structure of the photon Green's function will be considered elsewhere.

[1]An additional factor involving the line integral of the vector potential should be understood. It is needed to maintain gauge invariance and accounts for the nonvanishing commutator with the transverse electric field.

K^+ - K^0 MASS DIFFERENCE

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It has recently been established[1] that, in contrast to the π mesons, the K^0 is heavier than the $K^{\pm}$. This is in contradiction with perturbation theory,[2] and with the naive classical argument that the charged components of a mass multiplet should be heavier in proportion to the work done in assembling the charge. We wish to draw attention to the important difference that, whereas the π^0 has a vanishing charge density, the K^0, like the neutron, is a concentration of positive and negative charge,[3] giving total charge zero, but with a nonvanishing charge density. It is quite simple to construct physically reasonable charge distributions for K^0 and K^+ such that the neutral particle has the greater electrostatic energy. We produce such a model and show further that this classical interpretation of the self-energy is very closely related to the leading term in the field-theoretic calculation.

The familiar relation[4] between electric charge, isotopic spin, I, and hypercharge, Y, can be expressed in terms of the corresponding current densities, in an obvious notation, as

$$j_\mu{}^{el}(x) = j_\mu{}^{I_3}(x) + \tfrac{1}{2} j_\mu{}^{Y}(x). \tag{1}$$

The static charge distribution for a particular multiplet, χ, is the expectation value of the "zero" component of this operator for single-particle states in the rest system:

$$\begin{aligned}\rho(x) &= \langle \chi | j_0{}^{el}(x) | \chi \rangle \\ &= I_3 \rho_I(x) + \tfrac{1}{2} Y \rho_Y(x),\end{aligned} \tag{2}$$

where I_3 is the matrix appropriate to the multiplet and Y is the single-particle value.

For π^0 it follows that

$$\rho(x) = 0. \tag{3}$$

The neutral pion has a vanishing charge density, and the fact that the charged pions are heavier is consistent with the simple classical argument.[5]

For the case of the K meson ($Y=1$) the situation is quite different. Rewriting Eq. (2),

$$\rho_K(x) = \tfrac{1}{2}(1+\tau_3)\rho_1(x) + \rho_2(x), \tag{4}$$

it is seen that ρ_2 describes the charge density of the neutral K meson. We argue that by choosing a suitable charge distribution, say a negative "cloud" around a sufficiently small positive "core", one can easily give the neutral K meson a greater self-mass than the charged one. Choose, for example,

$$\rho_+(x) = (e^2/4\pi)\lambda^2 e^{-\lambda r}/r = \rho_1 + \rho_2, \tag{5a}$$

$$\rho_0(x) = (e^2/4\pi)[\lambda^2 e^{-\lambda r} - \mu^2 e^{-\mu r}]/r = \rho_2. \tag{5b}$$

Recalling that the classical self-mass is given (for the charged particle for example) by

$$\delta M_+ = \frac{e^2}{(2\pi)^2} \int d\vec{q}\, \frac{\lambda^2}{\lambda^2 + \vec{q}^2} \frac{1}{k^2} \frac{\lambda^2}{\lambda^2 + \vec{q}^2}, \tag{6}$$

it is found that the observed mass splitting is reproduced by taking λ to be a pion Compton wavelength and μ, the "core" radius, a little greater than two Σ-hyperon Compton wavelengths.

In the field-theoretic calculation, it is reasonable to suppose that the main contribution comes from intermediate states of the least possible energy, and this is borne out by the dispersion relation approach.[6] We need to calculate the contribution from the graph of Fig. 1. The cor-

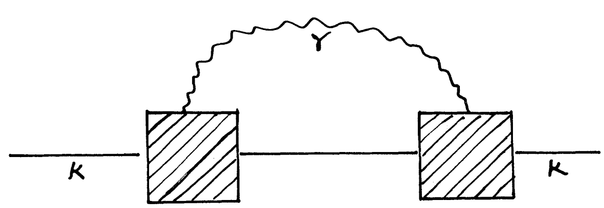

FIG. 1. Feynman diagram of the K-meson electromagnetic self-energy.

responding expression is ($q^2 = q^\mu q_\mu$, $p^2 = -M^2$)

$$\delta(M^2) = \frac{2e^2}{(2\pi)^3}\int d^4q \Gamma_\mu(p, p-q)\Delta_F'(p-q) \times \Gamma_\mu(p-q, p)\frac{1}{q^2}. \tag{7}$$

We make the choice (which is discussed further below)

$$\Gamma_\mu^{+}(p, p') = (p_\mu + p_\mu')\frac{\lambda^2}{\lambda^2 + (p-p')^2}, \tag{8a}$$

$$\Gamma_\mu^{0}(p, p') = [(p-p')^2(p_\mu + p_\mu') - (p_\mu - p_\mu')(p^2 - p'^2)] \times\left[\frac{\lambda^2}{\lambda^2 + (p-p')^2} - \frac{\mu^2}{\mu^2 + (p-p')^2}\right]\frac{1}{(p-p')^2}. \tag{8b}$$

These are the covariant quantum mechanical equivalents of the charge distributions (5a) and (5b). It will be shown below that it is consistent with the spirit of the proceedings to approximate the exact Feynman propagator by

$$\Delta_F'(p) = (p^2 + M^2 - i\epsilon)^{-1}. \tag{9}$$

The integral (7) is readily evaluated by standard techniques. Choosing the inverse "cloud" radius, λ, equal to m_π, as before, we obtain the following results:

$$\mu/\lambda = 8 \quad , 9$$

$$M_0 - M_+ = 2.7,\ 4.5 \text{ (Mev)}.$$

Recalling that the observed splitting[1] is about 3.5 Mev we see that the "core" radius corresponds to about one baryon Compton wavelength and that the classical and quantum mechanical estimates differ by a factor of about two.

We have stated that the two self-mass calculations are very closely related to each other. In fact the integral of Eq. (6) comes naturally out of the expression (7) in the limit that the K-meson mass, M, becomes very large. To see this we write Eq. (7) in the form comparable to Eq. (6),

$$\delta M_+ = \frac{e^2}{(2\pi)^3}\frac{1}{M}\int d^4q \frac{\lambda^2}{\lambda^2 + q^2}\frac{1}{q^2} \times\frac{q^2 - 4p\cdot q - 4M^2}{(p-q)^2 + M^2}\frac{\lambda^2}{\lambda^2 + q^2}, \tag{10}$$

and perform the q_0 integration by closing the contours in the upper half-plane. Consider only the pole at

$$(p-q)^2 = -M^2.$$

Evaluating the residue and keeping only the leading term in $1/M$, we are led back to the expression (6). This is not a surprising result. It says, merely, that the classical self-mass is the static limit of that part of the quantum mechanical expression where the (intermediate) K meson is restricted to the mass shell.

It remains now to justify our choice of vertex function and our use of the bare propagator, Δ_F. We construct the most general possible vertex from the two vectors,

$$A_\mu = (p_\mu + p_\mu')q^2 - q_\mu(p^2 - p'^2),$$

$$B_\mu = q_\mu/q^2 + (p_\mu + p_\mu')/(p^2 - p'^2). \tag{11}$$

Gauge invariance is identically satisfied by writing

$$\Gamma_\mu(p, p') = \tfrac{1}{2}(1+\tau_3)\{B_\mu[\overline{\Delta_F'^{-1}(p^2) - \Delta_F'^{-1}(p'^2)}] + A_\mu F\} + A_\mu G, \tag{12}$$

where F and G are functions of the three scalars

$$p^2 + M^2,\ p'^2 + M^2,\ q^2 = (p-p')^2.$$

In the absence of any knowledge of the functions $\Delta_F'(p^2)$ and F, it is convenient to rewrite Γ_μ as

$$\Gamma_\mu(p, p') = \tfrac{1}{2}(1+\tau_3)\{(p_\mu + p_\mu')F_1 + q_\mu F_2\} + A_\mu G. \tag{13}$$

It is clear that $G(0, 0, q^2)$ is just the charge structure of the K^0 meson as would be seen, say, in an electron scattering experiment. Similarly $F_1(0, 0, q^2)$ describes the charge and charge structure of the charged K meson. On the other hand $F_2(0, 0, q^2)$ must vanish as the interaction of a K^+ with an external field would not be gauge invariant. We now introduce the supposition that there is little probability that the meson can emit very energetic photons. That is to say, we suppose that the intermediate line in Fig. 1 is almost always near the physical meson mass. In this approximation we take F_1 and G as functions of q^2 only, and set F_2 equal to zero. This assumption also justifies our use of the "bare" propagator in the integral of Eq. (7).

The foregoing argument will not conceal from the astute reader the fact that we have attempted to construct the simplest possible field-theoretic model with a minimum number of arbitrary parameters and a ready classical interpretation. This we have done and shown that it leads to physically sensible results.

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*National Science Foundation post-doctoral fellow.

[1]Rosenfeld, Solmitz, Tripp, Crawford, and Cresti, Phys. Rev. Letters 2, 110 (1959); Crawford, Cresti, Good, Stevenson, and Ticho, Phys. Rev. Letters 2, 112 (1959).

[2]S. Gasiorowicz and A. Petermann, Phys. Rev. Letters 1, 457 (1958). The basis of our calculation is rather different in spirit from that of this reference. These authors performed lowest-order perturbation calculations with a cutoff introduced on a purely ad hoc basis, and made some estimates of the effect of higher-order corrections. We construct what seems to us a very plausible classical model and assume that the field-theoretic calculation will be dominated by the terms which correspond most closely to this classical model.

[3]This was apparently first observed by T. Nakano (unpublished). We are indebted to Professor Y. Yamaguchi for informing us of this work. See also G. Feinberg, Phys. Rev. 109, 1381 (1958).

[4]M. Gell-Mann, Phys. Rev. 92, 833 (1953); T. Nakano and K. Nishijima, Progr. Theoret. Phys. (Kyoto) 10, 581 (1953); J. Prentki and B. d'Espagnat, Phys. Rev. 99, 328 (1955).

[5]Actually the two-pion, one-photon vertex part vanishes identically, on or off the mass shell, as can be seen immediately from invariance under particle antiparticle conjugation. Quantum mechanically the neutral pion does have an electromagnetic self-energy, but this is either of higher order in e^2 or involves rather massive intermediate states.

[6]Riazzudin (to be published).

ERRATUM

OSCILLATORY MAGNETO-ACOUSTIC EFFECT IN METALS. T. Kjeldaas, Jr., and T. Holstein [Phys. Rev. Letters 2, 340 (1959)].

In Eq. (6) the expression in brackets should read

$$\left[\frac{\sigma_0\sigma_{yy}}{\sigma_{xx}\sigma_{yy}+\sigma_{xy}^2}-1\right].$$

On page 340, the first line of the right-hand column should read, "the numbers shown as ordinates by $mM^{-1}\tau^{-1}C_s^{-1}$."