Parametric Instabilities and Anomalous Heating of Plasmas near the Lower Hybrid Frequency

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It is shown both by theory and by computer experiments that a plasma subjected to a long-wavelength electric field oscillating near the lower hybrid frequency causes parametric instabilities which lead to an anomalous heating of both ions and electrons. Because the instability threshold is much lower than that for the corresponding electron plasma-wave problem, the saturation of the electric field can be much higher.

Plasma heating by anomalous absorption of a large-amplitude external electromagnetic wave has been of considerable theoretical and experimental interest.[1-5] We show by theory and by computer simulations that an electric field perpendicular to a dc magnetic field and oscillating at a frequency above but near the lower hybrid frequency can excite, at extremely low thresholds, decay and purely growing parametric instabilities.[2] These result in short-wavelength lower hybrid waves and ion waves and lead to a substantial plasma heating. This driving field might be associated with a whistler wave, with a lower hybrid wave propagating nearly perpendicular to a magnetic field,[6,7] or even with grids in the plasma. Such instabilities may occur naturally in shock-heated plasmas and Earth's magnetosphere.[8]

We consider the stability of a homogeneous plasma embedded in a uniform magnetic field $B_0\hat{z}$ and an oscillatory electric field $E_0\hat{y}\cos(\omega_0 t)$ in the dipole approximation; ω_0 is chosen close to the lower hybrid frequency $\omega_{\rm LH} \equiv \omega_{pi}/(1+\omega_{pe}^2/\Omega_e^2)^{1/2}$, where ω_{pj} and Ω_j are jth species plasma and gyrofrequencies, respectively. It is assumed that $\Omega_i \ll \omega_0 \sim \omega_{\rm LH} \ll \Omega_e$. The linearized Vlasov equations for ion and electrons in the lab frame are of the usual form with an external acceleration term $(q_j/M_j)E_0\cos(\omega_0 t)\hat{y}$.[5] The zeroth-order distribution function of the jth species is taken as Maxwellian in its own oscillating frame and the linearized equations are solved in the usual way.[5] The coupling coefficient $x = \vec{k}\cdot\vec{v}_0/\omega_0$ is assumed less than unity where $v_0 = (cE_0/B_0)\hat{x}$; ion orbits are taken as straight lines and $k\rho_e \ll 1$ is assumed. Keeping only the lowest-order terms in x and taking $k\lambda_{\rm De} \ll 1$ to simplify the form of the equation yields the following dispersion relation, which is of the standard form for three-wave mode coupling problems in the lab frame[2,8] (approximately the ion frame):

$$\epsilon_0 = \frac{x^2}{4}\chi_i^0\chi_e^0\left(\frac{1}{\epsilon_{-1}}+\frac{1}{\epsilon_1}\right),\qquad \epsilon_n = 1+\chi_i^n+\chi_e^n, \tag{1}$$

where

$$\chi_i^n = -\frac{\omega_{pi}^2}{(\omega+i\gamma_n+n\omega_0)^2-3k^2V_i^2}\quad (n=0,\pm 1), \tag{2a}$$

$$\chi_e^n = \frac{\omega_{pe}^2}{\Omega_e^2} - \frac{k_z^2}{k^2}\frac{\omega_{pe}^2}{(\omega+i\gamma_n+n\omega_0)^2-3k_z^2V_e^2}\quad (n=\pm 1), \tag{2b}$$

$$\chi_e^0 \approx 1/k^2\lambda_{\rm De}^2,$$

and γ_n are wave damping rates.

Electron motion via the $E_{-0}\times B_{-0}$ drift provides the dominant coupling here. In contrast to the high-frequency parametric instabilities where ω_0 is near ω_{pe} or Ω_e,[1,4] ion dynamics cannot be neglected in $\epsilon_{\pm 1}$, and this allows the excited high-frequency waves to heat ions directly.[9]

Equation (1) exhibits both decay and purely growing or two-stream-type instabilities. Using Eqs. (1) and (2) the minimum threshold[2] for the decay of the pump at ω_0 into a lower hybrid wave at

$$\omega_{Lk} = \omega_{\rm LH}\left\{1+\left[\frac{3k^2V_i^2}{\omega_{Lk}^2}+\frac{k_z^2}{k^2}\frac{M_i}{m_e}\left(1+\frac{3k_z^2V_e^2}{\omega_{Lk}^2}\right)\right]\right\}^{1/2}$$

and an acoustic wave at $\omega_s = kc_s$ is given by

$$\frac{V_0}{V_e} = 4\frac{\omega_0}{\omega_{pe}}\left[\frac{\gamma_0}{\omega_s}\frac{\gamma_1}{\omega_{Lk}}\left(1+\frac{\omega_{pe}^2}{\Omega_e^2}\right)\right]^{1/2}. \tag{3}$$

Above threshold where damping is not important the solution of Eq. (1) is given by $\omega = (\omega_s^2 + \frac{1}{2}ix \times \omega_{\rm LH}\omega_{Lk}\sqrt{\beta})^{1/2}$, where $\beta = (\omega_0-\omega_{Lk})/\omega_{Lk}$. It is readily seen that growth rates may exceed the acoustic frequency.

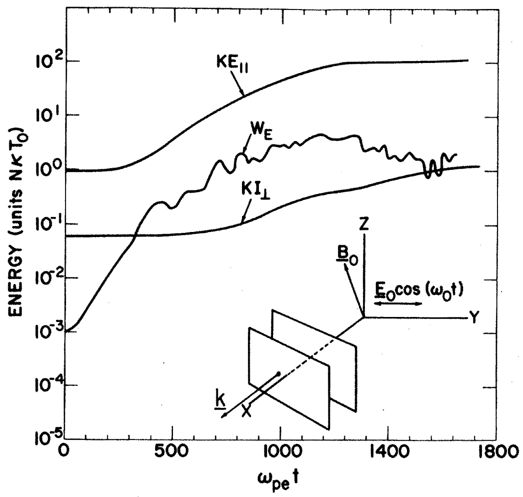

FIG. 1. Plots of total field energy W_E, electron parallel *thermal* energy $KE_\parallel$, and ion perpendicular *thermal* energy $KI_\perp$ in units of the initial electron thermal energy for a simulation with $k_z/k = 3(m_e/M_i)^{1/2}$, $\omega_0/\omega_{pe} = 0.115$, $\alpha = \frac{1}{4}$, $\omega_{pe} = \Omega_e$, $T_e/T_i = 16$, and $\lambda_{De} = 2$; 40 000 particles were used on a 256-cell grid. A sketch of the computer model used is shown.

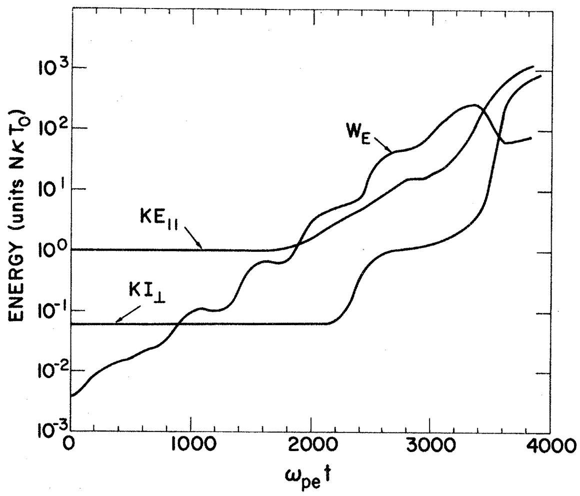

FIG. 2. Same as Fig. 1 except $k_z/k = (m_e/M_i)^{1/2}$, $\alpha = \frac{1}{2}$, with a grid of 512 cells.

The purely growing instability which occurs for $\omega_0 < \omega_{Lk}$ has a minimum threshold[2] given by

$$\frac{V_0}{V_e} \equiv 2\frac{\omega_0}{\omega_{pe}}\left(1 + \frac{T_i}{T_e}\right)^{1/2}\left[\frac{2\gamma_\perp}{\omega_{Lk}}\left(1 + \frac{\omega_{pe}^2}{\Omega_e^2}\right)\right]^{1/2}, \qquad (4)$$

and a growth rate well above threshold given by $\gamma \approx (\sqrt{-\beta}x\omega_{Lk}\omega_{\rm LH}/2)^{1/2}$. In arriving at Eq. (4), the kinetic form of $\chi_I^0 \approx \chi_e^0 T_e/T_i$ was used.

Note that the minimum threshold is that given for the modified two-stream instability,[10] but aided by a resonance effect. Furthermore, the threshold gets smaller as we let ω_0 go toward ω_{Lk}, i.e., reduce k_z/k. The optimum k_z/k tends to be $O(m_e/M_i)^{1/2}$. However, for very small k_z/k [less than $(m_e/M_i)^{1/2}$], the above results break down since we know that finite k_z is required to get any mode coupling. For a two-ion-species plasma, however, Kaw and Lee have considered coupling in the case of $k_z = 0$ and found another type of parametric instability.[11]

Verification of linear theory.—To determine the heating and saturation mechanism, we have carried out a number of numerical simulations using an electrostatic one-dimensional model of a type widely used in computational plasma physics[12] (see Fig. 1). Regarding linear theory, results of the simulation show that (1) the pump excites a wave near ω_0 and a low-frequency ion wave, (2) a power spectrum measurement verified the matching conditions, and (3) growth rates, which are the order of ω_s at the outset, are also in reasonable agreement with the linear theory.

Saturation and heating.—Nonlinear results for various pump field strengths are discussed. Two typical cases are shown in Figs. 1 and 2. Ions were unmagnetized, i.e., $\vec{V}\times\vec{B}_0$ force was neglected. These simulations as well as a number of other one-dimensional simulations suggest that the heating of ions and electrons as well as the saturation mechanism are quite different, depending on whether $k_z/k > (m_e/M_i)^{1/2}$ or $k_z/k \approx (m_e/M_i)^{1/2}$. The saturation mechanism appears also to depend on the strength of the pump.

(1) $k_z/k > (m_e/M_i)^{1/2}$. Figure 1 shows the results of a simulation with a strong pump where pump energy is comparable to the initial plasma thermal energy ($\alpha = \frac{1}{4}$). The electric field energy W_E grows more than 3 orders of magnitude from the initial noise level and stays more or less constant after $\omega_{pe}t = 1000$. As the unstable waves grow to sufficiently large amplitude, an anomalous heating of electrons sets in at $\omega_{pe}t \approx 500$, although ions remain relatively cold while acquiring a large amount of oscillatory motion.

The saturation level and the anomalous collision frequency ν^* are much higher than that for the corresponding electron plasma instability for the same strength of pump.[5] The anomalous collision frequency determined from the ratio of energy input through the relation $\nu^* = (d/dt)$(total plasma energy density)$[(E_0^2/8\pi)(\omega_{pe}^2/\Omega_e^2)]^{-1}$ is as large as $0.2\omega_{pe}$. This may result from the fact that, because the instability threshold is much lower here and the system appears to adjust back to marginal stability [i.e., Eq. (3)], the unstable waves can grow to a much larger amplitude for a given pump, resulting in a much higher collision frequency ν^*.

The overall saturation of the instability at $\omega_{pe}t \approx 1000$ appears to be due to the electron heating which effectively shuts off the growth by increasing the threshold until it equals the applied field. Using the new electron and ion temperatures in Eq. (3), we found that the plasma is near marginal stability at this point. However, the field energy does exhibit a quasisaturation at $\omega_{pe}t \approx 500$ which is associated with an electron trapping in the potential wells of modes 5–6, the most rapidly growing modes. Although this coherent trapped motion of the electrons becomes randomized, the first mode which has a much larger phase velocity continues to grow until stabilized by electron heating. Simulations with smaller fields ($\alpha = \frac{1}{20}$, $\frac{1}{150}$) show the same relative electron and ion heating and nearly the same saturation behavior. However mode 1, which eventually dominates, is stabilized by the formation of a tail on the electron distribution rather than by a heating of the whole distribution as we see in the strong pump case.

(2) $k_z/k \approx (m_e/M_i)^{1/2}$. Here, since both ions and electrons have phase velocities which imply comparable interaction with the unstable waves, heating of both species should be comparable and saturation should involve both species. Figure 2 shows the results of a simulation for a strong pump ($\alpha = \frac{1}{2}$). Both ions and electrons are heated by more than 3 orders of magnitude from the initial thermal level with $E_{\mathrm{sat}}^2 \approx 500E_0^2$. Since only mode one and to a much lesser extent modes two and three are unstable, and these modes have phase velocities much larger than the thermal velocities of ions and electrons, trapping does not cause saturation. The first mode continue to grow until $\omega_{pe}t \approx 3400$, where an abrupt wave breaking occurs associated with the subsequent saturation of the instability. In contrast, another simulation with a smaller pump ($\alpha = \frac{1}{50}$) allows higher k modes (4–5) to also grow and a more gradual wave breaking.

In addition to the above advantages for plasma heating, the lower hybrid parametric instabilities offer the advantage of little enhanced ion diffusion during the heating phase since the heating occurs on a time scale comparable to or faster than an ion gyroperiod. On the other hand, energy and perhaps particle transport due to electrons might be important because the turbulence is generated at $\omega_0 \sim O(\omega_{\mathrm{LH}})$.[13] In a real plasma, a second parametric instability driven by the electric field of the excited lower hybrid waves—i.e., $(\vec{E}_k \times \vec{B}_0)_y$ drift—may occur in the pump direction.[7] This problem is now under investigation by means of two-dimensional simulation.

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