sign). Using this value of $p$ to determine the analyzing power $\alpha$ of the polarimeter, one can then measure $R$ and $A$. In particular, say,

$$R_{\text{expt}} = \frac{A_{UD}}{p_x \alpha} \pm R_{\text{expt}} \left[ \left( \frac{\Delta A_{UD}}{A_{UD}} \right)^2 + \left( \frac{\Delta p_x}{p_x} \right)^2 + \left( \frac{\Delta \alpha}{\alpha} \right)^2 \right]^{1/2},$$

and since $A_{UD} \to 0$ with $\Delta A_{UD}/A_{UD} \approx 1$, the 5% uncertainty in $\Delta p_x/p_x$ and $\Delta\alpha/\alpha$ contributes only a small amount to the first determination of $R_{\text{expt}}$ and $A_{\text{expt}}$. Using $P^2 = 1 - R^2 - A^2$ determines $P$ with only a small uncertainty which, in turn, can be used to determine the original value of $p = 0.9$ with a smaller uncertainty than before. A second iteration of the calculation then arrives at a precise determination of $P$.

It is a pleasure to acknowledge helpful discussions with John L. Gammel about quadratic relations.

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# Failure of the Coexistence Model to Account for Observed Two-Neutron Pickup Cross Sections*

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We have studied the $(p,t)$ reaction on $^{18}$O and $^{42}$Ca at $E_p = 41.7$ MeV. The observed yields to the $0^+$, $T=0$ states in $^{16}$O and $^{40}$Ca are at variance with distorted-wave Born-approximation predictions using the coexistence model to describe the initial and final states of the targets. By use of wave functions which employ a more extensive set of configurations, satisfactory agreement with the observed yields is achieved for the reaction $^{18}\text{O}(p,t)^{16}\text{O}$.

The inclusion[1-4] of deformed multiparticle-multihole configurations into the set used to generate the low-lying states near doubly magic nuclei has led to spectacularly successful results. This *Ansatz,* often termed the coexistence model, produces acceptable energy spectra and gives good agreement with observed electric quadrupole transition rates. For example, in $^{40}$Ca the calculation of Gerace and Green[3,4] reproduces the twenty or so levels below 7 MeV and properly accounts for more than twenty $B(E2)$ values.[5] In $^{16}$O there is not such a large body of data but the model again seems quite successful.

In this Letter we wish to point out that this coexistence model fails to describe the results of two-neutron pickup experiments on the "doubly magic"-plus-two-neutron targets $^{18}$O and $^{42}$Ca. In the coexistence model these targets are described as superpositions of spherical two-particle and deformed four-particle, two-hole (4p-2h) states.

We will confine detailed discussion in this Letter to the two-neutron pickup yields to $J=0^+$, $T=0$ final states in $^{16}$O and $^{40}$Ca. Although both the reactions $^{18}$O($p,t$)$^{16}$O and $^{42}$Ca($p,t$)$^{40}$Ca have been studied previously by other researchers, it is felt that either the bombarding energy used was too low[6,7] or the energy resolution insufficient[8,9] to determine a reliable yield to all the levels of interest.

The 41.7-MeV proton beam of the Princeton University azimuthally-varying-field cyclotron was used to bombard an isotopically enriched (99%) $^{18}$O confined in a gas cell. The target was operated at 100 mm pressure with 1-mg/cm$^2$ Kapton windows. A silicon surface-barrier solid-state detector telescope was used to determine the scattered-particle type and kinetic energy. The energy resolution of the detected tritons was 60 keV, principally due to straggling in the windows and kinematic broadening due to the extended target. This resolution was not adequate to separate completely the relatively small yield to the 6.05-MeV $0^+$ level from that to the strongly excited $3^-$ state at 6.13 MeV. A peak-fitting routine AUTOFIT[10] was used to extract the yields and the degree of success may be judged by the characteristic $L=0$ angular distribution extracted for the 6.05-MeV level as is shown in Fig. 1. Altogether six $L=0$ yields were extracted corresponding to states in $^{16}$O at 0.00, 6.05, 11.26, 12.05, 16.33, and 22.72 MeV. The lowest four states, whose angular distribution are shown as points in Fig. 1, are certainly $T=0$. The yield observed corresponding to 22.72 MeV is to the lowest $0^+$, $T=2$ level. The isospin assignment to the 16.33-MeV level is as yet uncertain and hence will not be included in our discussion. The relative cross sections of the lowest four $0^+$, $T=0$ levels are listed in the last column of Table I as determined by integrating the observed cross section over the angular range shown in Fig. 1.

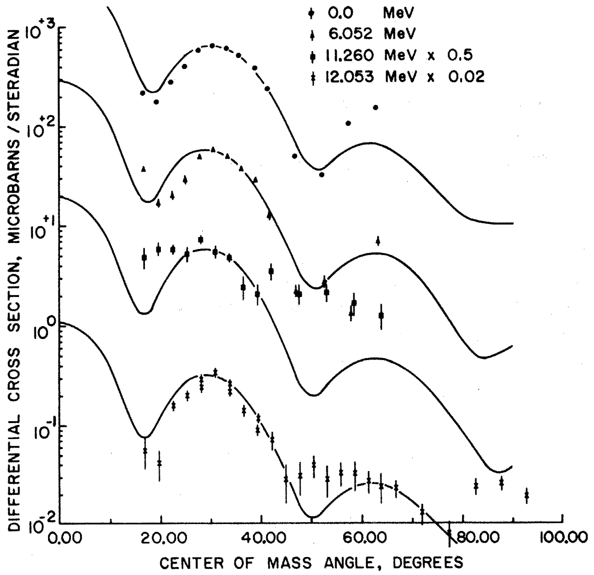

FIG. 1. The differential cross sections measured for the reaction $^{18}$O($p,t$)$^{16}$O at $E_p=41.7$ MeV to $J=0, T=0$ final states. The solid line shown with each set of data points is the angular distribution calculated using the distorted-wave Born approximation with form factors obtained from the results of Ref. 12.

The $^{42}$Ca($p,t$)$^{40}$Ca measurement was carried out using a 41.7-MeV proton beam to bombard an approximately 200-$\mu$g/cm$^2$, isotopically enriched (95%) $^{42}$Ca target. The target was prepared by vacuum evaporation onto a thin Formvar backing and transferred *in vacuo* into the scattering chamber. The energy resolution for the scattered tritons was of the order of 32 keV with several sources contributing equally. In this experiment four $L=0$ angular distributions are observed that are attributable to the reaction $^{42}$Ca($p,t$)$^{40}$Ca. They correspond to levels in $^{40}$Ca at 0.000, 3.357, 9.386, and 11.970 MeV. The lowest two levels are $T=0$ while the latter two are $T=1$ and $T=2$, respectively. There are other known $T=0, 0^+$ states in $^{40}$Ca at 5.21 and 7.30 MeV. The 5.20-MeV level is observed to have such small yield that no angular distribution could be obtained for it. Figure 2 shows the observed angular distributions, and the last column of Table I lists the relative integrated cross sections.

The third column of Table I shows the predicted amplitudes of various theories for the pickup of pairs of neutrons coupled to zero, leading to specific final states. The first two blocks present the results for $J=0$ pickup on $^{18}$O and $^{42}$Ca obtained with the wave functions generated[1-4] within the framework of the "coexistence" model. The pickup from the deformed components of the wave functions is calculated by projecting out the amplitude of pairs of neutrons coupled to $0^+$. Because of the restricted basis employed in the coexistence model, interference between particle and hole pairs is necessarily destructive in transitions to excited $0^+$ states. Using standard[11] distorted-wave Born-approximation procedures and generating the form factors by binding each of the picked up neutrons at one-half the separation energy, we obtain the relative cross sections shown in column 4 of Table I. The agree-

TABLE I. Comparison of observed relative $(p,t)$, $L=0$ yields to the predictions of the coexistence and the ZBM models.

| Physical energy level | Model energy level (dominant configuration) | Pickup amplitudes for $[nj^2]^0$ | | | Predicted rel. cross section | Observed Experimental rel. cross section |
|---|---|---|---|---|---|---|
| $^{18}O\ (p,t)\ ^{16}O$ | | | | | | |
| | ref. a | $[1d^2_{5/2}]$ | $[2s^2_{1/2}]$ | $[1p^2_{1/2}]$ | | |
| 0.00 MeV | 0.00 MeV (0p-0h) | -0.666 | -0.445 | .250 | 100. | 100. |
| 6.05 MeV | 6.07 MeV (4p-4h) | 0.129 | 0.082 | .654 | 0.76 | 7.74±.4 |
| 11.26 MeV[d] | 11.26 MeV (2p-2h) | 0.108 | .057 | .155 | 1.34 | 2.61±.4 |
| $^{42}Ca\ (p,t)\ ^{40}Ca$ | | | | | | |
| | ref. b | $[1f^2_{7/2}]^0$ | $[2p^2_{3/2}]^0$ | $[1d^2_{3/2}]^0$ | | |
| 0.00 MeV | 0.00 MeV (0p-0h) | -0.801 | -0.187 | 0.169 | 100 | 100 |
| 3.35 MeV | 3.50 MeV (4p-4h) | -0.140 | -0.025 | -0.399 | 2.7 | 21.3±1.0 |
| 5.20 MeV | 5.10 MeV (8p-8h) | - | - | - | | <1.0± |
| 7.30 MeV | 7.20 MeV (2p-2h) | 0.183 | 0.016 | -0.190 | 5.1 | <0.7± |
| $^{18}O\ (p,t)\ ^{16}O$ | | | | | | |
| | ref. c | $[1d^2_{5/2}]^0$ | $[2s^2_{1/2}]^0$ | $[1p_{1/2}]^0$ | | |
| 0.00 MeV | 0.00 MeV (0p-0h) | -0.858 | -.396 | .314 | 100 | 100 |
| 6.05 MeV | 6.194 MeV (4p-4h) | -0.094 | -0.015 | .463 | 14.4 | 7.74±.4 |
| 11.26 MeV[d] | 10.679 MeV (2p-2h,4p-4h) | -0.008 | -0.051 | .126 | 1.53 | 2.61±.4 |
| 12.05 MeV | 12.480 MeV (2p-2h,4p-4h) | 0.012 | 0.022 | -0.192 | 1.09 | 2.12±.3 |

[a] See Refs. 1 and 2.

[b] See Refs. 3 and 4.

[c] See Ref. 12.

[d] The 11.26-MeV level in $^{16}O$ is believed to be $0^+$ and has been chosen by the authors of Refs. 1 and 2 as the $0^+$ "2p-2h state." It is possible that this broad state is not $0^+$; however, even if it is not, the conclusions reached in the present article will stand.

ment with experiment is very poor particularly with respect to the yield to the first excited $0^+$ states. Furthermore, the calculated angular distributions for the excited $0^+$ states obtained using these wave functions are in poor agreement with the data because of the destructive interference of particle and hole pickup.

The last set of predictions shown at the bottom of Table I are those of Zuker, Buck, and McGrory[12] (ZBM) for $^{18}O(p,t)^{16}O$. They employ a much less restrictive base than is employed in the coexistence model as they close the $p_{3/2}$ shell at $N=Z=6$ and allow the nucleons above this number to be in the $1p_{1/2}$, $1d_{5/2}$, or $s_{1/2}$ orbits. Hence configurations such as $[{d_{5/2}}^{4,T=2}{p_{1/2}}^2]^{T=1,0^+}$ and $[{d_{5/2}}^6{p_{1/2}}^0]^{T=1,0^+}$ become possible for $^{18}O$. In fact the components of such configuration appear with sizable amplitude ($>0.1$) in the $^{18}O$ ground state. Employing this larger basis in calculating the pickup amplitude does not require destructive interference between particle and hole amplitudes, and in fact these amplitudes are coherent for the four lowest $0^+$ states. The angular distributions generated with the ZBM wave functions are shown in Fig. 1 as solid lines and are readily seen to reproduce the data quite well. The prediction of the relative magnitudes is shown in the next to last column of Table I, and contrary to the results obtained with the coexistence model the relative magnitudes are well reproduced. This extended shell-model approach has been started[13] in the $^{40}Ca$ region, but final results are not yet available. It should be most interesting to compare the results obtained here to these forthcoming predictions.

A few comments seem in order at this point. It is not yet clear just which of the additional degrees of freedom make the ZBM calculation superior to the coexistence-model calculation with regard to two-nucleon pickup. As mentioned above, in addition to the configurations employed in the coexistence model, the ZBM calculation

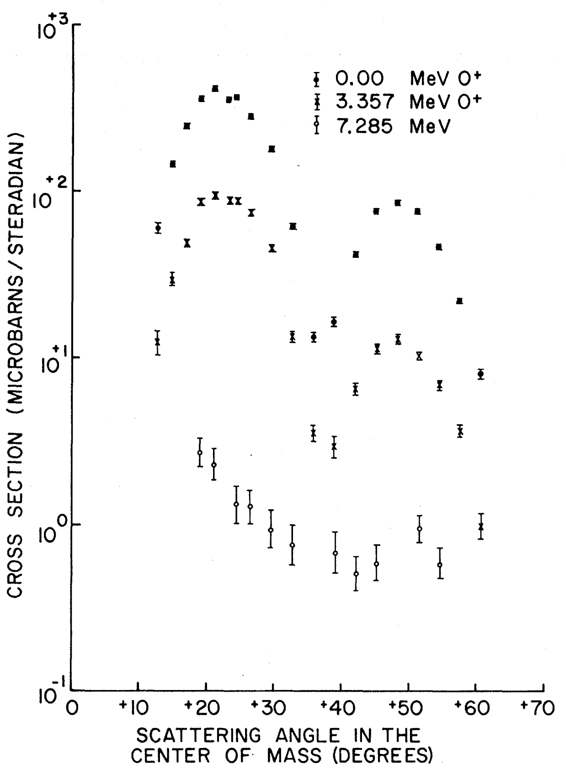

FIG. 2. The differential cross section measured for the reaction $^{42}$Ca($p,t$)$^{40}$Ca at $E_p=41.7$ MeV to $J=0, T=0$ final states. The yield to $0^+$ state at 5.212 MeV is not shown as only lower limits ($<1\ \mu b/sr$) could be extracted.

has large $[4p^{T=2}, 2h^{T=1}]^{T=1}$ and $[6p^{T=1}, 4h^{T=0}]^{T=1}$ amplitudes. These should overlap well with $[2p\text{-}2h]^{T_p=T_h=1}$ and $[4p^{T=0}, 4h^{T=0}]$ components in the final states. The complexity of the ZBM wave functions (e.g., 32 components in the $O^{18}$ ground state) makes direct comparison with the coexistence model difficult; however, we plan to investigate this matter further. It is tempting to ascribe the difference in the two results to the presence of higher isospin couplings in the ZBM calculation as these configurations may be added to the coexistence model without their having a strong effect on the $E2$ transition rates which are so well described by the physically attractive coexistence model.

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