momentum 85 ± 5 Mev/c, a proton of range 3.25 mm corresponding to a momentum of 103 ± 2 Mev/c, and a one-bubble stub. We interpret this event as the decay scheme

$$_{\Lambda}\mathrm{He}^4 \to \pi^- + p + \mathrm{He}^3.$$

Decay kinematics require the He^3 to have a range of 0.24 mm, approximately one bubble diameter. The recoil stub has been observed in the correct hemisphere for momentum conservation. The binding energy obtained is 3.5 ± 3.4 Mev and is compatible with the known binding energy[7] of the $_{\Lambda}\mathrm{He}^4$.

Alternate interpretations of event 83-1404 have been considered and rejected. Firstly, charge and baryon conservation at the K-interaction vertex preclude the possibility that the stub is a $_{\Lambda}\mathrm{H}^4$. Further, the event cannot be interpreted as a stopping $_{\Lambda}\mathrm{H}^3$, since this would require another proton of 3.1-mm range at the K vertex. It might be argued that perhaps the $_{\Lambda}\mathrm{H}^3$ decays in flight and that a very low energy proton is present at the K origin, but not visible. Such an interpretation, however, can be ruled out by further kinematical analysis which requires that the $_{\Lambda}\mathrm{H}^3$ be unbound by at least 10 ± 3.5 Mev.

In conclusion, our events are sufficiently unambiguous to establish the existence of reaction (1). This requires that the relative K^- - Λ parity be negative. We wish to stress that the validity of this conclusion depends on the assumptions (a), (b), and (c).

We would like to thank Dr. W. Fairbank and Dr. H. Blumenfeld for their invaluable aid and cooperation in the construction of the chamber, Dr. J. Murray and Dr. N. Horwitz for their help in using the coaxial separator, Dr. E. Lofgren, H. Heard, and the Bevatron crew, Dr. Alvarez, Dr. Powell, and the many other groups at the Berkeley Bevatron for their assistance during this experiment.

---

*Supported in part by the joint U. S. Atomic Energy Commission-Office of Naval Research program and the Office of Scientific Research.

†John Simon Guggenheim Memorial Fellow 1958-59, on leave of absence at University of Bologna, Italy.

‡Now at Department of Natural Philosophy, The University, Glasgow, Scotland.

‖On leave of absence from University College, Dublin, Ireland.

[1] Horwitz, Murray, Ross, and Tripp, University of California Radiation Laboratory Report UCRL-8269, 1958 (unpublished).

[2] R. H. Dalitz, Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics (Interscience Publishers, New York, 1956).

[3] M. Gell-Mann, Phys. Rev. 106, 1296 (1957).

[4] R. H. Dalitz and B. W. Downs, Phys. Rev. 111, 967 (1958). However, R. H. Dalitz, at the Kiev Conference, 1959 (unpublished), has pointed out that evidence for a spin-zero ground state is now not as conclusive, and the assignment of spin zero rests mainly on the Karplus-Ruderman argument.

[5] R. H. Dalitz, Kiev Conference, 1959 (unpublished); on the basis of new calculations, he concludes that it is very unlikely that the mass-4 hyperfragment has a bound spin-one excited state.

[6] As the slowing-down time of the hyperfragment is of the order of 14% of the lifetime of the free Λ, some decays may occur in flight and the stubs will exhibit a length less than the full range.

[7] Ammar, Levi-Setti, Slater, Limentani, Schlein, and Steinberg, Nuovo cimento (to be published).

LEPTONIC DECAY MODES OF THE K MESON*

K. Chadan and S. Oneda†
University of Maryland, College Park, Maryland
(Received August 26, 1959)

We wish to explore how far the so-called V-A Fermi interaction could account for the strangeness-nonconserving weak reactions. It has been noticed that the observed rates of the decays, $K^0 \to \pi^\pm + e^\mp + \nu$ and $K^+ \to \pi^0 + e^+ + \nu$, seem to be anomalously small compared with the universal rates.[1] Using a dispersion technique,[2] it has also been conjectured that the rate of $K \to \mu + \nu$ decay is not in contradiction with the experimentally indicated slow rates of hyperon decays into leptons.[3] It may be interesting and instructive to see how the other possible leptonic modes $K \to 2\pi + e + \nu$ appear. Possible modes will be (a) $K^+ \to \pi^+ + \pi^- + e^+ + \nu$, (b) $K^+ \to \pi^0 + \pi^0 + e^+ + \nu$, (c) $K^+ \to \pi^+ + \pi^+ + e^- + \bar{\nu}$, (d) $K_1^0 \to \pi^\pm + \pi^0 + e^\mp + \nu$, and (e) $K_2^0 \to \pi^\pm + \pi^0 + e^\mp + \nu$. We write the relevant Fermi interactions with the coupling constant f as follows: $(f/\sqrt{2}) J_\alpha^{(1)} j_\alpha + \mathrm{H.c.}$, $j_\alpha = \bar{e}\gamma_\alpha(1+\gamma_5)\nu + \bar{\mu}\gamma_\alpha(1+\gamma_5)\nu$. $J_\alpha^{(1)}$ denotes the strangeness-nonconserving baryonic currents.

If $J_\alpha^{(1)}$ is assumed to satisfy $\Delta S/\Delta Q = 1$,[4] where ΔQ and ΔS denote the change of charge and strangeness, respectively, of the current $J_\alpha^{(1)}$, the mode (c) is forbidden and the rates of the modes (d) and (e) are equal. This is also true when the $J_\alpha^{(1)}$ satisfies the $|\Delta I| = 1/2$ rule,[5] where $\Delta S/\Delta Q = 1$ is automatically guaranteed.

We could write the amplitudes of the various leptonic modes of K meson as follows, neglecting the electromagnetic corrections and the electron mass ($m_e = 0$):

$$K^+ \to \mu^+ + \nu, \qquad (f/\sqrt{2})\bar{\mu}\gamma_\alpha(1+\gamma_5)\nu k_\alpha L; \tag{1}$$

$$K^+ \to \pi^0 + e^+ + \nu, \qquad G(f/\sqrt{2})\bar{e}\gamma_\alpha(1+\gamma_5)\nu k_\alpha M; \tag{2}$$

$$K^+ \to \pi^0 + \pi^0 + e^+ + \nu, \quad (G^2/\sqrt{2})(f/\sqrt{2})\bar{e}\gamma_\alpha(1+\gamma_5)\nu k_\alpha N. \tag{3}$$

k_α denotes the four-momentum of the K^+ meson. We have extracted the pion coupling G which is responsible for the emission of pions in the final states. L, M, and N are the form factors and their dimensions are different from each other. Since these decays are assumed to proceed through the baryon and antibaryon loops, the characteristic length of the range of interactions would be of the order of the baryon Compton wavelength. (We take $\hbar = c = 1$ and denote this length as $\mathfrak{M}^{-1}$.) Then the dependence of the form factors, M and N, on the secondary pion energies will not be so important.[6] Assuming this, we need only one form factor, N, for the process (3) because of the symmetry with regard to the two final pions. The relations between these form factors will be approximately given by $M\mathfrak{M} \approx L$ and $N\mathfrak{M} \approx M$. Then the branching ratio of the process (2) to (1) will be

$$\frac{W(\pi^0 + e^+ + \nu)}{W(\mu^+ + \nu)} \approx \left(\frac{G^2}{4\pi}\right)\left(\frac{m_p}{\mathfrak{M}}\right)^2 \times 1.2\times 10^{-2}$$

$$\approx 1.6\times 10^{-1}\left(\frac{m_p}{\mathfrak{M}}\right)^2. \tag{4}$$

Taking $(G^2/4\pi) \approx 13.5$ and putting $\mathfrak{M} \approx$ baryon mass (m_p is the proton mass), the above ratio is close to the observed value (≈ 0.1). For the ratio in question we have found [using Eq. (7)]

$$\frac{W(\pi^0 + \pi^0 + e^+ + \nu)}{W(\pi^0 + e^+ + \nu)} \approx \left(\frac{G^2}{4\pi}\right)\left(\frac{m_p}{\mathfrak{M}}\right)^2 \times 7.4\times 10^{-5}$$

$$\approx 1.0\times 10^{-3}\left(\frac{m_p}{\mathfrak{M}}\right)^2, \tag{5}$$

which is much reduced compared with the surprisingly large branching ratio (4). The above order-of-magnitude estimate has been confirmed by two different methods. First let us compare directly the various rates in question. For the ratio of the rates the use of perturbation theory would be less controversial and also the unknown K-meson-baryon coupling will not enter. For the $J_\alpha^{(1)}$ current we assume the simplest form[7] $\bar{p}\gamma_\alpha \times (1+\gamma_5)\Lambda^0$ which satisfies $|\Delta I| = 1/2$. The form of the amplitude[6] for the $K^+ \to \pi^+(p_1) + \pi^-(p_2) + e^+(p_3) + \nu(p_4)$ decay is given by

$$(\sqrt{2}\,G)^2 (f/\sqrt{2})\,\bar{e}\gamma_\alpha(1+\gamma_5)\nu\,(p_{1\alpha}N_1 + p_{2\alpha}N_2) + \text{H. c.}, \tag{6}$$

which needs two form factors N_1 and N_2. Then the energy spectrum of the positron with energy p_3 (in the K-meson rest system) will be, except for the irrelevant constant factors,

$$\int_0^{(m_K^2-4m_\pi^2)/2m_K} p_3^2\,dp_3 \int_0^{(E^2-4m_\pi^2)/2E} q_4^2\,dq_4 \left(\frac{m_K}{m_K-2p_3}\right)^{1/2}\left(\frac{E^2-2Eq_4-4m_\pi^2}{E^2-2Eq_4}\right)^{1/2}\Big\{(E^2-Eq_4)(N_1+N_2)^2$$

$$-2m_\pi^2(N_1^2+N_2^2) - (E^2-2Eq_4)N_1N_2 + (E^2-2Eq_4-4m_\pi^2)\left[\frac{q_4}{3(E-2q_4)}(N_1-N_2)^2 - N_1N_2\right]\Big\}, \tag{7}$$

where $E = [m_K(m_K-2p_3)]^{1/2}$ and q_4 is the neutrino energy in the center-of-mass system of the π^+, π^-, and the neutrino. Now for the $K \to \mu + \nu$ and $K \to \pi + e + \nu$ the results are logarithmically divergent and we must use a cutoff.[8] However, the amplitudes for the $K \to 2\pi + e + \nu$ are convergent. We get the following crude estimates for the pseudoscalar

K meson:

$$W(\mu^+ + \nu):\ W(\pi^0 + e^+ + \nu):\ W(\pi^+ + \pi^- + e^+ + \nu):\ W(\pi^0 + \pi^0 + e^+ + \nu):\ W(K_{1,2}{}^0 \to \pi^\pm + \pi^0 + e^\mp + \nu)$$
$$= 1:\ 1.7\times10^{-1}:\ 8.3\times10^{-4}:\ 3.5\times10^{-4}:\ 2.9\times10^{-4}; \tag{8}$$

and for the scalar K meson:

$$W(\mu^+ + \nu):\ W(\pi^0 + e^+ + \nu):\ W(\pi^+ + \pi^- + e^+ + \nu):\ W(\pi^0 + \pi^0 + e^+ + \nu):\ W(K_{1,2}{}^0 \to \pi^\pm + \pi^0 + e^\mp + \nu)$$
$$= 1:\ 1.0\times10^{-1}:\ 9.0\times10^{-4}:\ 1.7\times10^{-4}:\ 1.2\times10^{-3}. \tag{9}$$

It is seen that the previous estimates (4) and (5) are reproduced. For the same K-meson–baryon coupling constant, the rate of $K \to \mu + \nu$ decay for the scalar K meson is about an order of magnitude smaller than that for the pseudoscalar K meson which is also found in the dispersion theory calculation.[2] However, the same tendency also holds for the other modes and so the above branching ratios are almost independent of the type of K meson. There is also an indication that the result is not so sensitive to the choice of the interactions adopted.[9] Next we shall try a different method. Let us compare the $K^+ \to \pi^+ + \pi^- + e^+ + \nu$ with the $K^+ \to \pi^+ + \pi^- + \pi^+$ decay. We could get[10] the latter process by just replacing the weak vertex (e, ν) in the former process by the weak vertex $\bar{\Lambda}^0 + p \to \pi^+$. We may use the effective weak vertex $g\bar{\Lambda}\gamma_\alpha(1+\gamma_5)p\,\partial_\alpha\varphi_\pi$ $(g^2/4\pi \approx 1.8\times10^{-15})$ which may come from the Fermi interaction $(f/\sqrt{2})J_\alpha{}^{(1)}\bar{p}\gamma_\alpha \times(1+\gamma_5)n$. Then we find for the branching ratio values of around a few percent (these are crude, and also the final-state interactions are neglected):

$$\frac{W(\pi^+ + \pi^- + e^+ + \nu)}{W(\pi^+ + \pi^- + \pi^+)} \approx \begin{cases} 4\times10^{-2} & \text{for } K(\text{pseudoscalar}) \\ 3\times10^{-2} & \text{for } K(\text{scalar}). \end{cases}$$

However, if we accept the fact[3] that the effective Fermi coupling constant for the hyperon decays into leptons is about an order of magnitude smaller than the usual Fermi coupling constant, then the above branching ratio should be reduced by a factor ≈ 10 and is in reasonably good agreement with the values obtained in (8) and (9), since experimentally we have found that $W(\pi^+ + \pi^- + \pi^+) \approx 1.2W(\pi^0 + e^+ + \nu)$.

Thus we would like to conclude that the frequencies of the mode $K^+ \to \pi^+ + \pi^- + e^+ + \nu$ would be approximately in the range between 10^{-2} and 10^{-3}% compared with the $K^+ \to \pi^0 + e^+ + \nu$ or $K^+ \to \pi^+ + \pi^- + \pi^+$ decay.

The positron (or neutrino) energy spectra are shown in Fig. 1 for the case $K^+ \to \pi^+ + \pi^- + e^+ + \nu$. The spectra are very similar for both types of K meson. This is because the spectrum is not sensitive to the value of N_1/N_2. As in the case with the $K^+ \to \pi^0 + \pi^0 + e^+ + \nu$ decay (where $N_1 = N_2 \approx$ constant), the $K^0 \to \pi^\pm + \pi^0 + e^\mp + \nu$ decay has also only one form factor ($N_1 = -N_2 \approx$ constant) since we have assumed the $|\Delta I| = 1/2$ rule for $J_\alpha{}^{(1)}$. So for these cases the spectra are unambiguous (see Fig. 2) and turn out to be almost the same as given in Fig. 1.

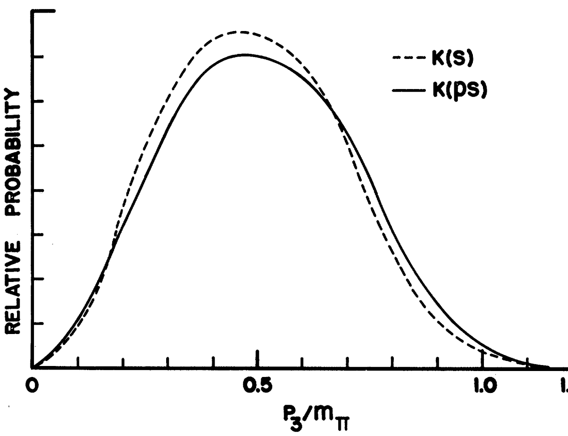

FIG. 1. The energy spectrum of the positron (or the neutrino) in the $K^+ \to \pi^+ + \pi^- + e^+ + \nu$ decay for the pseudoscalar and scalar K meson.

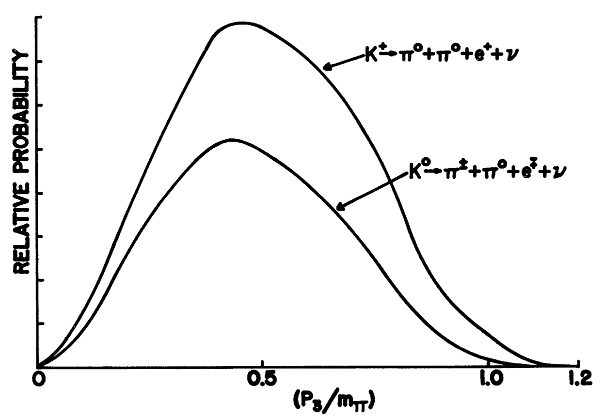

FIG. 2. The energy spectrum of the electron or positron in the decays, $K^+ \to \pi^0 + \pi^0 + e^+ + \nu$ and $K_{1,2}{}^0 \to \pi^\pm + \pi^0 + e^\mp + \nu$.

That is, all the spectra have a broad maximum around 70 Mev.

In the above discussion, no selection rules have been assumed for all the modes.[11] Experimentally there are already about a thousand events of the $K \to 3\pi$ decays. So far three noncoplanar events which look like anomalous $K^+ \to \pi^+ + \pi^- + \pi^+$ decays have been reported and interpreted as the $K^+ \to \pi^+ + \pi^- + \pi^+ + \gamma$ decay.[12,13] Dalitz has shown[14] that the ratio of the $K^+ \to \pi^+ + \pi^- + \pi^+ + \gamma$ to the $K^+ \to \pi^+ + \pi^- + \pi^+$ decay would be about 1.2×10^{-3} for emission of a photon whose energy is greater than 10 Mev. According to our estimate the frequency of the $K^+ \to \pi^+ + \pi^- + e^+ + \nu$ decay seems to be slightly larger than the $K^+ \to \pi^+ + \pi^- + \pi^+ + \gamma$ decay. As the decay $K^+ \to \pi^+ + \pi^- + e^+ + \nu$ is not so difficult a process to detect, experimental clarification may be possible in the future. Although the rates turn out to be not so large, they are still more than an order of magnitude larger than the predicted rates of the $K^+ \to e^+ + \nu$ decay which is an important process to prove the success of the V-A Fermi interaction.

---

*This research was supported in part by the U. S. Air Force through the Air Force Office of Scientific Research of the Air Research and Development Command.

†On leave of absence from the Institute of Theoretical Physics, Kanazawa University, Japan.

[1]S. Oneda, Nuclear Phys. 9, 476 (1958/59).

[2]B. Sakita, Phys. Rev. (to be published); C. H. Albright, Phys. Rev. (to be published).

[3]F. S. Crawford et al., Phys. Rev. Letters 1, 377 (1958); P. Nordin et al., Phys. Rev. Letters 1, 380 (1958); J. Leitner et al., Bull. Am. Phys. Soc. 4, 356 (1959).

[4]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958). The possible existence of this selection rule was once noticed by one of us from a slightly different standpoint. S. Oneda, Nuclear Phys. 4, 21 (1957).

[5]I. Yu. Kobzarev and L. B. Okun, Proceedings of the Padua-Venice Conference on Mesons and Recently Discovered Particles, 1957 (unpublished); S. Okubo et al., Phys. Rev. 112, 665 (1958).

[6]In the following calculations we neglect terms of the order $Q^2/\mathfrak{M}^2$ (Q is the available energy of the decays) which amounts to (at most) less than 20 %.

[7]This will be reasonable especially if the Λ is a more fundamental particle than other hyperons. See, for instance, S. Sakata, Prog. Theoret. Phys. (Kyoto) 16, 686 (1956).

[8]We have made use of the results of S. Oneda and Y. Tanikawa, Phys. Rev. 113, 1354 (1959), where an intermediate boson with mass $\approx 2300 m_e$ plays a role of relativistic cutoff. The result is not sensitive to the choice of the cutoff.

[9]Using the form of Fermi interaction proposed by Feynman and Gell-Mann[4] and assuming a global symmetric model of strong interactions, quite similar results were also obtained for the ratio (4). Lee, Shih, Ho, and Chu, Chinese Academy of Atomic Research, Phys. J. 15, 33 (1959).

[10]This seems to be the most important diagram for this decay.

[11]If we insist on the universality, we might introduce weak selection rules for the $K \to \mu + \nu$ and $K \to \pi + e + \nu$ decays. See, for instance, R. P. Feynman, Bull. Am. Phys. Soc. 4, 84 (1959). In this case, however, the branching ratios of $K \to 2\pi + e + \nu$ would be enhanced by a factor ≈ 10 unless we also forbid these processes. However, the $K \to 2\pi + e + \nu$ modes are allowed in the above model of Feynman.

[12]R. R. Daniel and Y. Pal, Proc. Indian Acad. Sci. A40, 114 (1954). Introductory summary by L. Leprince-Ringuet, Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics, 1956 (Interscience Publishers, New York, 1956).

[13]τ-meson-like phenomena, in which two charged pions and a μ meson appear, have also been reported. K. R. Dixit, Z. Naturforsch. 90, 335 (1954). In this note we have not discussed the μ-meson modes. The μ-meson modes will be as frequent as the electron modes. The phase space volume favors the latter, but the former modes have one more form factor since the muon mass cannot be neglected.

[14]R. H. Dalitz, Phys. Rev. 99, 915 (1955).