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PLASMA JET PIERCING OF MAGNETIC FIELDS AND ENTROPY TRAPPING INTO A CONSERVATIVE SYSTEM*

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To obtain a thermonuclear plasma inside a magnetic confinement system, we have either (a) to create the plasma in situ from cold gas by ionization and heating or (b) build up the plasma piecemeal by injection from some accelerator or hydromagnetic gun.

Both of these methods have characteristic requirements: For (a) as in the pinch effect, great input power is needed in order to get through the strongly radiating partly ionized plasma phase and after that it is difficult to raise temperatures of the ions much above a kilovolt by Joule heating on account of the high electrical conductivity of the plasma and the slow exchange rate between the electrons which receive the Joule heating and the ions which it is desired to heat. For (b) single particles cannot of course, be trapped into a static magnetic field and some method of circumventing the conservative property has to be found. Trapping by means of a rapidly operating magnetic trap door has been discussed by Post. Another nonadiabatic process can be the breakup of an energetic molecular ion such as $D_2^+ \rightarrow D^+ + D^+ + e$ by collision with an internal arc column as in the DCX experiment (Luce) or by collision with residual gas as in the OGRA experiment (Golovin). Energetic neutral D_2 can in principle be used instead of D_2^+. Still another process can be by interparticle collisions, as for example the injection of plasmoids by Bostick. Such plasmoids soak into the magnetic field and after developing the appropriate Hall polarization can continue across it. In this note, we propose a new method of achieving the nonadiabatic process which may prove very advantageous.

Consider a collisionless plasma of pressure p, confined on all sides by a magnetic field of magnitude $(8\pi p)^{1/2}$. Let the plasma now assume the improbable but possible configuration in which the particles all are directed at some small region on the wall. We see that the particle pressure there becomes greater than the magnetic field pressure, so that the field can be forced aside letting plasma escape. If now we review these events in reverse time sequence, a method emerges for the trapping of plasma into a magnetic field.

Consider the interaction of a broad parallel jet of cold plasma, density ρ_0, velocity v_0 in the x direction, electron mass m, ion mass M, incident on a magnetic field of magnitude B of large radial extent. The velocity v_0 is large, such that the cross section for ion-electron collisions with relative velocity v_0 is negligible (no collisions). These conditions resemble those for the plasma-field sheath described in the infinite-conductivity theory of the pinch (Rosenbluth and Garwin)[1] except that the particles here may be incident at angles other than normal to the magnetic field direction. According to this model, charge separation occurs, setting up an electric field which retards the ions and speeds up the electrons. The deflection of the ions is predominantly electrostatic, so that they are brought nearly to rest; the electrons reach an energy close to the original ion energy in the jet, and provide most of the electric current which screens off the magnetic field. The thickness of the sheath over which the magnetic field and plasma density drop reciprocally to zero is given by $d=v_0c(mM)^{1/2}/eB$. Particles enter and leave the sheath at the same angle.

We choose the stagnation pressure of the jet, $\rho_0 v_0^2$, to be greater than the magnetic field pressure so that yielding of the field occurs. (For simplicity in the order-of-magnitude discussion which follows, the field is chosen of large extent so that the magnetic field cannot pile up to larger values.) Let us consider the situation where the jet has pierced the magnetic field for some distance. We shall assume that the process of reflection off the curved end of the cavity can be reasonably approximated by uniform (cosine law) scattering from a plane end.

Let the rate of elongation of the cavity ("tip speed") be v. Writing the pressure balance equation for the end [remembering that for cosine scattering the mean backward particle velocity is one-half the particle speed $(v_0 - v)$ relative to the wall], then

$$\tfrac{3}{2}\,\rho_0(v_0 - v)^2 = B^2/8\pi,$$

or

$$\epsilon = \frac{\text{tip speed}}{\text{jet speed}} = 1 - \left(\frac{B^2}{12\pi\,\rho_0 v_0^2}\right)^{1/2} = 1 - B/B_c;$$

$$B_c^{\,2} = 12\pi\rho_0 v_0^2 = 7.7\times10^2 j\,\sqrt{E}, \qquad (1)$$

where j = deuteron equivalent current density in amp/cm^2 and E = deuteron energy in ev. Similar equations were derived for the neutral stream magnetic storm theory of Chapman and Ferraro[2] in 1930. We see that a critical value B_c of the magnetic field will bring ϵ and the penetration velocity to zero. Larger B give a negative ϵ, corresponding to extrusion of a previously penetrated beam. Using mass conservation and letting ρ be the internal density after scattering, we next write

$$\rho_0 v_0 = \rho[\tfrac{1}{2}(v_0 - v) - v] + \rho v,$$

or

$$\rho = 2\rho_0 v_0/(v_0 - v). \qquad (2)$$

The radial pressure on the walls, holding apart the magnetic field, using this density and the mean radial velocity $\frac{1}{2}(v_0 - v)$, turns out to be $\rho_0 v_0(v_0 - v)$ which is only $2v_0/3(v_0 - v)$ of the required value of $B^2/8\pi$. To meet this difficulty, (a) the cavity could expand leaving a neck at entry—this neck reflects back some of the outwardly escaping particles raising ρ to the value required to keep the walls from collapsing, or (b) the walls could contract to the point where they intercept some of the incoming beam, again raising the wall pressure. Resorting to the analogy of the cavitation of water by a jet of air, we find both cases reported in the literature of hydrodynamics together with a third possibility—Rayleigh-Taylor instability with mixing. We may expect the latter to occur in magnetic field piercing especially when the magnetic field direction is transverse to the jet direction, since in such a case the magnetic field-plasma boundary is necessarily curved in the unstable sense.[3] Such instabilities will incidentally trap plasma and may be responsible for the trapping reported by Coensgen and Ford.[4] For injection in the axial direction, the possibility of hydromagnetic stability appears to be greater; Helmholtz instability is minimized by the highly supersonic state of the jet.[5] It might be argued that one kind of trapping is as good as another, and that this process can be welcomed; as will appear however, we require the plasma to attain the maximum possible density with a minimum of intermixed magnetic field.

In order to exploit the trapping possibilities, consider a beam entering a magnetic field such as a picket fence or cusped geometry[6] which after the initial increase to some value less than B_c, falls again. The beam can pierce the barrier forming a neck but in doing so must raise its pressure to a value larger than the field pressure further on. Consequently the jet cavity can expand against the field further down stream, forming a balloon (Fig. 1). Any jet which makes a clear traverse of the peak field without deflection, can be scattered off a $B > B_c$ "anvil" field further on, though this is not essential to the argument since scattering at the entry reduces the beam piercing power. The size of the plasma balloon will be governed by the balance between input flow and loss by back flow and diffusion into the magnetic field.

If the jet is now stopped, pressures fall and as the magnetic walls close in, the neck closes in faster, pinching off the balloon and leaving a plasma bubble trapped inside. The picket fence seems particularly well adapted to this method, since the field has the appropriate valley in the center of the machine, though the method seems adaptable with trivial modifications to Stellarator and mirror machine geometries.[7] Two types of

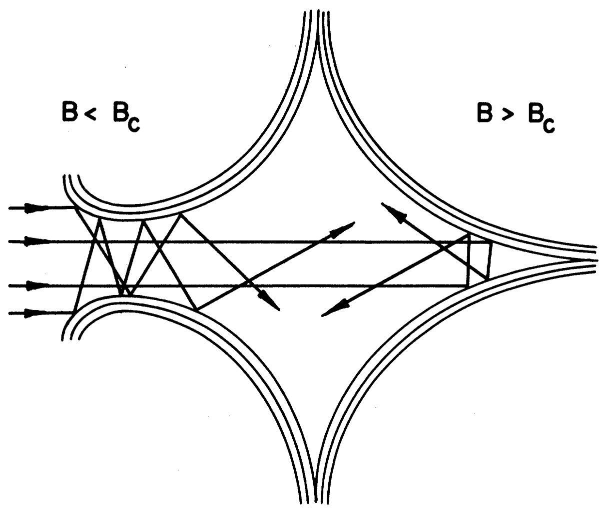

FIG. 1. Jet piercing into a picket fence magnetic field.

operation come to mind: (1) pulsed inflation whereby the plasma bubble pinched off during the off-beam period by the collapsing walls of the entry channel is left to react inside the machine; (2) steady operation whereby a large volume is kept full against losses by mixing and back-streaming. In the latter case, the prime consideration for a thermonuclear reactor becomes the chance of an ion reacting compared with its chance of finding the exit channel and escaping. At first sight it might appear that this ratio might be made arbitrarily advantageous by reducing the jet to needle-like dimensions as from an accelerator. However, there is a limit to the narrowness of the beam set by (1) breakdown of the infinite plane sheath approximation and (2) large pressure balance errors when the sheath thickness d becomes comparable with the jet diameter; and furthermore boundary layer instability may erode the jet during the penetration, limiting the ratio of the penetration depth of the unscattered beam to the jet diameter to some constant value as in the hydrodynamic analog.

The expression for the total current carried by one kind of particle in a deuterium jet of cross-sectional area $\alpha^2 d^2$, using the current density from Eq. (1), turns out to be (independently of B)

$$I = 1.35\times10^{-15}\alpha^2 mc^2\sqrt{E}, \quad (E \text{ in ev}).$$

We do not yet know how low α can be. Making a reasonable choice of $\alpha = 10$, we have

$$I = 0.3\sqrt{E} \text{ amperes.}$$

We are naturally interested in injecting plasmas for which no further heating is required, and we see that for $E = 90$ kev, $I \sim 100$ amperes. This is a current outside present accelerator practice by a factor of 100 but should not be too difficult, if one uses a condenser power supply. More immediately available seems to be a hydromagnetically accelerated plasma jet. Taking tentative estimates of particle density 10^{16} cm^{-3}, $v_0 = 10^7$ cm/sec for quantities already achieved,[8] we find the critical magnetic field for piercing to be a sizable 6500 gauss. This hydromagnetic gun already produces a volume of jet adequate for inflating several liters to a density of 10^{16} cm^{-3}. A 30-fold increase of the present velocity would bring the plasma to the optimum temperature for a DT thermonuclear reactor.

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[1]M. Rosenbluth and R. Garwin, Los Alamos Scientific Laboratory Report LA-1850, 1954 (unpublished); M. Rosenbluth, Magnetohydrodynamics, edited by R. Landshoff (Stanford University Press, Stanford, 1957).

[2]S. Chapman and V. Ferraro, Nature 126, 129 (1930); S. Chapman, Revs. Modern Phys. 9, 44 (1937).

[3]M. Rosenbluth and C. L. Longmire, Ann. Phys. 1, 120 (1957).

[4]F. H. Coensgen and F. C. Ford (private communications).

[5]Pointed out by T. Northrop (private communication)

[6]J. L. Tuck, Washington Report No. 184, 1954 (unpublished), p.77; J. L. Tuck, Washington Report No. 289, 1955 (unpublished), p.7; C. L. Longmire, Washington Report No. 289, 1955 (unpublished), p.11; H. Grad, Washington Report No. 289, 1955 (unpublished), p.115.

[7]The picket fence geometry, long known to have great hydromagnetic stability, has received little experimental study mainly because of its obvious disadvantage with respect to the other main geometries, pinch, Stellarator, mirror machine, of leakier confinement. The prediction of strong cyclotron radiation from a hot plasma containing a magnetic field by B. A. Trubnikov and V. S. Kudryavtsev [Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958 (United Nations, Geneva, 1959), Vol. 31, p. 93], may make it of primary importance to avoid this radiation by using plasmas containing zero or little magnetic field. Only the picket fence has enough hydromagnetic stability to confine such a plasma. The other devices are theoretically limited to β (= plasma pressure/magnetic field pressure) less than about 0.1.

[8]J. Marshall (to be published).