[8]R. F. Blunt, J. Chem. Phys. 44, 2317 (1966).
[9]J. Ferguson, H. J. Guggenheim, and D. L. Wood, J. Chem. Phys. 40, 822 (1964).
[10]Y. Tanabe and S. Sugano, J. Phys. Soc. Jap. 9, 753 (1954).
[11]A. D. Liehr, J. Phys. Chem. 67, 1314 (1963).
[12]W. E. Vehse and W. A. Sibley, Phys. Rev. B (to be published).
[13]W. A. Sibley, S. I. Yun, and W. E. Vehse, to be published.
[14]W. A. Sibley and O. E. Facey, Phys. Rev. 174, 1076 (1968).
[15]M. Ikeya and N. Itoh, J. Phys. Soc. Jap. 29, 1295 (1970).
[16]M. Ikeya, Phys. Status Solidi (b) 51, 407 (1972).
[17]S. C. Jain and K. Lal, in *Non-Metallic Crystals*, edited by S. C. Jain and L. T. Chadderton (Gordon and Breach, New York, 1969), p. 219.

New Domain-Wall Configuration for Magnetic Bubbles

A. Rosencwaig, W. J. Tabor, and T. J. Nelson
Bell Laboratories, Murray Hill, New Jersey 07974
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A theory based on the hypothesis that the perimeter wall of a magnetic bubble may be composed of alternate Bloch and Néel segments is shown to account quantitatively for the anomalous dependence of diameter on bias field of the recently observed "hard" bubbles in garnet films.

Recently a new class of cylindrical magnetic domains or bubbles has been observed.[1] These new bubbles, designated as hard bubbles, have static and dynamic properties substantially different from those of normal bubbles. Their most anomalous static property is their much larger range of stability in both applied-field and diameter variations. Furthermore any material that allows these hard bubbles also exhibits a range of intermediate bubbles that collapse at fields and diameters intermediate between those of the hard and normal bubbles.

In this Letter we wish to show how the static properties of these hard and intermediate bubbles can be quantitatively explained by assuming the presence of Néel segments in the Bloch wall that forms the perimeter of the bubbles.[2,3]

We therefore consider a hard bubble as having a wall as shown in Fig. 1(a). Here the magnetization is up (+) within the bubble and down (−) outside. The wall itself is composed of a set of Bloch segments of opposing polarity. These Bloch segments are separated from one another by 180° walls which are essentially Néel segments. In Fig. 1(b) we show the central spins through three Bloch segments separated by two Néel segments. We define δ as the width of the Bloch wall, y as the length of a Bloch segment, and x as the length of a Néel segment.

As long as the sense of rotation of the spins around the bubble perimeter is maintained as either clockwise or counterclockwise, then this segmented configuration is frozen in; that is, it is stable against small spin perturbations. If we consider that both the Bloch and Néel segments are walls of uniform rotation,[4] then the wall energy density (energy per unit surface area of the perimeter wall) stored in each segment can be written as

$$\sigma_B = \pi^2 A/\delta + \tfrac{1}{2}K\delta,$$
$$\sigma_N = \pi^2 A/\delta + \tfrac{1}{2}K\delta + \pi^2 A\delta/2x^2 + \tfrac{1}{2}\pi M^2\delta. \qquad (1)$$

Here A is the exchange constant, K the uniaxial anisotropy (with $\frac{1}{2}K$ being the effective anisotropy for spin rotation along the dimension δ),

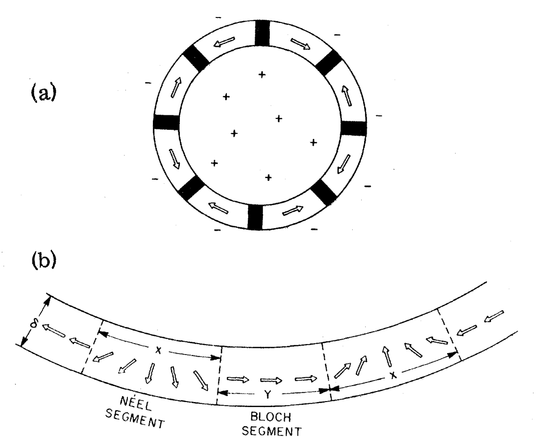

FIG. 1. (a) Top-view representation of a cylindrical magnetic bubble showing the perimeter Bloch wall with Néel segments (dark regions). (b) Representation of the spins in three Bloch wall segments separated by two Néel segments.

and M is the magnetization. The Bloch energy density σ_B is the usual one where we have ignored any small demagnetizing terms. The first two terms in the Néel energy density σ_N are the exchange and anisotropy energy densities arising from the spin rotation across the width δ. The exchange energy density arising from the spin rotation across the length x can be shown to be given by the third term. Note that there is no anisotropy energy arising from this spin rotation. The last term in σ_N is an approximate demagnetizing energy density and is derived by considering both magnetization components, parallel and perpendicular to the perimeter wall, in the Néel segment.

The total wall energy is now given by

$$E=\sigma_B(2nyh)+\sigma_N(2nxh), \tag{2}$$

where $2n$ is the total number of Néel (and Bloch) segments present and h is the bubble height. Since $2n(x+y)=\pi d$, where d is the bubble diameter, Eq. (2) can be rewritten as

$$E=(\pi^2A/\delta+\tfrac{1}{2}K\delta)\pi dh+(\pi^2A/2x+\tfrac{1}{2}\pi M^2x)2nh\delta. \tag{3}$$

To find the configuration that will have the minimum energy we minimize the above expression with respect to both δ and x obtain

$$x_0=\pi(A/\pi M^2)^{1/2} \tag{4}$$

and

$$\delta=\pi A\left[\tfrac{1}{2}K+\frac{n}{d}\left(\frac{\pi A}{x}+M^2x\right)\right]^{-1/2}. \tag{5}$$

The Bloch wall width δ is not only a function of the material parameters A and K but also of the bubble parameters n, d, and x. The width δ decreases if n increases, if d decreases, or if x differs from the equilibrium value x_0. For $n=0$ or d large we find that δ and E are the usual expressions that one would obtain for a simple Bloch wall.[4]

As long as the total bubble perimeter $\pi d>2nx_0$, then the length of each Néel segment in this noncompression region is $x_{nc}=x_0$, the equilibrium length. The Bloch wall width[5] and total wall energy in this region are then given by

$$\delta_{nc}=\pi A[\tfrac{1}{2}K+2n(\pi M^2A)^{1/2}/d]^{-1/2} \tag{6}$$

and

$$E_{nc}=(\pi^2A/\delta_{nc}+\tfrac{1}{2}K\delta_{nc})\pi dh+2nh\delta_{nc}[\pi(\pi M^2A)^{1/2}]. \tag{7}$$

For any given n, E_{nc} decreases to zero as $d\to0$. The dimensionless derivative of this energy is

$$R_{nc}=(4\pi^2M^2h^2)^{-1}\partial E_{nc}/\partial d=(4\pi^2M^2h^2)^{-1}(\pi^2A/\delta_{nc}+\tfrac{1}{2}K\delta_{nc})\pi h, \tag{8}$$

where we have made use of the fact that $\partial E_{nc}/\partial\delta=0$. We find that R_{nc} is essentially constant for $d/h>1$ and then starts to increase as $d/h\to0$. A normal bubble ($2n=0$) has E decreasing as a straight line to zero. Thus its R is a constant ($=l/h$) at any value of d,[6] where the material length $l=\sigma_w/4\pi M^2$, with σ_w the Bloch wall energy density.

It is clear that the bubble cannot remain forever in the noncompression region as $d\to0$, since it will eventually reach the point where $\pi d=2nx_0$. At this point, the Néel segments will come into contact with one another and any further decrease in d will tend to compress these segments. In this region the Bloch segment length $y=0$. We will assume a uniform compression and thus set the Néel segment length in this compression region to $x_c=\pi d/2n$. The Bloch wall width and total wall energy are now given by

$$\delta_c=\pi A(\tfrac{1}{2}K+2n^2A/d^2+\tfrac{1}{2}\pi M^2)^{-1/2} \tag{9}$$

and

$$E_c=\left(\frac{\pi^2A}{\delta_c}+\tfrac{1}{2}K\delta_c\right)\pi dh+h\delta_c\left(\frac{2\pi n^2A}{d}+\tfrac{1}{2}\pi^2M^2d\right). \tag{10}$$

Since $\delta_c\propto d$ for small d, E_c will not go to zero but will approach an asymptotic value of $2^{3/2}\pi^2nAh$ as $d\to0$. The dimensionless derivative in this region is found to be

$$R_c=\frac{1}{4\pi^2M^2h^2}\left[\left(\frac{\pi^2A}{\delta_c}+\tfrac{1}{2}K\delta_c\right)\pi h-2\pi n^2Ah\frac{\delta_c}{d^2}+\tfrac{1}{2}\pi^2M^2h\delta_c\right]. \tag{11}$$

One finds that $R_c\to0$ as $d\to0$.

R can be determined experimentally by measuring the bubble diameter d for various values of the ap-

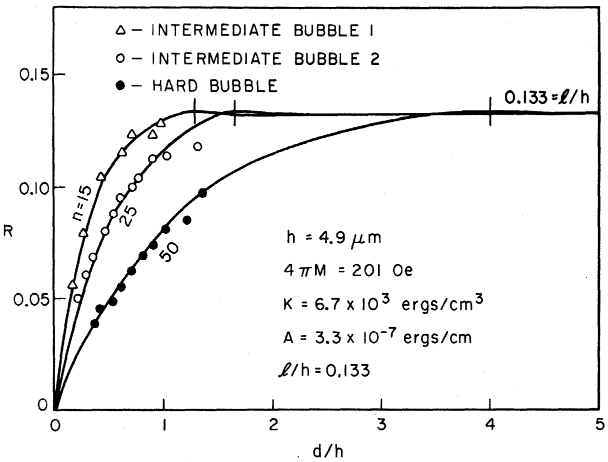

FIG. 2. Theoretical curves for R and experimental R values for both intermediate and hard bubbles in an (ErGdGa) iron-garnet film. The region to the left of the short vertical line is the compression region, while that to the right is the noncompression region. The value of A shown takes into account the presence of Ga.

plied bias field H, and then calculating R by using the equilibrium relation

$$R+\frac{d}{h}\frac{H}{4\pi M}-F\left(\frac{d}{h}\right)=0,$$

where the second term is the dipole energy term and $F(d/h)$ is Thiele's force function.[7] For a normal bubble R has been found to be a constant $(=l/h)$ for any d, while the intermediate to hard bubbles show an R that decreases as d decreases.

Measurements of R as a function of d/h have been made in various garnet films for both hard and intermediate bubbles.[8] In Fig. 2 we show theoretical curves for R and experimental R values for both a hard bubble and two intermediate bubbles in an (ErGdGa) iron-garnet film. In making the theoretical curves we have joined R_c and R_{nc} where they intersect (position of short vertical line). One can readily show that at this point the energies E_c and E_{nc} are also equal. We note from Fig. 2 that in the noncompression region where $y>0$, even the hard bubble will appear as essentially a normal bubble since the calculated R here is approximately equal to the l/h of the normal bubble. Good agreement with the experimental data is obtained if one considers an $n\sim 15$ for the weaker intermediate bubble, $n\sim 25$ for the stronger intermediate bubble, and $n\sim 50$ for the hard bubble. Thus this segmented wall model appears to account adequately for the anomalous dependence of bubble diameter on bias field of these hard and intermediate bubbles.

[1]W. J. Tabor, A. H. Bobeck, G. P. Vella-Coleiro, and A. Rosencwaig, Bell Syst. Tech. J. 51, 1427 (1972).

[2]P. J. Grundy, D. C. Hothersall, G. A. Jones, B. K. Middleton, and R. S. Tebble, Phys. Status Solidi (a) 9, 79 (1972), have also reported the presence of bubbles with Néel segments in cobalt films.

[3]A similar model has been proposed by A. P. Malozemoff, Appl. Phys. Lett. 21, 149 (1972). However, this model is incomplete in that no account is taken of the crucial effect that these Néel segments have on the wall width.

[4]Various treatments of domain walls with uniform rotation can be found in the literature such as S. Middelhoek, *Ferromagnetic Domains in Thin Ni-Fe Films* (Drukkerij Wed. G. Van Svest N. V., Amsterdam, The Netherlands, 1961), Chap. 4.

[5]Only one wall width is considered even though the Néel and Bloch segments may have different widths in the noncompression region. If different widths are present, the wall width has an undulation which gives rise to an added exchange energy. When the Néel segments are close to each other, this new exchange energy will tend to equalize the Néel and Bloch wall widths. Our approximation of equal widths is then reasonable. The approximation is less satisfactory when the Néel segments are far apart, but now such a discrepancy will have only a negligible effect on R_{nc}.

[6]Experimental data by F. B. Hagedorn, W. J. Tabor, J. E. Geusic, H. J. Levinstein, S. J. Licht, and L. K. Shick, Appl. Phys. Lett. 19, 95 (1971), indicates that $R=l/h$ for a normal bubble at least until the point of collapse.

[7]A. A. Thiele, Bell Syst. Tech. J. 50, 725 (1971).

[8]The dependence of the experimental derivative R on H is a more sensitive test of the model than the raw data for d versus H. This is so since R is a constant for all normal bubbles, thereby clearly highlighting the anomalous dependence of R on H for hard bubbles.