for scalar and pseudoscalar K, respectively.

Thus, the K parity can be determined, in principle, by the measurement of the sign of this correlation.

The determination of $\langle\sigma_{1n}\sigma_{2n}\rangle$ in reaction (1) presents two main practical difficulties: in the first place, reaction (1) occurs only in about 1% of all K^- capture reactions; secondly, if the final system is Λn, this measurement will involve the determination of the spin direction of n, which may present special problems. The spin direction of the Λ may be determined from the angular distribution of the decay products. If the final system is $\Sigma^- p$, the determination of the spin direction of the Σ^- may be difficult because of the apparently small value of the asymmetry parameter in Σ^- decay.

It is perhaps interesting to observe that if, in the more frequent reactions

$$K^- + d \to Y + N + \pi, \tag{5}$$

it turns out to be possible to isolate those events in which the Y and N are produced with sufficiently small relative momentum so that they are predominantly in a relative s state, then a similar theorem would hold for their spin correlation. In that case, in Eqs. (2a) and (2b) the vector $\vec{k}$ would represent the momentum of the pion relative to the YN system and the results of Eqs. (4a) and (4b) would be interchanged because of the pseudoscalar nature of the pion field. Reactions of type (5) have the advantage that they are more frequent than (1) and they would permit the investigation of the spin correlations of the Λp and the $\Sigma^+ n$ systems, which may avoid some of the difficulties mentioned above.

Finally we mention the reaction

$$\Sigma^{\pm} + He^4 \to {}_\Lambda He^5 + \pi^{\pm}, \tag{6}$$

as a possible means of determining the relative Λ-Σ parity. (We assume spin $\frac{1}{2}$ for ${}_\Lambda He^5$.) When the Σ has a polarization $\vec{P}_0$, the differential cross section for reaction (6) is given by

$$d\sigma/d\Omega = (d\sigma/d\Omega)_0 (1 \pm \vec{P}_0 \cdot \vec{P}_1), \tag{7}$$

where $\vec{P}_1$ is the polarization of the ${}_\Lambda He^5$ in the case $\vec{P}_0 = 0$ and $(d\sigma/d\Omega)_0$ is the corresponding differential cross section. The +(-) sign in Eq. (7) applies in the case in which the relative intrinsic parity of the initial and final systems is even (odd). Thus, the relative intrinsic parity of the final and initial systems in (6) can be determined, in principle, by a measurement of $\vec{P}_1$ and the left-right asymmetry for $\vec{P}_0 \neq 0$. The derivation of Eq. (7) follows the same lines as the theorems recently given by Bilenky[2] and Bohr.[3]

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ELECTROMAGNETIC TRANSITIONS BETWEEN μ MESON AND ELECTRON*

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The existence of the ordinary μ decay, $\mu \to e + \nu + \bar{\nu}$, seems to prove that the muon and electron do not differ in any quantum numbers.[1] It follows that weak electromagnetic transitions between muons and electrons could occur, if there is a mechanism to produce them. For example, one such mechanism would exist if the μ decay was not caused by a direct $\bar{\mu}e\bar{\nu}\nu$ Fermi interaction but instead involved a virtual charged boson. This particular possibility seems ruled out, since the predicted[2] rate for $\mu \to e + \gamma$ would be considerably greater than the upper limit set by recent experiments.[3,4] The purpose of this note is to discuss phenomenologically (without attachment to any specific mechanism) other kinds of electromagnetic transitions between muon and electron that may be possible even if $\mu \to e+\gamma$ is somehow suppressed.

Since muon and electron both have spin $\frac{1}{2}$, the general μ-e-γ interaction may be described by four form factors, which correspond roughly to distributions of $E0$, $M0$, $E1$, and $M1$ transition moments. They are defined by[5]

$$\langle e|J_\lambda(0)|\mu\rangle = -ie(2\pi)^{-3}\bar{u}_e\left[f_{E0}(q^2)\left(\gamma_\lambda + iq_\lambda\frac{m_\mu}{q^2}\right) + f_{M0}(q^2)\left(\gamma_5\gamma_\lambda + i\gamma_5\frac{q_\lambda m_\mu}{q^2}\right) + f_{E1}(q^2)\gamma_5\sigma_{\lambda\eta}\frac{q^\eta}{m_\mu} + f_{M1}(q^2)\sigma_{\lambda\eta}\frac{q^\eta}{m_\mu}\right]u_\mu. \quad (1)$$

Here J_λ is the electromagnetic current, and $q_\lambda = p_{e\lambda} - p_{\mu\lambda}$ is the momentum transfer. The form factors f_{E0}, f_{M0}, f_{E1}, f_{M1} are dimensionless functions of

$$q^2 = q_\lambda q^\lambda = -\vec{q}^2 + (q^0)^2.$$

Equation (1) follows from Lorentz invariance, current conservation, and the Dirac equations for u_e and u_μ, with the electron mass neglected.

In order to avoid an infinity in (1) for $q^2 \to 0$, we must have

$$f_{E0}(0) = 0, \quad f_{M0}(0) = 0. \quad (2)$$

Furthermore, if ϵ_λ is the polarization four-vector of a real photon with momentum $-q_\lambda$ ($q^2 = 0$), we must have $q_\lambda\epsilon^\lambda = 0$. We therefore have the familiar result that the $E0$ and $M0$ terms cannot contribute to the rate of emission of a real photon, which is given by

$$\omega(\mu \to e+\gamma) = m_\mu \alpha \xi_1^2,$$

$$\xi_1^2 = \tfrac{1}{2}[|f_{E1}(0)|^2 + |f_{M1}(0)|^2]. \quad (3)$$

Thus the observed absence of $\mu \to e+\gamma$ decay tells us nothing about the possible existence of an $E0$ or $M0$ form factor, or about the possibility that $f_{E1}(q^2)$ or $f_{M1}(q^2) \neq 0$, although $f_{E1}(0) = f_{M1}(0) = 0$.

Actually, because of the presumably "chiral" character of lepton interactions, it is not difficult to find examples of theories in which we expect that f_{E0} and f_{M0} would be much greater than f_{E1} and f_{M1}. Suppose, for instance, that in addition to the usual μ-decay Fermi interaction, the muon and electron also have a Fermi interaction of the form

$$H = g\bar{\psi}_e\gamma_\lambda(1+\gamma_5)\psi_\mu\bar{\psi}_f\gamma^\lambda(1+\gamma_5)\psi_f, \quad (4)$$

with some charged particle f.[6] This interaction will generate, through "vacuum polarization" type diagrams, monopole form factors given by

$$f_{E0}(q^2) = -f_{M0}(q^2) = -\frac{gq^2}{12\pi^2}\left[\tfrac{5}{6} + D - V\left(\frac{q^2}{4m_f^2}\right)\right], \quad (5)$$

where m_f is the mass of f, D is a logarithmically divergent constant, given in terms of a cutoff λ by

$$D = \ln\left(\frac{m_f^2+\lambda^2}{m_f^2}\right) + \frac{m_f^2}{m_f^2+\lambda^2} - 1, \quad (6)$$

and V is[7] the vacuum polarization function

$$-V(x) = \tfrac{5}{3} - \frac{1}{x} + \left(1 - \frac{1}{2x}\right)\left(1+\frac{1}{x}\right)^{1/2}\ln\left|\frac{(1+1/x)^{1/2}-1}{(1+1/x)^{1/2}+1}\right|$$

$$\to \tfrac{4}{5}x \text{ as } x \to 0. \quad (7)$$

For example, if we take $q^2 = m_\mu^2$, and $\lambda = m_p$, then $f_{E0} = -f_{M0} = f$:

$$f = -g(m_\mu^2/12\pi^2)\times 4.4.$$

On the other hand, in this model f_{E1} and f_{M1} are of order ge^2, since a virtual photon must be emitted and absorbed to produce any dipole transition moment.[6] This is a direct consequence of the "chirality conservation" of the μ-e covariant in (4), and would still hold if the factor

$$\bar{\psi}_f\gamma^\lambda(1+\gamma_5)\psi_f$$

were replaced by a more complicated function of the f-field operators. It should be mentioned that, realistically, f could only be a muon if the coupling constant in (4) is comparable with the Fermi constant. This is because we must have $m_f > m_\mu/2$ in order to avoid the unobserved decay process $\mu \to e+f+\bar{f}$, and furthermore f can only interact weakly with nuclei in order to avoid the unobserved nuclear absorption process $\mu + N \to e + N'$ in amounts comparable to ordinary μ capture.

In order to test the possible existence of the $E0$ and $M0$ terms in (1) or of the $E1$ and $M1$ terms for $q^2 \neq 0$, it is necessary to consider cases where the $\mu \to e$ transition is accompanied by an interaction with a virtual photon.[8] For example, a μ^- bound in a $1S$ state around a nucleus N may absorb a virtual photon from the Coulomb field of the nucleus, forming an electron and a recoiling nucleus N'. The kinematics of this will be almost identical to those in ordinary μ^- absorption, the electron and N' being emitted with opposite momenta of magnitude $m_\mu c = 106$ Mev/c, providing that the excitation of the nucleus is small.

The rate of such a transition is given by

$$\omega_{N\to N'} = 16 m_\mu Z_{\rm eff}{}^4 \alpha^5 \xi_0{}^2 Z |F_{N'N}|^2, \qquad (8)$$

where $F_{N'N}$ is defined by

$$\langle N' | J^0(0) | N\rangle = (2\pi)^{-3} Ze F_{N'N}(q^2), \qquad (9)$$

and

$$\xi_0{}^2 = \tfrac{1}{2}[|f_{E0}(m_\mu{}^2) + f_{M1}(m_\mu{}^2)|^2 + |f_{M0}(m_\mu{}^2) + f_{E1}(m_\mu{}^2)|^2]. \qquad (10)$$

[We are using a plane wave for the electron, a constant wave function for the muon, and take the density of states for the μ^- at the nucleus to be $Z_{\rm eff}{}^4\alpha^3 m_\mu{}^3/\pi Z$. Only virtual "timelike" photons are assumed to contribute. Terms of order m_μ/m_N or $(m_{N'} - m_N)/m_\mu$ are neglected throughout.]

For experimental purposes, it is best to compare (8) with the rate for ordinary μ^- absorption[9]:

$$\omega_{\rm abs} \simeq (1/2\pi^2) Z_{\rm eff}{}^4 \alpha^3 \xi_\beta{}^2 m_\mu, \qquad (11)$$

$$\xi_\beta{}^2 = \tfrac{1}{2}(|C_V|^2 + |C_{V'}|^2 + 3|C_A|^2 + 3|C_{A'}|^2) m_\mu{}^4. \qquad (12)$$

We have then

$$\omega_{N\to N'}/\omega_{\rm abs} = 32\pi^2\alpha^2 Z |F_{N'N}|^2 \xi_0{}^2/\xi_\beta{}^2 \simeq 0.017 Z |F_{N'N}|^2 \xi_0{}^2/\xi_\beta{}^2. \qquad (13)$$

The matrix element (9) is just the Born approximation nuclear matrix element for electron scattering ($e + N \to e + N'$) at momentum transfer

$$(q^2)^{1/2} \simeq 2E_e \sin(\theta/2) = m_\mu. \qquad (14)$$

The cross section for this process in Born approximation is

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm lab} \simeq \frac{Z^2\alpha^2\cos^2(\theta/2)|F_{N'N}|^2}{4E_e \sin^4(\theta/2)}. \qquad (15)$$

If the Born approximation gave a correct description of electron scattering, one could use the experimental data directly to compute the variation of $\omega_{N\to N'}/\omega_{\rm abs}$ from nucleus to nucleus. Since the Born approximation is not accurate over the whole periodic table, it seems more reasonable to use the information obtained about nuclear charge distributions in the electron scattering experiments to compute $F_{N'N}$. For the elastic processes, where $N = N'$, we estimate F_{NN} using a "Fermi shape" for the nuclear charge density,[10] with radius $1.08\times10^{-13}A^{1/3}$ cm and surface thickness $t = 2.5\times10^{-13}$ cm. Then setting $(q^2)^{1/2} = 5.36\times10^{12}$ cm^{-1}, we obtain

$$F_{NN} = \frac{3}{\Theta^3}\frac{(1.10\sin\Theta - 0.86\Theta\cos\Theta)}{1 + 0.916/\Theta^2}, \quad \Theta \equiv 0.579A^{1/3}. \qquad (16)$$

The factor $Z|F_{NN}|^2$ in (13) then reaches a maximum of about 6 for nuclei near Cu, falls off slowly for heavier elements to about 2.5 for Pb, where our approximations are not valid anyway, and falls off rapidly for lighter elements, where the shape probably differs from the Fermi shape anyway.

In addition to the coherent process where $N = N'$, we must also consider the possibility of inelastic processes, where N' is some excited state of N. Here we can make use of a theorem known in the study of muon and electron scattering.[11] We make the following assumptions: (1) Born approximation, (2) closure approximation, (3) neglect of structure and mesonic effects, and (4) neglect of nucleon-nucleon correlations in N, except for the Pauli principle. Then

$$\textstyle\sum_{N'\neq N} |F_{N'N}|^2 \leq 1/Z, \qquad (17)$$

so that

$$\textstyle\sum_{N'} |F_{N'N}|^2 Z \leq 1 + Z|F_{NN}|^2. \qquad (18)$$

This factor then reaches a maximum of about 7 for nuclei near Cu. Since the coherent process $N \to N$ occurs for such nuclei at least six times more frequently than all inelastic processes (providing the above assumptions are justified), the electron emitted will have a very sharp spectrum, peaked at 105 Mev.

It should be noted that the ordinary μ^- capture, which involves the change of a proton into a neutron, cannot lead to the same nucleus, and thus gets no contribution from the coherent process.

The best experimental evidence on possible electrons from μ capture was obtained for copper.[12] This experiment showed that

$$\omega_{N\to N'}(\mathrm{Cu})/\omega_{\mathrm{abs}}(\mathrm{Cu}) < 5\times 10^{-4},$$

so that, taking 7 as the value of $\sum_{N'}|F_{N'N}|^2 Z$, we have

$$\xi_0^2/\xi_\beta^2 \lesssim 4\times 10^{-3}. \qquad (19)$$

This indicates the amount by which the μ - e - γ form factors must be reduced compared to the μ decay coupling on the basis of present experiments. For comparison, the intermediate vector boson theory gives an answer of the order of $\xi_0^2/\xi_\beta^2 \sim (1/64\pi^2)^2$ for this ratio. In view of the mysteries surrounding the relation between muons and electrons, a more intensive study of whether electrons come from μ capture would be desirable.

The experiment of Steinberger and Wolfe was performed to test the possible existence of μepp and μenn Fermi interactions. The ratio of $\mu + N \to e + N'$ to ordinary μ absorption would, for vector coupling of equal strength to p and n, be proportional to $\sum_{N'} A^2 |F_{N'N}|^2/Z$. According to our calculations, this factor is about equal to 30 for Cu. Since this enhancement factor was taken to be about 4 in reference 12, the sensitivity of that experiment as a test of equal μepp and μenn vector interactions is about 7 times better than quoted.

Another experimental test of $\mu \to e$ transitions involving virtual photons would be the internal conversion process $\mu \to e + (e^+ + e^-)$. The rate for this process depends in a complicated way on the form of the functions f_{E0}, f_{M0}, f_{E1}, f_{M1} since it does not take place at fixed momentum transfer. (We cannot use the approximation of Kroll and Wada[13] here, as was done in reference 2, since we are supposing that the decay $\mu \to e + \gamma$ does not occur.) An order-of-magnitude estimate gives

$$\omega(\mu\to 3e)/\omega(\mu\to e+\nu+\bar{\nu}) \sim (e^4/4)(\xi_0^2/C_V^2).$$

Experimentally,[14] this ratio is less than 10^{-5}, so that $\xi_0^2/\xi_\beta^2 \lesssim 10^{-3}$. Therefore the two types of experiment are about equally sensitive as tests of μ - e - γ interactions, but the former has a simpler dependence on the form of the interaction.

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1 See J. Schwinger, Ann. Phys. 2, 407 (1957) for a possible alternative to this conclusion.

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4 See however, M. Ebel and F. J. Ernst (to be published).

5 We use Dirac matrices γ_μ satisfying $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$ $(\mu = 0, 1, 2, 3)$, $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$, and $\sigma_{\mu\nu} = (1/2i)[\gamma_\mu, \gamma_\nu]$. The spinors are normalized by $\bar{u}u = m/E$.

6 Such an interaction with $f = \mu$ has been discussed by Gamba, Marshak, and Okubo (to be published) who conclude that it cannot be present in strength equal to the Fermi coupling because it generates $\mu \to 3e$ by the mechanism we discuss. We thank Dr. Marshak for a discussion on this point.

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8 After this work was done, we received a communication from Dr. T. Kinoshita discussing the μ-e transition in a Coulomb field by this mechanism. His conclusions are quite similar to ours. We thank Dr. Kinoshita for his communication. See also S. P. Rosen (to be published) who also discusses the μ-e transition in a Coulomb field, through the μ-e-γ interaction.

9 We are indebted to Dr. Juliet Lee for a discussion of the relevant experimental problems.

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