of $\vec{H}$ in the plane. We expect that b will increase as $\vec{H}$ departs from the [100] axis; this would ordinarily mean fewer subharmonics would be resolved. The fact that more are resolved can be explained by assuming that c increases faster than b. If the Fermi surface overlaps the zone edge near the center of the (111) face, m^* of the normal branch should have singularities when $\vec{H}$ is near the [110] direction, since then open orbits are possible. The normal branch thus "maps out" the Fermi surface. In addition to the normal branch, several anomalous branches may appear associated with the (111) contact areas [$\vec{k}_H{}^0 = \pi a^{-1}(1,1,1)$].

Quite different line shapes are expected depending on whether $C = (\gamma c/\delta)^2$ or $B = b^2$ is larger. When more subharmonics are resolved, so that C is larger, the masses are more unique, so that the asymmetrical lines predicted by Azbel' and Kaner in the ellipsoidal case should be more in evidence. In particular the asymmetry should be preserved in the higher subharmonics by (3) whereas (2) leads to a progressively symmetrical broadening.

Chambers[11] has argued that in the extreme anomalous limit Z should be independent of $\vec{H}$ for $\vec{H}$ normal to the surface. In this geometry τ in (1) should be replaced by $\tau/(1 + i\omega'\tau)$, where (for circular polarization) $\omega' = \omega - \omega_C$. From (1) and (3) it can be seen, in agreement with Chambers' conclusion, that the decrease in τ is exactly compensated by an increase in the number of effective carriers. Under these conditions, however, Galt *et al.*[4] have observed large effects in Zn very similar to those obtained under classical conditions in Bi.[12] It is possible that these effects result from higher-order corrections, such as the variation of $\vec{E}$ in the skin depth, which are neglected in the ineffectiveness treatment. In any case the observed masses should result from tubes with $\vec{v}_D \cong 0$. With $\omega\tau \cong 10^2$, τ replaced by $1/\omega$ and $\gamma \cong 1$, we find as in the transverse case $v_D \lesssim 10^{-2} v_F$. Thus the same masses should be observed as in the transverse case; this conclusion agrees qualitatively with the results of Galt *et al.*[4] in Zn.

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CYCLOTRON RESONANCE IN COPPER[*]

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We are reporting preliminary results of a study of cyclotron resonance in copper. Observation of cyclotron resonance in a sample of natural copper consisting of a small number of single crystals was first reported by Langenberg, Kip, and Rosenblum.[1] The present experiments were done with an artificially grown single crystal and, because of improved experimental techniques and a more satisfactory sample, have yielded significantly more detailed results. These are the first observations of a resolved cyclotron resonance in a monovalent metal.

The experiments were done at 4.2°K and at a microwave frequency of 24 kMc/sec using a conventional microwave bridge. One plane surface of the coin-shaped sample was cut within two degrees of a (110) plane and electropolished. This surface formed one end wall of a TE_{011} cylindrical cavity. The sample and cavity were mounted in a magnetic field so that they could be rotated together about an axis normal to the sample surface, keeping the magnetic field in the plane of the sample. Experiments were done both with and without magnetic field modulation. The results obtained with the two methods were consistent. Most of the results presented here are based on the field modulation data because these data were generally better.

The experimental results without field modulation showed a general decrease in sample surface resistance with increasing magnetic field. This decrease was estimated to be approximately 30% from zero field to the maximum value of 16 kilo-oersteds.

Well-resolved resonances were observed for all orientations of the magnetic field in the (110) plane. Only single masses were observed when the magnetic field was parallel to the ⟨100⟩, ⟨110⟩, and ⟨111⟩ axes; however, for most arbitrary orientations, at least two masses were well resolved. For some orientations, the anisotropy is such that the mass continuity is lost when making changes in orientation as small as two degrees.

Cyclotron masses were calculated from the period $\Delta(1/H)$ of the derivative maxima, as shown in Fig. 1. The masses for H parallel to the principal crystal axes and their behavior with respect to orientation are given in Table I; the value of $m^*(111)$ is in excellent agreement with the value estimated by Shoenberg[2] from the temperature dependence of the de Haas–van Alphen effect. Although the observed masses lay between extreme limits of $0.5m_e$ and $5.0m_e$ (both values observed at the same orientation, with H 40 degrees from a ⟨100⟩ axis), the masses for most orientations were between $1.1m_e$ and $1.4m_e$. Experimental errors in the masses themselves were usually less than 2%; the largest uncertainty in quoting masses for specific orientations where the anisotropy was large arises from small uncertainties in orientation.

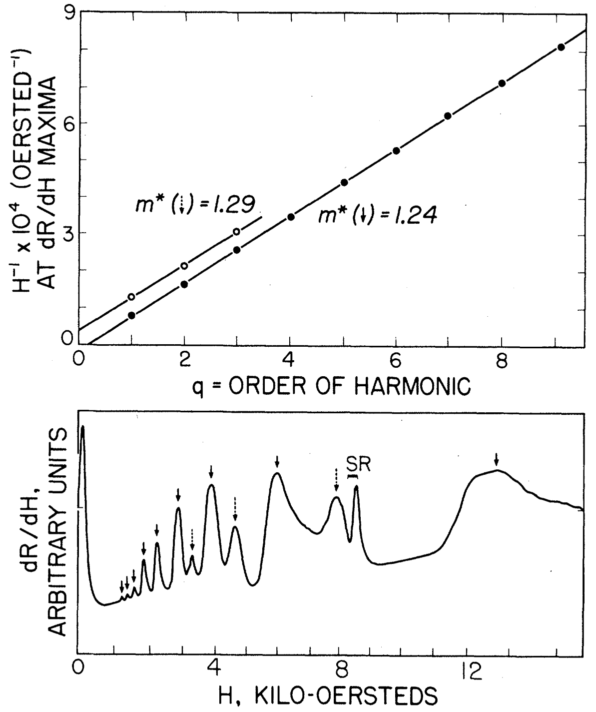

FIG. 1. The lower panel shows a recorder trace of absorption derivative for H in a (110) plane 10° from a ⟨100⟩ axis. J_{rf} is in a (110) plane and 45 degrees from H, 55 degrees from the ⟨100⟩ axis. $f = 24.47$ kMc/sec. No significance should be attached to the envelope of the derivative peaks since the lock-in detector phase varied with magnetic field. The upper panel is a plot of reciprocal magnetic fields at the absorption derivative maxima against integers; $m^* = e/[\omega c \Delta(1/H)]$, where $\Delta(1/H)$ is the slope of the $1/H$ plot. Note the large "phase" discrepancy between the two nearly equal masses.

Table I. Cyclotron masses for copper with H in (110) plane parallel to principal crystal axes.

Axis	Mass	Behavior of mass with respect to orientation
⟨100⟩	$(1.32 \pm 0.02)m_e$	maximum
⟨111⟩	$(1.30 \pm 0.03)m_e$	increasing as H rotated toward ⟨110⟩ axis
⟨110⟩	$(1.12 \pm 0.02)m_e$	minimum

For finite relaxation times (we estimate $\omega\tau \sim 50$ in the present case), small corrections should be applied to these experimental masses because absorption derivative maxima rather than absorption minima have been used and because the absorption minima themselves do not occur at exactly the cyclotron fields. No attempt has been made to apply these corrections since the latter one depends upon the "character" of the mass, i.e., whether the associated carriers are on a region of the Fermi surface where the mass is a relative minimum or maximum, or ellipsoidal (Azbel' and Kaner[3]), and this is not yet known with any certainty. We believe the net mass correction to be small in any case, certainly less than 5 %, since the two corrections tend to cancel one another, $\omega\tau$ is large, and there is no detectable curvature in the $1/H$ plots.

These resonances are perhaps the first in any metal for which the relationship with the Fermi surface appears fairly clear and from which further detailed information about the surface may be derived. Pippard,[4] on the basis of anomalous skin-effect experiments, has proposed a model of the Fermi surface of copper which lies entirely in the first Brillouin zone and is very nearly spherical except for "humps" which contact the hexagonal zone boundaries around $\langle 111\rangle$ directions; Olson and Rodriguez[5] also conclude that the surface contacts the zone boundary, from an analysis of magnetoresistance measurements. The very complexity of our data seems to argue against the existence of a single, closed convex surface. If Pippard's model is at least qualitatively correct, the following features should be apparent in the resonances: for H parallel to a $\langle 100\rangle$ axis, closed orbits are available to carriers having small momentum along the field, and those having no momentum along the field should display a resonance, since by symmetry they have an external mass. If the resonance due to these "equatorial" orbits is followed as H is rotated toward a $\langle 110\rangle$ axis, the associated masses should change only slowly over the nearly spherical part of the Fermi surface, but should have singularities at the three orientations for which the "equatorial" orbits just intersect the edges of the contact areas. Our data are in excellent agreement with these comments. One of the expected mass singularities has been observed in detail with H $18^\circ \pm 1^\circ$ from a $\langle 110\rangle$ axis, as compared to $14\frac{1}{2}^\circ$ predicted by the Pippard model; the present data are not complete enough for us to observe the other two expected singularities.

For H along the $\langle 100\rangle$ and $\langle 110\rangle$ axes only 8 or 9 harmonics are resolved and the shape of the derivative peaks is fairly symmetrical, while for H along arbitrary axes 12 harmonics are often observed and the peaks have the asymmetrical "N" shape characteristic of the Azbel'-Kaner theory for ellipsoidal (unique) masses and high $\omega\tau$; in general, the peaks become increasingly symmetrical as the order of the harmonic increases. The implications of these results are considered by Phillips.[6]

An interesting unexplained feature of the resonance is that at some orientations the "phase" of the oscillations is not that predicted by Azbel' and Kaner; i.e., if the period of the oscillations is fitted to the Azbel'-Kaner curve, the positions of the peaks are shifted with respect to the theoretical curve. This phase shift is most easily observed by noting the intercept of the $1/H$ plots (see Fig. 1). In at least one case a continuous and rather rapid variation of the phase is observed as the magnetic field orientation is changed. A phase discrepancy has also been observed by Aubrey and Chambers[7] in bismuth and by Galt[8] et al. in zinc.

This work is being continued and a complete report will be published at a later date.

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