are

$$f_+ = \sum_{I=0}^{2S} p_I (2I+1) I(I+1)/4, \quad (6)$$

$$f_- = \sum_{I=1}^{2S} p_I p_{I-1} I[(2S+1)^2 - I^2]/(p_I + p_{I-1}), \quad (7)$$

where p_I is the equilibrium probability that each state of energy E_I is occupied.

Consider now the temperature dependence of the resistivity contributions. For $W/kT < 1$, Eqs. (6) and (7) become

$$f_\pm \cong \tfrac{1}{2} S(S+1)[1 \pm S(S+1) W/3kT].$$

Therefore, the net temperature-dependent contribution to the resistivity is

$$\Delta\rho = \alpha S^2 (S+1)^2 (B_+ - B_-) W/6kT. \quad (8)$$

Since $B_+ > B_-$ [assuming only that $J(q)$ is well behaved], the resistivity will increase with decreasing temperature. One can show that this increase will be monotonic all the way to 0°K as long as

$$B_+/B_- > 2(4S+1)/(4S-1). \quad (9)$$

If the inequality (9) is not satisfied, the resistivity will go through a maximum before reaching its 0°K value. In either case the competition between (8) and the phonon resistivity should produce a minimum.

The magnitude of the anomalous resistivity increase also agrees with experiment for reasonable values of J and W. A detailed account of this work will be published elsewhere.

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ATTENUATION OF SOUND IN A GERMANIUM CRYSTAL AT ULTRA-HIGH FREQUENCIES AND LOW TEMPERATURES*

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We have measured the ultrasonic attenuation of compressional and shear waves in a high-purity crystal of germanium at frequencies up to 650 Mc/sec and at temperatures down to 1.5°K. At room temperature there is some evidence for the dislocation loss mechanism, but at low temperatures the attenuation is very small.

Previous measurements of ultrasonic attenuation at high frequencies in germanium have been at room temperature, and limited to 90 Mc/sec in the guided-wave method[1] and to 300 Mc/sec in the usual, unbounded medium, method.[2] We have added an ultra-high-frequency pulsed oscillator to our equipment to extend the frequency range to 700 Mc/sec. The low temperatures were achieved in a conventional liquid helium cryostat, the electrical energy passing down a bifilar line to the transducer, which was bonded to the parallel-sided specimen.

The specimen was of n-type germanium, with room temperature resistivity 45 ohm-cm and net donor concentration 1×10^{12}/cc. Its flat faces