CYCLOTRON RESONANCE IN ALUMINUM*

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Cyclotron resonance has been observed in aluminum at 36 kMc/sec and 4.2°K, with effective masses in the range from about $0.1m_0$ to $0.4m_0$.

The surface resistance is measured by a calorimetric method with an experimental arrangement similar to one described previously by the author.[1] The microwave current direction in the plane surface of the fixed sample is vertical; the magnetic field can be rotated in a horizontal plane but is normally set parallel to the sample plane. The samples are of zone-refined aluminum of nominal purity approaching 99.9999% supplied by the A.I.A.G. Research Laboratories, Switzerland. After rolling and punching out disks of convenient size, recrystallization occurs at room temperature,[2] and the grain size is large enough to cover the end of the cavity with a single crystal. After annealing at about 600°C for 3 hours to complete the recrystallization and further relieve mechanical strains, the sample is electropolished.

Figures 1 and 2 show the variation of the surface resistance with magnetic field up to 4.5 kilo-oersteds. For each orientation there is evidence for cyclotron resonance of two distinct effective masses. The resonances most clearly resolved, in some cases with subharmonics up to the fifth, are indicated by arrows. These show the positions of the minima of the resistance according to the theoretical formulation of cyclotron resonance for a free electron metal given by Mattis and Dresselhaus[3]; their Eq. (27) for the surface impedance (see also Rodriguez[4]) differs slightly from the corresponding Eq. (2.4) obtained by Azbel' and Kaner[5] using a different approximation procedure.

Figure 3 shows the effect of tilting the magnetic field out of the sample plane. The minimum at about 3 kilo-oersteds broadens when the inclination ψ equals $\pm 2°$ and disappears completely at $\psi = \pm 4°$, while the minimum at about 1.5 kilo-oersteds is still well resolved at $\psi = \pm 4°$. This behavior corroborates the interpretation of the minima as being due to different groups of carriers, the one having about twice the effective mass of the other. According to the alternative interpretation that the two minima correspond to the fundamental and second subharmonic of the same group of carriers, the minimum at the lower field due to carriers describing cyclotron orbits at half the microwave frequency would evidently disappear first.

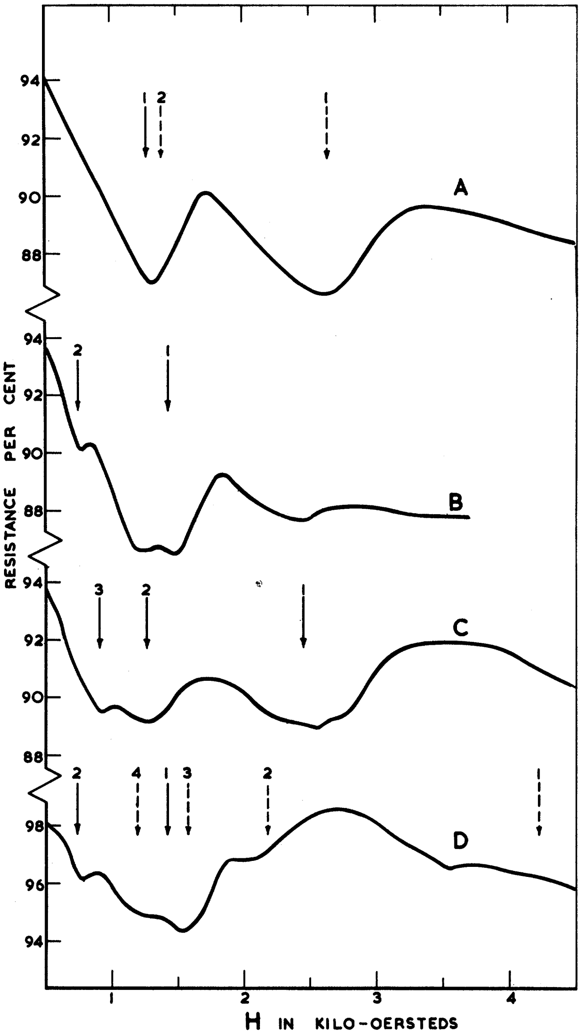

FIG. 1. Cyclotron resonance in aluminum at 36 kMc/sec and 4.2°K in magnetic fields up to 1 kilo-oersted. The ⟨110⟩ axis lies in the sample plane and the normal makes an angle θ with ⟨001⟩. The magnetic field is in the sample plane at 90° to the current direction. Arrows show positions and order of subharmonics calculated for a free electron metal with $\omega\tau = 10$ and effective mass ratios as follows:

A, $\theta = 7°$, $H \parallel \langle 110 \rangle$, $\downarrow 0.11$, $\downarrow 0.23_5$.
B, $\theta = 7°$, $H \perp \langle 110 \rangle$, $\downarrow 0.12_5$.
C, $\theta = 18°$, $H \parallel \langle 110 \rangle$, $\downarrow 0.21_5$.
D, $\theta = 18°$, $H \perp \langle 110 \rangle$, $\downarrow 0.12_5$, $\downarrow 0.37$.

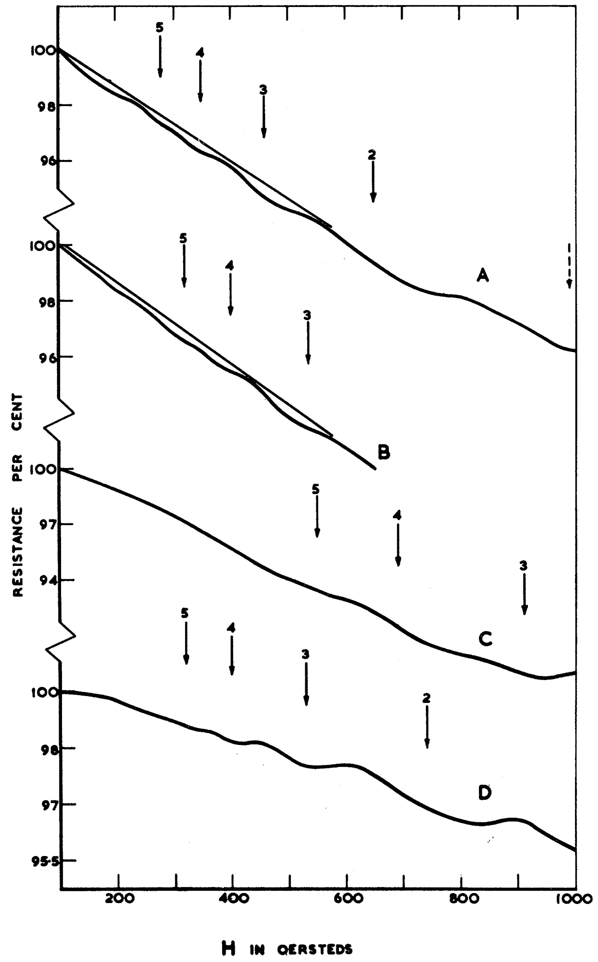

FIG. 2. Cyclotron resonance in aluminum at 36 kMc/sec and 4.2°K in magnetic fields between 0.5 and 4.5 kilo-oersteds. The orientations and effective mass ratios are as for Fig. 1.

An approximate estimate of the relative amplitudes of successive subharmonics shows them to be roughly in agreement with the theoretical values for a free-electron gas with $\omega\tau \sim 10$ (Mattis and Dresselhaus[3]). The fundamentals have amplitudes between one and two percent of the resistance in zero field. This value is somewhat higher than, but in order of magnitude agreement with, the expected contribution from the 3.6×10^{-3} hole per atom deduced by Gunnersen[6] from his measurements of the de Haas–van Alphen effect in aluminum (see also Heine[7]). The effective mass $0.125m_0$ agrees reasonably well with Gunnersen's value $0.15m_0$ for the same orientation.

The effective masses of the low-mass carriers agree qualitatively with the values found by Langenberg and Moore.[8] Resonances corresponding to their high-mass carriers were not observed in fields up to about 6 kilo-oersteds, probably because of too low an amplitude of the lower subharmonics. The effective mass $0.11m_0$ for orientation A agrees well with the value $0.11m_0$ for the same orientation calculated by Heine from

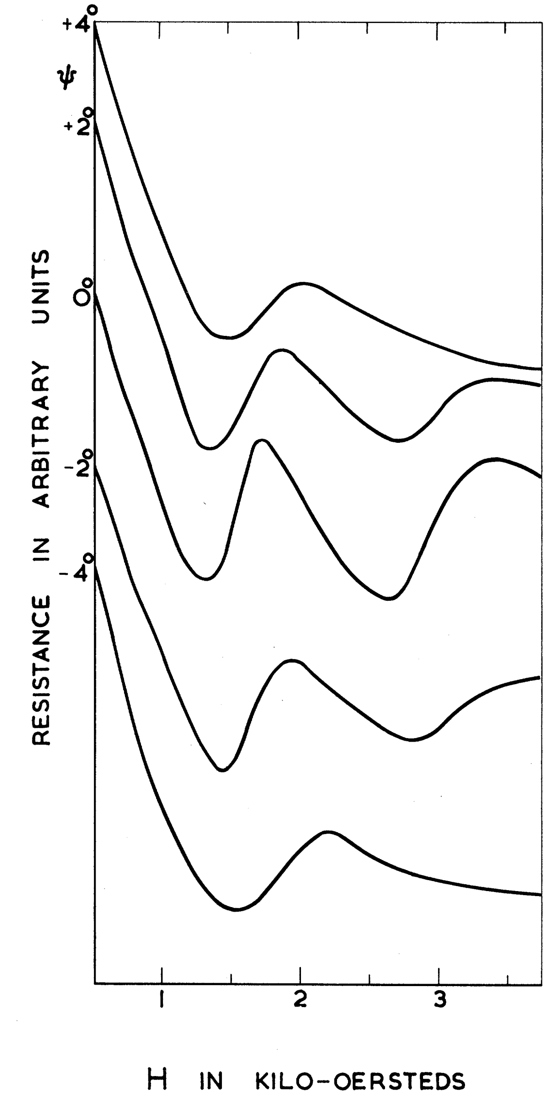

FIG. 3. Cyclotron resonance in aluminum at 36 kMc/sec and 4.2°K for the orientation A of Figs. 1 and 2, as the magnetic field is tilted to make an angle ψ with the sample plane.

the shape of the hole pockets (but compare $0.15m_0$ observed experimentally by Gunnersen from the temperature variation of the de Haas–van Alphen oscillations).

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DETECTION OF COHERENT BREMSSTRAHLUNG FROM CRYSTALS*

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When fast electrons (of several hundred Mev or more) pass through a crystal we expect the bremsstrahlung to be enhanced[1] and linearly polarized[2] if the electrons travel in a direction nearly parallel to some densely occupied rows of atoms. In simple terms, the reason is that an electron of energy $E_1 = \gamma_1 mc^2$ ($\gamma_1 \gg 1$) has a speed of $v = c[1 - 1/(2\gamma_1^2)]$ and can travel a distance $A = 2\gamma_1^2 \bar{\lambda}$ before falling back by $\bar{\lambda}$ behind a quantum (of wavelength $\lambda = 2\pi\bar{\lambda}$) which it is racing. This "coherence length" A can be several times the distance a between atoms in a row, and if an electron travels parallel to such a row one would expect A/a nuclei to cooperate coherently in the emission of a bremsstrahlung quantum; compared to a crystal not so aligned, the radiation should be stronger by a factor A/a. This factor is reduced[1] by thermal vibrations and by multiple scattering but may be still well above unity.

In estimating A, we have ignored the fact that after emitting the quantum the electron has less energy $E_2 = \gamma_2 mc^2 = E_1 - \hbar c/\bar{\lambda}$. The semiclassical argument above cannot allow for this; but the uncertainty principle gives the right answer. If both the electron and the emitted quantum go in the forward direction then the momentum that has to be taken up by the lattice is calculated as $P_L = (mc/2)(\gamma_2^{-1} - \gamma_1^{-1})$. The place where such a small momentum has been deposited cannot be located better than within $\hbar/P_L = 2\gamma_1\gamma_2\bar{\lambda}$; this must be the correct expression for A and indeed agrees with the previous expression if γ_1 and γ_2 are taken as equal.

Alignment has to be accurate to about $r_B/Z^{1/3}A$ or the enhancement will be reduced (r_B = Bohr radius $= 5.3 \times 10^{-9}$ cm). Deliberate misalignment of that order will produce fairly strong linear polarization of the quanta,[2] and such radiation may be of value for the study of photonuclear reactions; indeed this should give greater intensity than one gets in the fringe[3] of a beam from an ordinary target.

We have found, experimentally, about the expected enhancement, using a small crystal of germanium. It was mounted in the Cornell synchrotron in such a way that the 1-Bev electrons, spiralling in when the rf accelerating voltage was turned off, would cut across one of its edges, approximately in the [110] direction (face diagonal) on which there is a nucleus every 4.0 A. This produced the usual narrow beam of bremsstrahlung; the central portion (one milliradian around the axis) was selected by a lead collimator and cleared of charged particles by a magnetic field (in vacuo). It then passed successively through two plastic scintillation counters S and H, separated by 5 cm of lead and preceded by 3 mm of lead. Because of showers developing in the lead, the second counter H would be more sensitive to hard quanta whereas S would respond about equally to all quanta above 50 Mev. An increase in the ratio S/H should indicate, despite fluctuations in beam intensity, the expected enhancement of the softer x-rays as the crystal approached the correct alignment.

The crystal was aligned within about 2° on the basis of its Laue pattern; it could then be turned in small steps about a horizontal and a vertical