be expressed as

$$P_{\pi\pi}+P_{D\pi}\equiv(\sigma_{\pi\pi}+\sigma_{D\pi})/\sigma_{\pi}=P_{\pi\pi}+bZ,$$

where $P_{\pi\pi}$ is a constant. This sum of effects is plotted as a function of atomic number in Fig. 3. A linear least-squares fit of these data yields the results

$$P_{\pi\pi}=\sigma_{\pi\pi}/\sigma_{\pi}=(2.4\pm1.6)\times10^{-5},$$

and

$$P_{D\pi}=(0.38\pm0.19)\times10^{-5}Z.$$

Although the prime objective of the experiment was to measure $P_{\pi\pi}$ it is obvious that the method adopted enables one to determine the three effects separately. This is important as it has been pointed out[1] that the major uncertainty in a single measurement in a low-Z target would be due to the correction for the direct process. The measured values of $P_{D\pi}$ and $P_{\pi\pi}$ are consistent with predicted values. The latter, $P_{\pi\pi}$, is estimated via a formula applicable in the high-energy limit[1] as $P_{\pi\pi}\sim2\times10^{-5}$. Another possible interfering effect is indicated by Volkovyskii.[5] An attempt at estimating this contribution suggests that it is negligible at an energy of 6.6 MeV. Work is in progress to determine ratios in several other targets as well as to improve the statistical significance of the results reported above.

The authors gratefully acknowledge the financial support given by the National Research Council of Canada and the Province of Ontario. We also wish to acknowledge the assistance of the McMaster nuclear reactor staff.

[1]D. H. Wilkinson and D. E. Alburger, Phys. Rev. C 5, 719 (1972).

[2]P. Fettweiss and M. Saidane, Nucl. Phys. 139A, 113 (1969).

[3]L. Nichol, A. Lopez, A. Robertson, W. V. Prestwich, and T. J. Kennett, Nucl. Instrum. Methods 81, 263 (1970).

[4]S. De Benedetti and R. W. Findley, in *Handbuch der Physik*, edited by S. Flügge (Springer, Berlin, 1958), Vol. 45.

[5]R. Yu. Volkovyskii, Yad. Fiz. 2 878 (1965) [Sov. J. Nucl. Phys. 2, 878 (1965)].

Gauge Theory of Strong, Weak, and Electromagnetic Interactions*

I. Bars,† M. B. Halpern, and M. Yoshimura†‡

Department of Physics and Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

(Received 13 July 1972)

Starting from the recent $U(3)\otimes U(3)$ gauge model of strong interactions, we discuss the very natural synthesis with Weinberg's model of leptons. As a result of the interplay of the two models (a) neutral $\Delta S=1$ currents are eliminated without enlarging the number of quarks, and (b) a natural $(3,\overline{3})\oplus(3,\overline{3})$ symmetry breaking emerges for the hadrons. Incorporation of alternative lepton theories is mentioned.

Although there are now a number of renormalizable gauge models of leptons,[1] only one class of such theories has been constructed for hadrons.[2] We have asked the following question: Can we effect a marriage between the hadron and lepton theories, such that, for simplicity, we have just $[(3,\overline{3})\oplus(\overline{3},3)$ broken] $U(3)\otimes U(3)$ (three quarks) for the hadrons, and only observed leptons? The answer, modulo anomalies, is a very natural yes. Our removal of anomalies, for reasons mentioned below, involves fermionic doubling; this will be discussed after presentation of the model, along with inclusion of other lepton models.

Our approach to the synthesis is orderly: Starting with the $U(3)\otimes U(3)$ gauge model of hadrons, and Weinberg's $SU(2)\otimes U(1)$ model, we study embedding the latter in progressively larger "primed"[2] groups. Details of this study (and intermediate models) will appear in a larger paper. Here we sketch the emerging picture: The structure of the hadron theory requires the "primed" group at least as large as $SU(3)\otimes SU(3)$,[2] and Weinberg's model can be embedded therein. Unfortunately, models of this type have trouble with strangeness-changing processes. When we embed the leptons in $U(4)\otimes U(4)$, however, everything falls together beautifully, and it is this model we now present.

Groups and representations.—The hadronic group, entirely local, is $U(3)_L\otimes U(3)_R$. We represent its generators $F_{\alpha L}$, $F_{\alpha R}$ by $\frac{1}{2}\lambda^{\alpha}$ ($\alpha=0,\ldots,8$, left or right), being the usual 3×3 SU(3) matrices. The local leptonic group is $\mathrm{SU}(2)_L\otimes\mathrm{U}(1)$, embedded in a "primed" $\mathrm{U}(4)_L\otimes\mathrm{U}(4)_R$. We call the latter's generators $F_{\beta L}'$, $F_{\beta R}'$ ($\beta=0,\ldots,15$), but only four are realized locally. These are $\tilde{F}_{kL}'$ ($k=1,2,3$) and $\tilde{Y}=\tilde{F}_{3R}'+\frac{1}{3}(\tilde{F}_{0R}'+\tilde{F}_{0L}')$, with representations

$$\tilde{t}_{\beta}=R\begin{bmatrix}\frac{1}{2}\tau_{\beta} & 0\\ 0 & \frac{1}{2}\hat{\tau}_{\beta}\end{bmatrix}R^{-1},\quad R=\begin{bmatrix}1&0&0&0\\0&\cos\theta&\sin\theta&0\\0&-\sin\theta&\cos\theta&0\\0&0&0&1\end{bmatrix},\tag{1}$$

where τ_{β} ($\beta=0,1,2,3$, left or right) are the usual Pauli matrices, $\hat{\tau}_{\beta}\equiv\tau_2\tau_{\beta}\tau_2$, and the tilde operation R is the Cabibbo rotation. The charge operator is

$$Q\equiv(F_{3L}+F_{3R})+\tfrac{1}{3}\sqrt{3}(F_{8L}+F_{8R})+(F_{3L}'+F_{3R}')+\tfrac{1}{3}(F_{0L}'+F_{0R}').\tag{2}$$

Neutral operators do not rotate under R: $\tilde{t}_3=t_3$, $\tilde{Y}=Y$, etc.

Local transformations.—We represent the general local operator transformation in a unified supermatrix notation,

$$\exp[i(\alpha_L\cdot F_L+\alpha_R\cdot F_R+\beta\cdot F_L'+\gamma Y)]\to S=\begin{bmatrix}S_L&0&0&0\\0&S_R&0&0\\0&0&\tilde{S}_R'&0\\0&0&0&\tilde{S}_L'\end{bmatrix},\tag{3}$$

where, e.g., $\tilde{S}_L'=\exp[i(\tilde{t}\cdot\beta+\frac{1}{3}t_0\gamma)]$, $\tilde{S}_R'=\exp[i(t_3+\frac{1}{3}t_0)\gamma]$, etc.

Fields.—Let V_{α}^{μ}, A_{α}^{μ}, W_k^{μ}, B^{μ} be (respectively) the strong vector and axial vector fields and the weak gauge bosons. Defining

$$V_{L,R}^{\mu}\equiv\sum_0^8(V_{\alpha}^{\mu}\mp A_{\alpha}^{\mu})\tfrac{1}{2}\lambda^{\alpha},\quad \tilde{W}\equiv\sum_1^3\tilde{t}_kW_k,$$

we specify the transformation properties of all gauge bosons at once in terms of a diagonal supermatrix V^{μ} (like S), with entries[3]

$$V^{\mu}\colon[fV_L^{\mu},fV_R^{\mu},g'(t_3+\tfrac{1}{3}t_0)B^{\mu},g\tilde{W}^{\mu}+\tfrac{1}{3}g't_0B^{\mu}].$$

Then $\mathfrak{U}V_{\mu}\mathfrak{U}^{-1}=S(V_{\mu}-i\partial_{\mu})S^{-1}$. Similarly, for Weinberg's leptons, we introduce ($\nu_L=\nu_e$, $\nu_R=\nu_{\mu}^{c}$, D = doublet, S = singlet)

$$\psi_D=\begin{bmatrix}\nu_L&\mu_R^{+}&0\\e_L^{-}&-\nu_R&\\&0&?\end{bmatrix},\quad \psi_S=\begin{bmatrix}0&\mu_L^{+}&0\\e_R^{-}&0&\\&0&?\end{bmatrix},\quad l=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&\tilde{\psi}_S&\frac{1}{2}\sqrt{2}\tilde{\psi}_D^{c}\\0&0&\frac{1}{2}\sqrt{2}\tilde{\psi}_D&0\end{bmatrix}.\tag{4}$$

The question marks will be replaced later by heavy leptons to eliminate anomalies and ψ_D^{c} is the charge conjugate of ψ_D; then $\mathfrak{U}l\mathfrak{U}^{-1}=SlS^{-1}$. There is a great deal of freedom in the quark assignment. We will take here the simplest case, three fractionally charged quarks: (T = transpose) $q^T=(q_L,q_R,0,0)$, $\mathfrak{U}q\mathfrak{U}^{-1}=Sq$. The supermatrix notation is most symmetric for the scalar mesons,

$$M=\begin{bmatrix}0&\Sigma&0&M_L\\\Sigma^{+}&0&M_R&0\\0&M_R^{+}&0&\tilde{\varphi}^{+}\\M_L^{+}&0&\tilde{\varphi}&0\end{bmatrix},\quad \mathfrak{U}M\mathfrak{U}^{-1}=SMS^{-1}.\tag{5}$$

Here, $\Sigma=\sigma+i\pi$ is the usual $(3,\bar{3})\oplus(\bar{3},3)$ multiplet of scalars and pseudoscalars; $M_{L,R}$ are *three-by-four* complex matrices, being the scalars of Ref. 2, now with an extra fourth column. These scalars are the *only* connections of the weak and electromagnetic interactions with the strong, and will give mass to strong vector mesons[2]; the Weinberg scalar field $\varphi=\varphi_0t_0+i\varphi\cdot t$, which we see (in this notation) is to the leptonic system what Σ is to the hadronic. The covariant derivatives can be read from the covariant momentum operator

$$\mathcal{P}^{\mu} \equiv P^{\mu} + f(V_L{}^{\mu}\cdot F_L + V_R{}^{\mu}\cdot F_R) + gW^{\mu}\cdot \tilde{F}_L{}' + g'B^{\mu}Y \tag{6}$$

by commuting this operator with each field. We find $\Delta_\mu q \equiv \partial_\mu q - iV_\mu q$, $\Delta_\mu l \equiv \partial_\mu l - i[V_\mu, l]$, $\Delta_\mu M \equiv \partial_\mu M - i[V_\mu, M]$, and the usual $F_{\mu\nu}$'s for each gauge meson. Our locally invariant Lagrangian is

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}\mathrm{Tr}(F_L{}^{\mu\nu}F_{\mu\nu}{}^L + F_R{}^{\mu\nu}F_{\mu\nu}{}^R) - \tfrac{1}{4}\mathrm{Tr}(F_{\mu\nu}{}^B F_B{}^{\mu\nu}) - \tfrac{1}{4}\mathrm{Tr}(F_{\mu\nu}{}^W F_W{}^{\mu\nu}) - i\bar{q}\Delta^{\mu}\gamma_\mu q - i\,\mathrm{Tr}(\bar{l}\Delta^{\mu}\gamma_\mu l)\\ &- \tfrac{1}{4}\mathrm{Tr}\{(\Delta^{\mu}M)^{+}\Delta_\mu M\} + \alpha(\bar{q}_L\Sigma q_R + \mathrm{H.c.}) + \mathrm{Tr}(\tilde{\bar{\psi}}_D\tilde{\varphi}\tilde{\psi}_S\tilde{G} + \mathrm{H.c.}) + V(M_L) + V(M_R) + V(\Sigma) + V(\varphi)\\ &+ \mathrm{Tr}(G_2\tilde{\varphi}^{+}M_L{}^{+}M_L\tilde{\varphi} + G_2M_R{}^{+}M_R\tilde{\varphi}^{+}\tilde{\varphi}) + \mathrm{Tr}(G_1\tilde{\varphi}^{+}M_L{}^{+}\Sigma M_R + \mathrm{H.c.}),\end{aligned} \tag{7}$$

where the $V(\cdots)$'s are the usual quartic and quadratic terms,[3] and the G "insertions,"[2] which are 4×4 diagonal matrices with entries

$$G: \frac{2}{\lambda}(m_e, m_\mu, ?, ?),\quad G_1: \frac{\sqrt{2}}{2\lambda}\left(\frac{f_\pi m_\pi{}^2}{\kappa_1{}^2}, \frac{f_\pi m_\pi{}^2}{\kappa_1{}^2}, \frac{2f_K m_K{}^2 - f_\pi m_\pi{}^2}{\kappa_2{}^2}, d\right),\quad G_2: (a, a, b, c),$$

do not spoil the unified gauge invariance. The interpretation of the parameters in these insertions will be clarified in the following paragraph.

Spontaneous breakdown and symmetries.—A detailed study of the complicated scalar system will be presented in a larger paper. Here we sketch the general ideas. First, we use 21 degrees of gauge freedom (all but Q) to eliminate the 3×3 submatrices of $M_L - M_L{}^{+}$ and $M_R - M_R{}^{+}$, and all the components of φ except φ_0. Next, in order to give masses to all the gauge fields except the photon, we assign vacuum expectation values $\langle\varphi\rangle \equiv \lambda t_0$, $\langle M_L\rangle = \langle M_R\rangle \equiv \kappa$. These then generate a linear term in Σ (last term in $\mathcal{L}$). Thus Σ itself acquires a vacuum expectation value $\langle\Sigma\rangle \equiv v$, which is the *usual* $(3,\bar{3})\oplus(\bar{3},3)$ hadronic symmetry breaking in the spirit of Gell-Mann, Oakes, and Renner.[4] It further turns out that the system allows the following arbitrary vacuum expectation values:

$$\kappa = \begin{bmatrix}\kappa_1 & 0 & 0 & 0\\ 0 & \kappa_1 & 0 & 0\\ 0 & 0 & \kappa_2 & 0\end{bmatrix},\quad v = \frac{1}{\sqrt{2}}\begin{bmatrix}f_\pi & 0 & 0\\ 0 & f_\pi & 0\\ 0 & 0 & 2f_K - f_\pi\end{bmatrix}, \tag{8}$$

and no Goldstone bosons. Except for d, the interpretation of the parameters in G_1 and v is standard,[5] while G_2, d, and $V(\cdots)$ can be adjusted to give arbitrarily large masses to φ_0 and the remaining scalars in M_L and M_R; hence with Ref. 2, we continue to regard these as unobservable entities. Actually, the model is perhaps more satisfactory with $\kappa_1 = \kappa_2$, leaving ω-φ splitting until higher order in strong interactions. For this case we preserve the Weinberg sum rules[6] and so we specialize to $\kappa_1 = \kappa_2$ below.

Photon system and diagonalization.—Our spontaneous breakdown is such that the only unbroken gauge symmetry is generated by Q. Rewriting the covariant momentum (6), we isolate the (massless, universal) photon as the coefficient of Q:

$$\begin{aligned}&A^{\mu} = \cos\eta(\sin\varphi W_3{}^{\mu} + \cos\varphi B^{\mu}) + \sin\eta(\tfrac{1}{2}\sqrt{3}V_3{}^{\mu} + \tfrac{1}{2}V_8{}^{\mu}),\\ &e = g\sin\varphi\cos\eta;\quad \tan\varphi = g'/g,\quad \tan\eta = 2g\sin\varphi/\sqrt{3}f.\end{aligned} \tag{9}$$

With $f^2/4\pi \sim 2$ and g, g' small, we obtain approximately Weinberg's $e \sim gg'/(g^2+g'^2)^{1/2}$. This diagonalization induces electromagnetic mixing of bare ρ_0, φ, and ω, such that the physical particles have order e^2/f couplings directly to the leptonic electromagnetic currents. This simulates vector-dominated electromagnetic form factors in lowest order, and gives a *hadronic* correction to the muonic $\frac{1}{2}(g-2)$ which agrees with previous estimates.[7] To keep the usual universality of weak interactions, we do not diagonalize the $W^{\pm}$–strong-vector-meson mixings involved in the term

$$\mathcal{L}' = gf\,\mathrm{Tr}\{V_L(M_L+\kappa)\tilde{W}(M_L{}^{+}+\kappa)\}. \tag{10}$$

Thus, charged currents at low energy proceed via *vector dominance* in lowest order.

$\Delta S = 1$ neutral currents.—Because our Cabibbo rotation rotates only $W^{\pm}$, we find *no* neutral $\Delta S = 1$ currents. In this model, then, although we need four "things" to eliminate such currents, they are the columns of the unobservable $M_{L,R}$, and *not* extra quarks.

Fermions and anomalies.—As thus far presented, the model has anomalies.[8] Further, there appears to be no way, in the presence of both strong and weak vector mesons, to cancel hadronic against leptonic anomalies. Thus, we mention a flexible doubling scheme that for hadrons is in the spirit of dual models. We introduce q', $\psi_{S,D}'$[9] that couple to gauge bosons just as q, $\psi_{S,D}$, but with the opposite sign of γ_5. In the leptonic system anomalies are canceled without complication. To avoid suppressing $\pi_0 \to 2\gamma$, however, we also need a new Σ', which transforms like Σ, but by choice, couples only to q'. It is then easy to arrange, with other terms in $\mathcal{L}$ like $\mathrm{Tr}[\tilde{\varphi}^+ M_L{}^+ \Sigma' \times M_R G_1']$ and $(\alpha' \bar{q}_L' \Sigma' q_R' + \mathrm{H.c.})$, that the masses of q', Σ' are high with negligible effect on V, A masses. Then, $\pi^0 \to 2\gamma$ proceeds only through q. To get an extra factor of 3 in amplitude,[10] there are a number of choices—the simplest being the introduction of two more "pairs" of canceling quarks (like q, q') with large mass.[11]

Other lepton models.—Among the other lepton models in the literature, one stands out as fitting our hadrons as well as Weinberg's. This is the "second" model of Prentki and Zumino[1] which may confront neutral current measurements more successfully than Weinberg. The Prentki-Zumino leptons fit into our $\psi_{D,S}$ using the lower right-hand corners as well. All other details are essentially the same as above. Not all leptonic theories fit our hadron theory, however. For example, the model of Georgi and Glashow,[1] if it fits at all, appears unnatural. In the first place we need a $\mathrm{U}(5) \otimes \mathrm{U}(5)$ hadronic group (five quarks before anomaly cancelation), and worse, their scalar field transforms such that, without further scalars, we cannot find a $(3, \bar{3}) \oplus (\bar{3}, 3)$ symmetry breaking term like $\mathrm{Tr}[\tilde{\varphi}^+ M_L{}^+ \Sigma M_R]$.

Conclusions.—To the best of our knowledge, our unified model is consistent with known low-energy data, including vector-meson dominance at low energies for electromagnetic and weak form factors, and accepted theoretical ideas about broken hadron symmetries, etc.—in the presence of explicit hadron dynamics.

The question of deep inelastic scaling for our model (in perturbation theory) remains to be investigated. Although it turns out that the current algebra generally resembles algebra of fields, we do not expect worse scaling properties than other renormalizable (longitudinally damped) theories.[12]

We find it very encouraging that a unified renormalizable gauge theory of strong, weak, and electromagnetic interactions exist in which all three forces derive from a single, stringent principle: gauge invariance.

We acknowledge helpful conversations with K. Bardakci, M. Suzuki, and D. Levy.

*Work supported in part by the U. S. Atomic Energy Commission.

†Participating guest, Lawrence Berkeley Laboratory, Berkeley, Calif. 94720.

‡Address after September 1972: Department of Physics, University of Pennsylvania, Philadelphia, Pa. 19104.

[1]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967), and 27, 1688 (1971); M Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972); B. W. Lee, to be published; J. Prentki and B. Zumino, to be published.

[2]K. Bardakci and M. B. Halpern, to be published. The "primed" or "leptonic" groups here are the "global" groups of this reference.

[3]For different ninth-meson couplings, let

$$f V_{L,R} \to f \sum_{1}^{8} (V \mp A) \tfrac{1}{2}\lambda + f'(V_0 \mp A_0)\tfrac{1}{2}\lambda_0.$$

The ninth axial vector meson seems needed to avoid a Goldstone boson in $(M - M^+)_{L,R}$, thus we cannot use the term $\det\Sigma + \det\Sigma^+$. As an *effective* Lagrangian then our model has a π-η degeneracy.

[4]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); S. L. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968). We emphasize that our ability to construct this important last term in $\mathcal{L}$ depends on the φ, Σ symmetry observed above. A similar mechanism was suggested by Weinberg (Ref.1).

[5]S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. 41, 531 (1969). Actually, there is freedom to induce $\langle\sigma_3\rangle$ as well, if desired, for a lowest order model of $\eta \to 3\pi$.

[6]S. Weinberg, Phys. Rev. Lett. 18, 507 (1967); T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Lett. 18, 1209 (1967).

[7]M. Gourdin and E. de Rafael, Nucl. Phys. B10, 667 (1969).

[8]D. Gross and R. Jackiw, to be published; C. Bouchiat, J. Iliopoulos, and Ph. Meyer, to be published.

[9]ψ_S' must also include (heavy) neutrino entries on the diagonal, which do not couple to gauge bosons. With minor rearrangement, the new heavy leptons fit in Eq. (4) in place of the question marks.

[10]S. Adler, in *Lectures on Elementary Particles and Quantum Field Theory*, edited by S. Deser *et al.* (Massachusetts Institute of Technology Press, Cambridge, Mass., 1970).

[11]An alternate mechanism uses two "pairs" of integrally charged quarks with Y providing a "charm." Only one triplet couples to Σ by choice.

[12]S. Adler and W.-K. Tung, Phys, Rev. Lett. 22, 978 (1969).