SPECIFIC HEAT MEASUREMENTS AND THE ENERGY GAP IN SUPERCONDUCTORS*

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It has recently been pointed out by Boorse[1] that the low-temperature ($T_c/T \gtrsim 4$) specific heats of various superconductors seem to deviate from the theoretical curve calculated by Bardeen, Schrieffer, and the author.[2] We should like to suggest that the theory of superconductivity proposed in reference 2 contains a possible explanation of these deviations.

If instead of the constant electron-electron interaction used in reference 2 for the detailed calculation of electrodynamic and thermal properties, one uses the electron-electron interaction $V(\vec{k}, \vec{k}')$ which is nonconstant and anisotropic as might be expected in actual metals, one finds the results of reference 2 altered only in that the square of the energy gap $\epsilon_0^2(T)$ is everywhere replaced by $\mathfrak{F}\mathfrak{F}^*(\vec{k}, T)$, while the criterion for the existence of the superconducting state is that the integral equation

$$\mathfrak{F}(\vec{k}) = -\tfrac{1}{2}\sum_{\vec{k}'} \mathfrak{F}(\vec{k}')V_{\vec{k}\vec{k}'}\frac{\tanh[(\beta/2)E(\vec{k}',T)]}{E(\vec{k}', T)} \qquad (1)$$

have a nonzero solution for $\mathfrak{F}$. Here $\beta = 1/kT$, $E(\vec{k}',T) = (\epsilon^2 + \mathfrak{F}\mathfrak{F}^*)^{1/2}$, and $V_{\vec{k}\vec{k}'} = V_{\vec{k}'\vec{k}}$ due to arguments of time-reversal invariance.

It is seen that $\mathfrak{F}\mathfrak{F}^*$, which now plays the role of the square of the energy gap, is a function of $\vec{k}$ as well as of T; thus the energy of a single-particle excitation depends upon the direction as well as the magnitude of its momentum vector.[3] This might result in the appearance of single-particle excitations which seem [e.g., in absorption experiments] to lie below the "energy gap." If, for example, in a particular material there was a small region of the Fermi surface over which the energy gap was smaller than that over the rest of the surface, the single-particle excitations in this region might appear to lie below the "energy gap" determined by the gross behavior of the entire surface.

Further one would expect anisotropies in measurements which pick out a direction in a crystal and which are sensitive to excitations from selected portions of the Fermi surface. One such would be the absorption of ultrasound in a superconductor. Recently Morse, Olsen, and Gavenda[4] have in fact observed a variation of the rate of ultrasonic attenuation with direction in a single crystal of tin.

With regard to the low-temperature specific heat measurements, the "upward curvature" of the specific heat of aluminum pointed out by Boorse in the region $T_c/T \gtrsim 4$ would be another consequence of a variation in the energy gap over the Fermi surface. To provide a very simple illustration: if the energy gap had a value ϵ_{01} over one region of the Fermi surface and a value of ϵ_{02} over the rest of the surface, the logarithm of the electronic specific heat of the superconductor would be expected to assume roughly the following form at low temperatures:

$$\ln\left(\frac{C_{es}}{\gamma T_c}\right) \simeq \text{constant} + \tfrac{3}{2}\ln\left(\frac{T_c}{T}\right) + \ln\left[a_1 \exp\left(-\frac{\epsilon_{01}}{kT_c}\frac{T_c}{T}\right) + a_2 \exp\left(-\frac{\epsilon_{02}}{kT_c}\frac{T_c}{T}\right)\right]. \qquad (2)$$

If $a_1 \gg a_2$ and $\epsilon_{01}/kT_c > \epsilon_{02}/kT_c$, the first term in the square brackets would dominate the second for T_c/T small, while the second would dominate for T_c/T large. The logarithm of the specific heat would then change its slope in going from the region of dominance of the first term to that of the second, producing an upward curvature.

One test of these ideas could be made by a direct comparison of the variation of the energy gap in a given material as determined by the ultrasonic attenuation and by the low-temperature electronic specific heat.

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[1]H. A. Boorse, Phys. Rev. Letters 2, 391 (1959). See also for references to the measurements.

[2]Bardeen, Cooper, and Schrieffer, Phys. Rev. 108, 1175 (1957).

[3]This fact, which has been recognized by several other authors, among them P. W. Anderson, A. B. Pippard, and V. Heine, was discussed at the International Conference on the Electronic Properties of Metals at Low Temperatures, Geneva, New York, August 25-29, 1958 (unpublished). Anderson has also pointed out the relevance of a possible anisotropy of the energy gap to specific heat measurements.

[4]Morse, Olsen, and Gavenda, preceding Letter [Phys. Rev. Letters 3, 15 (1959)].