of the relative intensity of the inner high-field line and the outer low-field lines labels them as the $-3/2 \rightarrow -5/2$ and $-1/2 \rightarrow -3/2$ transitions, respectively, and establishes the positive sign of a for Fe^{3+} in octahedral water coordination.

Further details including the relation of the crystal field splitting parameters to ferrimagnetic anisotropy as well as a more detailed summary of splitting of S-state ions will appear in forthcoming publications.

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CANTED SPIN ARRANGEMENTS*

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The aim of this Letter is to discuss two spin systems where canted spin arrangements are very likely to occur. These are (1) the ordered iron aluminum alloys[1]; (2) the tin-substituted yttrium iron garnets.[2]

(1) Iron aluminum alloys.—A very attractive model for exchange interactions in this system has been proposed by Arrott and Sato.[1] In the "ordered" alloys there are three sublattices. One of them, A, is only partially filled up with iron, and has the saturation magnetization $M_A = I\frac{1}{2}(1-2c)$ where I is the saturation magnetization of pure iron, and $c(<\frac{1}{2})$ the aluminum percentage. The other two sublattices B' and B'' are completely filled with iron and have magnetizations $M_{B'} = M_{B''} = \frac{1}{4}I$. The assumed spatial arrangement is made of planes $B'AB''AB'\ldots$ normal to a (100) direction of the bcc structure. The $B'B''$ coupling takes place through Al atoms and is antiferromagnetic. The AB' and AB'' couplings are taken as ferromagnetic. The resultant molecular fields may be written, in the notation of Yafet and Kittel,[3]

$$H_A = -n(M_{B'} + M_{B''}),$$
$$H_{B'} = -n(M_A - \gamma_2 M_{B''}),$$
$$H_{B''} = -n(M_A - \gamma_2 M_{B'}); \qquad (1)$$

n and γ_2 are negative here. $|\gamma_2| = 4c/\mu$ where μ is the absolute value of the ratio between ferromagnetic exchange integrals as defined in reference 1. A discussion similar to that of reference 3 then gives the following results:

	Ground state	Upper transition point
$0<c<c'$	Ferro	Ferro
$c'<c<c''$	Triangular	Ferro
$c''<c<\frac{1}{2}$	Triangular	Antiferro

c' and c'' are defined by the equations

$$\mu(1-2c') - 4c' = 0, \qquad (2)$$
$$\mu^2(1-2c'') - 8(c'')^2 = 0. \qquad (3)$$

Typical values are quoted below:

	$\mu = 2.00$	$\mu = 1.60$	Exp.[1]
c'	0.25	0.22	0.28
c''	0.36	0.33	0.33

The ground state is never antiferromagnetic (the different, incorrect result of reference 1 stems from the use of the Ising model). The spontaneous magnetization at $T = 0$ is

$$M_0 = I(1-c), \qquad c < c'$$
$$M_0 = \tfrac{1}{2}I(1-2c)(1+\mu/4c), \qquad c > c'. \qquad (4)$$

The M_0 vs c curve displays a typical change in slope which is indeed observed (Fig. 1). The low-temperature susceptibility, due to changes

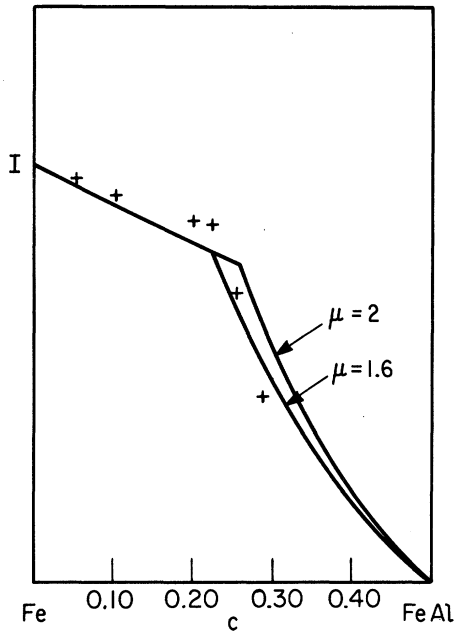

FIG. 1. Spontaneous moments in ordered iron aluminum alloys at $T=0$. Experimental points by Arrott and Sato.[1] Theoretical curves for two typical values of the exchange integral ratio μ.

of the angle between sublattice magnetizations, has the value $1/n\gamma_2$. (Detailed experimental data on this quantity are not yet available.)

(2) Sn-substituted garnets.—Pure YIG has two iron sublattices, A (octahedral) and B (tetrahedral), for which the molecular field equations may be written as[4]

$$H_A = -7000M_A - 15\,800M_B,$$
$$H_B = -15\,800M_A - 4200M_B. \qquad (5)$$

(In all equations M refers to magnetization per unit volume.) In the Sn-substituted garnets, we assume that (a) all Sn goes into the A lattice,[2] according to the symbolic formula

$$\{Y_{3-x}Ca_x\}[Sn_xFe_{2-x}](Fe_3)O_{12},$$

and (b) all exchange interactions have the values corresponding to YIG.

A natural way of further dividing A and B into sublattices is the following: The A sites form a bcc lattice and may be split into two simple cubic lattices $A'A''$. The B sites form a more complicated structure but may still be divided into two lattices $B'B''$ such that a B' site, for instance, has four nearest neighbors, all of them B''. The crystallographic positions of the B' sites in such an arrangement are the following: (3/8, 0, 1/4) (1/4, 3/8, 0)(0, 1/4, 3/8)(1/8, 0, 3/4)(3/4, 1/8,0) (0, 3/4, 1/8) plus six other positions deduced from the preceding ones by the translation (1/2, 1/2, 1/2). The molecular fields are taken to be

$$H_{A'} = n(\alpha_2 M_{A''} - M_{B'} - M_{B''}),$$
$$H_{A''} = n(\alpha_2 M_{A'} - M_{B'} - M_{B''}),$$
$$H_{B'} = n(-M_{A'} - M_{A''} + \gamma_2 M_{B''}),$$
$$H_{B''} = n(-M_{A'} - M_{A''} + \gamma_2 M_{B'}), \qquad (6)$$

where from Eq. (5) we may put $n\alpha_2 = -14\,000$, $n\gamma_2 = -8400$, $n = +15\,800$, $\alpha_2 = -0.88$, $\gamma_2 = -0.53$. The parameter $y = M_A/M_B$ of reference 3 takes the value $y = \frac{1}{3}(2-x)$. As $\alpha_2 < 1$ the A lattice never splits up in the ground state. The B lattice does split up, however, and the arrangement is then

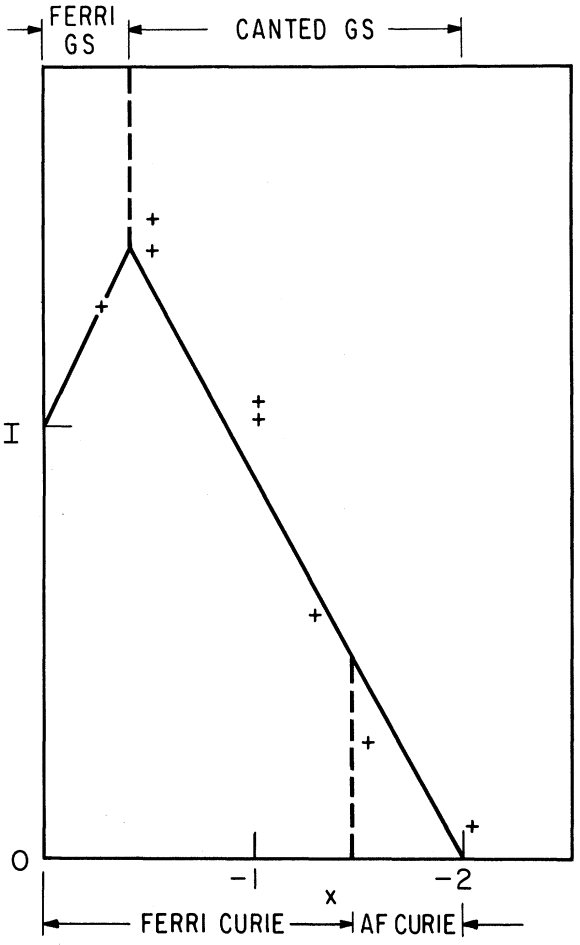

FIG. 2. Spontaneous moments in Sn-substituted YIG at $T=0$. Experimental points by Geller et al.[2]

triangular $AB'B''$, when $y < |\gamma_2|$, or $x > 0.41$. The spontaneous magnetization at $T = 0$ is

$$M_0 = I(1+x), \qquad y > |\gamma_2|$$
$$= I(2-x)(1/|\gamma_2| - 1), \quad y < |\gamma_2|, \qquad (7)$$

I being here the saturation magnetization of YIG. The resulting curve (in which there is no adjustable parameter) agrees rather well with experiment (Fig. 2). These considerations can be extended to higher temperatures. The upper Curie point corresponds to ferrimagnetic order when $y > \gamma_2^2/(2 - \alpha_2\gamma_2)$, $(x < 1.45)$, and to antiferromagnetic order at higher x.

More information on these materials could be obtained from neutron diffraction, and from high-field susceptibilities (as in the manganite series).[5]

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ANTIPHASE ANTIFERROMAGNETIC STRUCTURE OF CHROMIUM*

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In the course of a neutron diffraction study of single crystals of chromium grown by the strain-anneal method[1] it was observed that the antiferromagnetic superstructure reflections exhibited characteristic splittings. An analysis of these splittings has led to an interpretation in terms of an antiphase antiferromagnetic domain structure in analogy with antiphase domains in ordered alloys.[2]

The region around the (100) and (111) reciprocal lattice points was systematically explored by the traverses shown in Fig. 1. From the experimental observations, the intensity distribution in reciprocal space shown in Fig. 2 was deduced. This distribution can be characterized in the following manner: (a) The size of the individual

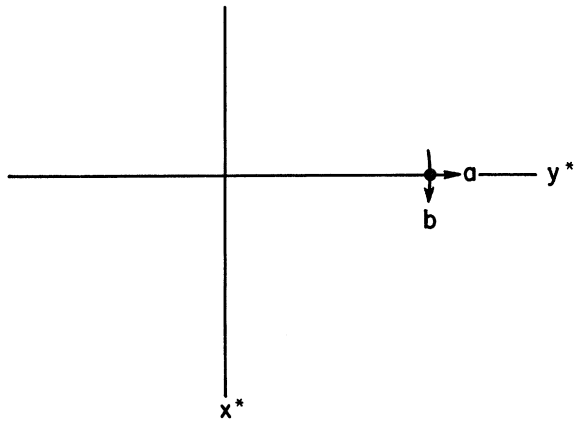

FIG. 1. Traverses of reciprocal lattice point. (a) Crystal and counter rotation in $\theta, 2\theta$ relationship, (b) Crystal rotated and counter fixed at $2\theta_{\text{Bragg}}$.

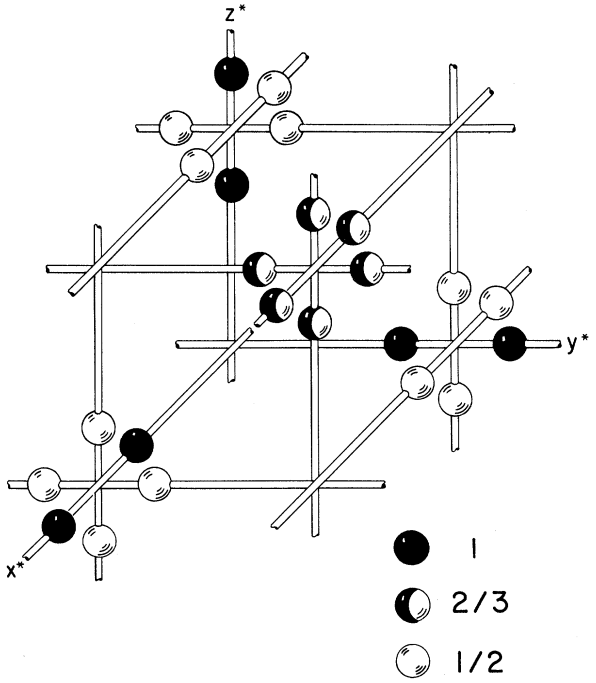

FIG. 2. Intensity distribution about the (100) and (111) reciprocal lattice points. The relative intensities are shown in the diagram.