the same model which was developed to explain the static properties. The velocity component perpendicular to the applied field gradient follows naturally from the Landau-Lifshitz-Gilbert equation of motion without the introduction of any arbitrary assumptions. The dependence of the bubble velocity components on the drive field is correctly explained qualitatively by the introduction of the f factor, but a quantitative comparison with experiment must await a detailed derivation of the dependence of f on the drive field.

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Effect of Bloch Lines on Magnetic Domain-Wall Mobility*

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(Received 2 August 1972)

Certain magnetic domain walls in a garnet film show a much reduced mobility from that of normal domain walls in the film and are interpreted to contain vertical Bloch lines. Theory shows that forward wall motion involves sideways propagation of Bloch lines and that in the high Bloch-line density limit, the mobility for small α is reduced to α^2 of the conventional value, where α is the Gilbert damping parameter.

Until recently the theory of ferromagnetic or ferrimagnetic domain wall motion in a constant field has been confined to the case of a uniform Bloch wall propagating in an infinite medium. It has therefore involved only one-dimensional solutions of the Landau-Lifshitz equation. In this case, Döring[1] has shown that the velocity of the wall is governed by the precession of the magnetization within the wall width about the demagnetizing field of the wall, and Walker[2] has calculated an upper limit of this velocity. More recently, the theory has been extended by Slonczewski[3,4] and by Schlömann[5] to cases of thin films where two-dimensional solutions are required because surface stray fields vary along the thickness of the film. In one of these cases, interacting of a horizontal (parallel to film plane) Bloch line with the stray field was invoked to account for critical-velocity observations of Argyle, Slonczewski, and Mayadas.[4]

In this paper we consider a new kind of wall motion involving the propagation of Bloch lines. We have studied such effects experimentally in garnet films with anomalous cylinder (bubble) and stripe domains. The static behavior of these domains has been interpreted in a previous paper[6] (I), and also by Tabor *et al.*,[7] in terms of vertical (parallel to easy axis) Bloch lines interacting along the perimeter of the domain. We will argue that the mobility of such walls is a two-dimensional problem, involving forward propagation of the wall and sideways propagation of Bloch lines.[8] However, in contrast to the earlier studies in films,[3-5] the surface of the specimen plays no essential role in the dynamic phenomenon, which should therefore occur as well in a bulk specimen, given the initial presence of Bloch lines.

In I, a garnet film 7 μm thick and of composition $(Yb_{0.15}Eu_{0.65}Y_{2.2})(Ga_{1.1}Fe_{3.9})O_{12}$ was shown to sustain classes of bubbles and stripes with different characteristic dependences of dimensions on field and different characteristic collapse fields. We chose to study the mobility of the bubbles with the highest collapse field (132 Oe in Fig. 2 of I) because we could easily differentiate them from other bubbles by lifting the bias field to just under this collapse field and thus eliminating all other bubbles and stripes. We measured

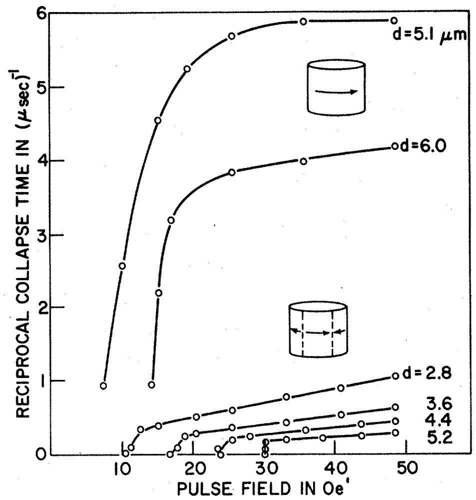

FIG. 1. Reciprocal collapse time, in units of reciprocal microseconds, versus pulsed field in oersteds for two types of bubbles in the garnet film described in I. The upper two curves are for normal bubbles and the lower four for bubbles collapsing statically at the highest field value, as shown in Fig. 2 of I. In both cases, curves are shown for several different starting diameters d, as controlled by varying the bias field.

the time T of a pulsed field required to collapse the bubbles as a function of the field strength H and initial diameter d.[9] The results are shown in Fig. 1 along with those for "normal" bubbles, which, as argued in I, presumably contain no Bloch lines.

The results for the normal bubbles are typical of low-damping garnet films, indicating a high initial mobility and showing a velocity saturation of the type studied previously.[4] Since the initial mobility is higher than can be measured by bubble collapse, it was measured instead by a photometric technique as described by Seitchik, Doyle, and Goldberg,[10] but with greater time resolution.[11] A mobility of 3000 cm/sec Oe and a Gilbert damping parameter $\alpha = 0.02$ was extracted from the decay of the underdamped oscillations of stripe domains in response to a pulse field.

The results of Fig. 1 for the bubbles with Bloch lines show dramatically longer collapse times as compared to the normal bubbles. To interpret these data properly, we should extend the dynamic collapse theory of Callen and Josephs[12] to include the Bloch-line exchange energy described in I; this is complicated by the interplay of elliptic and radial instability. So, as a first approximation, we calculate the average wall velocity from $(d-d_c)/2T$, where the collapse diameter d_c is taken to be the static collapse diameter 1.5 μm, as found in Fig. 2 of I. The nonlinear parts of the lower four curves in Fig. 1 are attributable to bubble potential effects and the linear parts, extrapolating roughly through zero, give a mobility of 1.3 cm/sec Oe, consistently within ± 50% for all d.

The mobility of the normal domain walls, which we assume contain no Bloch lines, can be understood in terms of either of the equivalent conventional formulas

$$\mu = \gamma\Delta\alpha^{-1} \equiv \Delta M\gamma^2\lambda^{-1}, \tag{1}$$

where $\Delta = (A/K)^{1/2}$ is the Bloch-wall thickness, A the exchange stiffness, K the uniaxial anisotropy, γ the gyromagnetic ratio, and M the spontaneous magnetization. The damping can be expressed either in terms of the Gilbert parameter α or the Landau-Lifshitz parameter λ. If we assume that the damping coefficinet λ/γ^2 scales linearly with composition, then we may estimate its value from mobility determinations in other compositions. Consulting the table of Vella-Coleiro, Smith, and Van Uitert,[13] we find the value $\lambda/\gamma^2 \sim 6\times10^{-8}$ Oe sec. With $A = 3\times10^{-7}$ erg/cm, $K = 10^4$ erg/cm^3, and $M = 16$ G, we predict a mobility of 1500, in reasonable accord with the experimental value for the normal domain walls.

In order to explain the effect of Bloch lines, we consider a domain wall with Bloch lines lying straight along the z axis; x is the distance perpendicular to z along the (possibly curved) wall. One dynamical variable is the normal wall displacement $q(x,t)$. The second dynamical variable is the angle $\psi(x,t)$ between the x axis and the wall-surface magnetization of magnitude $\pi\Delta M$ lying in the xq plane. The Bloch lines may be represented by a ψ varying monotonically with x, provided that all the Bloch lines have the same handedness. To apply this model to a bubble of circumference C and Bloch-line number n, which is even and of either sign, we ignore the change in circumference with time and set the boundary condition

$$\psi(x+C,t) = \psi(x,t) + \pi n. \tag{2}$$

The Landau-Lifshitz-Gilbert equation reduces to the following coupled equations for high-Q ($\equiv K/2\pi M^2$) materials[14]:

$$\dot q = 2\pi\gamma\Delta M\sin 2\psi - 2\gamma\Delta A M^{-1}(\partial^2\psi/\partial x^2) + \alpha\Delta\dot\psi, \tag{3}$$

$$\dot\psi \doteq \gamma H_z + 2\gamma A^{1/2}K^{1/2}M^{-1}(\partial^2 q/\partial x^2) - \alpha\Delta^{-1}\dot q, \tag{4}$$

where the dot means the derivative with respect to t. The first of these equations expresses the proportionality of wall velocity to the torque per unit area acting on ψ in which the essential terms are from wall demagnetization, Bloch-line exchange energy, and viscous damping, respectively. Stray fields emanating from surface poles, which are important to the problem of horizontal Bloch-line motion,[3,4] are neglected here. The second equation shows that the precession rate of ψ is proportional to the pressure on the wall, to which the essential contributions are from the applied field, surface tension, and damping, respectively.

The steady-state solution of Eqs. (3) and (4), for a small constant applied field H_z, depends on the value of n in the boundary condition (2). For the simple case $n=0$, neither ψ nor $\dot{q}$ depends on x or t. Thus, according to Eq. (3), the velocity is imparted entirely by the demagnetizing torque. Moreover, in this case Eq. (4) reduces to the mobility relation $\dot{q}=\mu H_z$ with μ given by (1), in agreement with previous theory. However, in the case of large Bloch-line density $|n|/C$, the large exchange energy dominates the magnetostatic energy. Thus, neglecting the magnetostatic torque, we find the solutions[8]

$$\psi=\pi n C^{-1}+\gamma H_z(1+\alpha^2)^{-1}t, \tag{5}$$

$$\dot{q}=\mu H_z=\Delta\gamma(\alpha+\alpha^{-1})^{-1}H_z. \tag{6}$$

We can check that the demagnetizing torque contributes nothing to the time average of $\dot{q}$ in first order by substituting (5) in (3) and integrating with respect to t. Equations (1) and (6) coincide with the upper and lower bounds, respectively, on the mobility as predicted from conservation of energy.[14]

Equation (5) can be interpreted as a sideways propagation of the Bloch lines at velocity $\gamma H_z C/n\pi(1+\alpha^2)$. The direction of propagation depends on the handedness of the Bloch lines, since the sign of the velocity depends on the sign of n. Although such propagation has not been seen directly, it may account for the remarkable phenomenon of stripe rotation which has been seen recently in several laboratories.[15] When the z field supporting an anomalous stripe domain of the type described in I is changed or pulsed, the stripe is observed to rotate in the plane. Some stripes rotate clockwise, others anticlockwise, for a given field sense. This rotation may be attributed to coupling between the long axis of the stripe and the circulation of Bloch lines around the perimeter. A more quantitative study of this phenomenon is in progress.

Equations (1) and (6) predict a reduction in mobility for bubbles containing many Bloch lines by a factor of α^2 for low-loss ($\alpha \ll 1$) bubbles. From the experimental normal-wall mobility of 3000 cm/sec Oe and damping coefficient of $\alpha=0.02$, we predict a Bloch-line–bubble mobility of 1.2 cm/sec Oe, which is fortuitously close to the measured value of 1.3 cm/sec Oe, considering the 20–50% error bars which must be placed on all the above experimental values.

A more detailed theory considers both demagnetizing and exchange torque consistently. In the limit of well-separated Bloch lines, one finds the initial mobility[3]

$$\mu=\gamma\Delta/(\alpha+\pi^2\Lambda|n|/2C\alpha), \tag{7}$$

where $\Lambda=(A/2\pi)^{1/2}M^{-1}$ is the characteristic Bloch-line thickness in analogy with the Bloch-wall thickness Δ. This result resembles (6); yet for vanishing Bloch-line density $|n|/C$, it reduces to the conventional Eq. (1). We have not determined the precise number of Bloch lines in our low-mobility bubble. The quantization of static characteristics shown in Fig. 2 of Ref. 6 indicates at least four Bloch lines, but there could easily be a few more since it is not known how many Bloch lines it takes before the static diameter of a bubble is measurably affected. On the other hand, the good agreement of (6) with experiment and the approximate independence of mobility on starting bubble diameter indicate that the assumptions of (6) are fulfilled, namely, that the demagnetizing energy $2\pi^2M^2\Delta d$ is indeed less than the Bloch-line exchange energy $2n^2\pi A\Delta/d$, which implies $n \gtrsim 20$ for our sample. We have also assumed that the wall thickness $\Delta=(A/K)^{1/2}$ is independent of Bloch-line density; this is true as long as the usual Bloch-wall energy $4(AK)^{1/2}$ is greater than the Bloch-line exchange energy, that is, as long as $n<100$, which appears to be the case.

The authors thank Dr. E. A. Giess for growing the garnet film.

*Research partially supported by the U. S. Air Force Office of Scientific Research under Contract No. F44620-72-C-0060.

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Observation of Polarization-Analyzing Power Inequality in the Reaction $^{88}Sr(p,p'\gamma)^{88}Sr$ Using Polarized Protons*

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(Received 1 August 1972)

The difference between the polarization and the analyzing power in the inelastic scattering of protons at isobaric analog resonances in ^{89}Y has been determined by measuring the spin-flip probability of a polarized beam. The differences are large, and are sensitive to the structure of the resonances.

In a recent Letter[1] it was suggested that spin-flip measurements with a polarized incident proton beam be used to determine the difference between the polarization (p_z) and the analyzing power (A_z) for inelastic scattering at isobaric analog resonances (IAR's). Here we present the first results of such measurements.

The defining equations for the differential cross section $d\sigma/d\Omega$, p_z, A_z, the spin-flip probability S, and the spin-flip asymmetry ΔS are as follows:

$$\begin{aligned} d\sigma(\theta)/d\Omega &= \tfrac{1}{2}(\sigma^{++}+\sigma^{+-}+\sigma^{-+}+\sigma^{--}) \equiv \tfrac{1}{2}\sigma(\theta),\\ p_z(\theta)\sigma(\theta) &= \sigma^{++}+\sigma^{-+}-\sigma^{+-}-\sigma^{--},\\ A_z(\theta)\sigma(\theta) &= \sigma^{++}+\sigma^{+-}-\sigma^{-+}-\sigma^{--},\\ S(\theta)\sigma(\theta) &= \sigma^{+-}+\sigma^{-+},\\ \Delta S(\theta)\sigma(\theta) &= \sigma^{+-}-\sigma^{-+} = \tfrac{1}{2}(A_z-p_z)\sigma(\theta). \end{aligned} \tag{1}$$

The z axis is chosen along the direction $\hat{k}_{\rm in}\times\hat{k}_{\rm out}$, where $\hat{k}_{\rm in}$ and $\hat{k}_{\rm out}$ are the incident and outgoing proton directions.[2] Then $\sigma^{+-}(\theta)$, for instance, is the partial differential cross section for scattering from a state with incident proton spin in the positive z direction to a final state with outgoing proton spin in the negative z direction. Only four of the five quantities defined above are independent, since there are only four incoherent partial cross sections. For elastic scattering, time-reversal invariance requires that p_z and A_z be equal. No such requirement exists for inelastic scattering, but until now there has been little experimental evidence for a difference between p_z and A_z.[3] Differences occur if the cross section for spin flip from + to − is different from the cross section for spin flip from − to +.

In the present experiment we have measured $A_z(\theta)$, $S(\theta)$, and $\Delta S(\theta)/S(\theta)$ for inelastic scattering to the 1.84-MeV 2_1^+ state of ^{88}Sr at three resonances: those at 7.00 MeV ($\frac{5}{2}^+$), 7.08 MeV ($\frac{3}{2}^+$), and 7.53 MeV ($\frac{3}{2}^+$) incident proton energy. The differential cross sections had been measured previously[4] for the two lower-energy resonances, and we measured that for the 7.53-MeV resonance. Our measurements were performed using the polarized proton beams[5] of the tandem Van de Graaff accelerators at both Rutgers Uni-