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[22]This is somewhat implausible for the widest channels.

New Macroscopic Quantum Effect in a Type-II Superconductor

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A series of structures is observed in the field dependence of the current threshold of a type-II superconductor in a longitudinal magnetic field. In the mixed state the structures appear in the field $H=H_{c2}/N$ (H_{c2} is the upper critical field; N is an integer), while for the case $H>H_{c2}$, they are observed when $H=N^{1/2}H_{c2}$. These phenomena are discussed on the basis of the Ginzburg-Landau equation. The experimental data in the mixed state are interpreted as evidence for the existence of some new quantum states other than the Abrikosov state.

It is well known that type-II superconductors exhibit considerable enhancement of the conduction-current threshold and peculiar magnetization behavior when there is an impressed dc conduction current in a longitudinal magnetic field.[1-6] In the case of an ac conduction current a decrease of ac power loss in the field is also observed.[7-9]

I would like to report on new macroscopic quantum phenomena found in the measurement of the current threshold of a type-II superconductor near the critical temperature T_c in longitudinal magnetic fields. The material used in our experiment is commercial Nb–25% Zr (Westinghouse) wires of 0.025 cm diam. The current threshold is measured by increasing the conduction current in the specimen which has been cooled through T_c to 10°K in a static longitudinal field H. The specimen temperature is chosen near T_c in order to decrease the influence of the flux-pinning effect.

Figure 1 gives the observed relation between threshold current J_c and applied field H. In a field less than the lower critical field H_{c1}, where the superconductor is in the Meissner state, the conduction current reaches threshold when the resultant surface field becomes H_{c1}.[2] In the mixed state ($H_{c1}<H<H_{c2}$), however, a series of structures appears when $H=H_{c2}/N$ (N is an integer), where very small threshold current is observed, while when $H>H_{c2}$, the structures appear in the field $H=N^{1/2}H_{c2}$.

In order to discuss these phenomena, we investigate the Ginzburg-Landau (GL) equation. Since the temperature is near T_c, a linearized equation

$$\{[\nabla-(2ie/\hbar c)\vec{A}]^2+\xi^{-2}\}\psi=0 \tag{1}$$

can be used, where $\vec{A}$ is the vector potential; ξ, the coherence length; and ψ, the order parameter. In the frame in which the applied magnetic field is in the z direction, the vector potential is

$$\vec{A}=(-Hy,0,0).$$

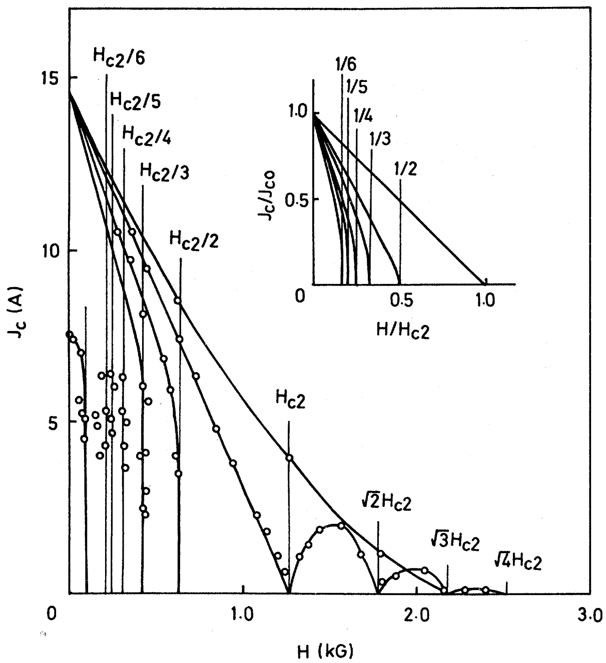

FIG. 1. Observed field dependence of the threshold current. The inset shows the theoretically derived characteristics.

Equation (1) obviously has a solution of the form

$$\psi = \exp[i(k_x x + k_z z)]F(y),$$

where $F(y)$ has to satisfy the equation

$$(\partial^2/\partial u^2 + \mu - u^2)F = 0 \qquad (2)$$

with

$$u = (2|e|H/\hbar c)^{1/2}(y - y_0), \quad y_0 = -\hbar c k_x/2eH,$$

$$\mu = (\hbar c/2|e|H)(\xi^{-2} - k_z^2).$$

Just as for the one-dimensional Schrödinger equation for simple harmonic oscillator, the solutions of Eq. (2) are finite everywhere only when $\mu = 2n + 1$ (n is an integer). This condition is rewritten as

$$(\hbar k_z)^2/4m + (n+\tfrac{1}{2})\hbar|e|H/mc = (\hbar/\xi)^2/4m \qquad (3)$$

or

$$|k_z| = k_n(H) = \xi^{-1}[1 - (2n+1)H/H_{c2}]^{1/2}. \qquad (4)$$

For each quantum state the order parameter may assume an expression of the form

$$\psi = C_{n+}(x, y)\exp(ik_n z) + C_{n-}(x, y)\exp(-ik_n z),$$

where the coefficients $C_{n\pm}$ are determined by boundary conditions. The current in the z direction is given by

$$J_z = -(ie\hbar/2m)(\psi\,\partial\psi^*/\partial z - \text{c.c.}),$$

since the vector potential $\vec{A}$ lacks a z component. We then have the maximum current for a quantum state,

$$J_c = (e\hbar|\psi|^2/m)k_n. \qquad (5)$$

With use of the well-known field dependence of the order-parameter amplitude,[10]

$$\left|\frac{\psi}{\psi_0}\right|^2 = \frac{1 - H/H_{c2}}{\beta_A(1 - 1/2\kappa^2)}$$

(ψ_0 is the order-parameter amplitude at $H = 0$, $\beta_A = 1.16$, and κ is the GL parameter), we have from Eqs. (4) and (5)

$$\frac{J_c}{J_{c0}} = \left(1 - \frac{H}{H_{c2}}\right)\left[1 - (2n+1)\frac{H}{H_{c2}}\right]^{1/2}, \qquad (6)$$

with

$$J_{c0} = \frac{e\hbar\psi_0^2/m\xi}{\beta_A(1 - 1/2\kappa^2)}.$$

Equation (6) shows that $J_c = 0$ when $H = H_{c2}/(2n+1)$, while our experiment shows that the structures in the mixed state appear when $H = H_{c2}/N$; thus we must reconsider our discussion so as to eliminate the discrepancy.

Equation (3) can be viewed as expressing that a Cooper pair has a quantized diamagnetic momentum $(n+\frac{1}{2})\hbar|e|/mc$. In the Abrikosov state one may associate a paramagnetic momentum $\frac{1}{2}p\hbar|e|/mc$ with a Cooper pair, which corresponds to the vortex state of p flux quanta (p an integer). We arbitrarily introduce these two quantum numbers and associate a resultant momentum $(2n - p + 1)\hbar|e|/2mc$ with a Cooper pair. With this stipulation the threshold current takes the form

$$\frac{J_c}{J_{c0}} = \left(1 - \frac{H}{H_{c2}}\right)\left[1 - (2n - p + 1)\frac{H}{H_{c2}}\right]^{1/2}. \qquad (7)$$

We see from Fig. 1 that in the mixed state only the threshold current with the lowest $2n - p + 1$ value is realized when $H/H_{c2} \neq 1/N$, and states with higher $2n - p + 1$ values are only observed for the case $H/H_{c2} = 1/N$. Apparently this rule does not fit well in the region with small applied field and large current. In this region, however, the self-field induced by the conduction current is comparable to or larger than the applied field. Therefore, some corrections for the observed data of the current and the field are necessary to find out their real values. Table I illustrates J_c-H characteristics given by Eq. (7) for some sets

TABLE I. Derived field dependence of the threshold current fitted to the experiment.

$h=H/H_{c2}$	J_c/J_{c0}	n, p
$1>h>1/2$	$1-h$	0, 1
$1/2>h>1/3$	$(1-h)(1-2h)^{1/2}$	1, 1
$1/3>h>1/4$	$(1-h)(1-3h)^{1/2}$	2, 2
$1/4>h>1/5$	$(1-h)(1-4h)^{1/2}$	2, 1
$1/5>h>1/6$	$(1-h)(1-5h)^{1/2}$	3, 2
$1/6>h>1/7$	$(1-h)(1-6h)^{1/2}$	3, 1

of the parameters (n, p) which are selected to have the equation fit the observed field dependence of the threshold current. The curves of the inset in Fig. 1 show the derived characteristics. In the selection of the parameters, we assume $p \geqslant 1$ and take the lowest set if there are plural sets for a value of $2n-p+1$. Since the Abrikosov state corresponds to $n=0$ and $p=1$, the above interpretation of the experiment means that when $H_{c1}<H<H_{c2}$, there exists new quantum states other than the Abrikosov one. LeBlanc[11] has reported structure in the axial magnetization of an Nb wire carrying a critical current which exhibits a dependence on the longitudinal magnetic field below H_{c2} similar to that described in this article.

The behavior of superconductors when $H>H_{c2}$ is usually attributed to time-dependent phase fluctuations and surface superconductivity.[12] In order to investigate the observed structures in this region, we use the diffusion-type time-dependent GL equation,[13]

$$\{-D^{-1}\partial/\partial t+[\nabla-(2ie/\hbar c)\vec{A}]^2+\xi^{-2}\}\psi=0,$$

where the diffusion constant D is given by

$$D\approx\xi v_F \tag{8}$$

in the dirty limit, v_F being the Fermi velocity. Looking for a solution of Eq. (8) in the form

$$\psi=\exp(-\omega t)\exp[i(k_x x+k_z z)]G(y),$$

we again arrive at the one-dimensional equation of a harmonic oscillator. The equation has finite solutions when

$$H/H_{c2}=(\omega\xi^2/D+1-k_z^2\xi^2)/(2n+1). \tag{9}$$

Here we make an assumption that the decay frequency ω takes only the discrete values

$$\omega=q\Delta/\hbar, \tag{10}$$

where Δ is the energy gap and q an integer. Using the relation $\xi=\hbar v_F/\pi\Delta$ and assuming $n=1$, we find from Eqs. (8)–(10) that $k_z=0$ is realized when

$$H/H_{c2}=1+q/\pi,$$

which takes the values 1.32, 1.64, 1.96, $\cdots$ according as $q=1, 2, 3, \cdots$. These values are in good agreement with the fields where the structures are observed.

From Fig. 1 we see that the surface superconductivity appears in the fields where, for the bulk superconductivity, $k_z=0$.

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