Off–Energy-Shell Effects in Deuteron-Induced Deuteron Breakup

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An improved phenomenological fit to the experimental data on the reaction $d+d \to d+n+p$ has been achieved based on one-nucleon-exchange quasifree nucleon-deuteron scattering by including a large off–energy-shell correction in the nucleon-deuteron scattering amplitudes.

The detailed studies[1-7] of neutron–charged-particle and charged-particle–charged-particle coincidence spectra from the reaction $d+d \to d+p+n$ at low energies demonstrated that both d-n and d-p coincidence spectra are dominated by a broad peak associated with nucleon-deuteron quasifree scattering. For some angular settings of the detectors the enhancement corresponding to low relative n-p energies have been observed.[5,8] A previous attempt[6] to understand these spectra based on one-nucleon-exchange quasifree N-d scattering met with partial success. Four graphs were considered with either a neutron or proton in the target or the beam deuteron acting as the spectator particle. The limitations of this model of one-nucleon-exchange quasifree N-d scattering were best revealed by the fact that the calculated spectral shapes were much broader than the measured ones and that the normalization factor (of approximately 0.1) was somewhat dependent on the bombarding energy and the detector angles.

In the present work these defects have been overcome by including a large off–energy-shell correction in the nucleon-deuteron scattering amplitudes which are parametrized by a separable form factor for the initial and final relative nucleon-deuteron momentum, so

$$t_{Nd}(E, P_f, P_i) = (P_f^2+\beta^2)^{-2}\tau_{Nd}(E)(P_i^2+\beta^2)^{-2}. \quad (1)$$

The choice of form factor

$$v(P^2) = (P^2+\beta^2)^{-2} \quad (2)$$

is dictated largely by experience with separable potentials. An exponent of 1 would correspond to a Yamaguchi potential; the exponent 2, which is used, corresponds to a potential which is not singular at short range, unlike the Yamaguchi potential. It is not intended, however, to relate nucleon-deuteron scattering to a potential but merely to parametrize its off-shell dependence. The choice of range parameter β was dictated by the proposition that the elastic and inelastic scattering occurs with a minimum momentum transfer, i.e., the reactions are extremely peripheral and take place at a deuteron-deuteron separation which is dictated by the dimensions of the deuteron itself. For this reason the tentative choice is made that the interaction range determining the off-shell corrections to the nucleon-deuteron amplitude is the deuteron radius and β is set equal to α (where $\alpha^2/m = 2.2$ MeV). This choice provides a good fit to the data, and no effort to improve the fit by varying this parameter has been made.

Figure 1 shows the measured d-p coincidence spectrum measured at $E_d = 12.5$ MeV with $\theta_d = 20°$ and $\theta_p = 18°$ as the projection on the deuteron energy axis. The solid curve marked by *IA* is the normalized prediction for one-nucleon-exchange quasifree N-d scattering with on–energy-shell N-d amplitudes with a normalization factor of 0.095. If we include off–energy-shell corrections of the form

$$v(P^2) = (P^2+\beta^2)^{-1}, \quad (3)$$

the calculated spectral shape is narrower and the normalization factor is increased to 0.3 (dashed curve marked Y in Fig. 1). With the form factor of Eq. (2) the impressive agreement with experimental data was achieved (dash-dotted curve marked YY in Fig. 1) with a normalization factor of 1 within the uncertainties in experimental data. The agreement is not so good for the part of the spectrum corresponding to low-energy deuterons. With the inclusion of the Coulomb scattering amplitude in the two graphs where the neutron acts as a spectator particle, the shape of the spectrum remains the same, although the absolute cross section is reduced (normalization factor 1.37 for the data of Fig. 1).

The form factor of Eq. (2) satisfactorily reproduces the bombarding energy dependence of the peak cross section for p-d quasifree scattering for $\theta_d = -\theta_p = 20°$ (data are from Ref. 6, Table 1). The normalization factor is close to 1. The angular dependence of the peak cross section is re-

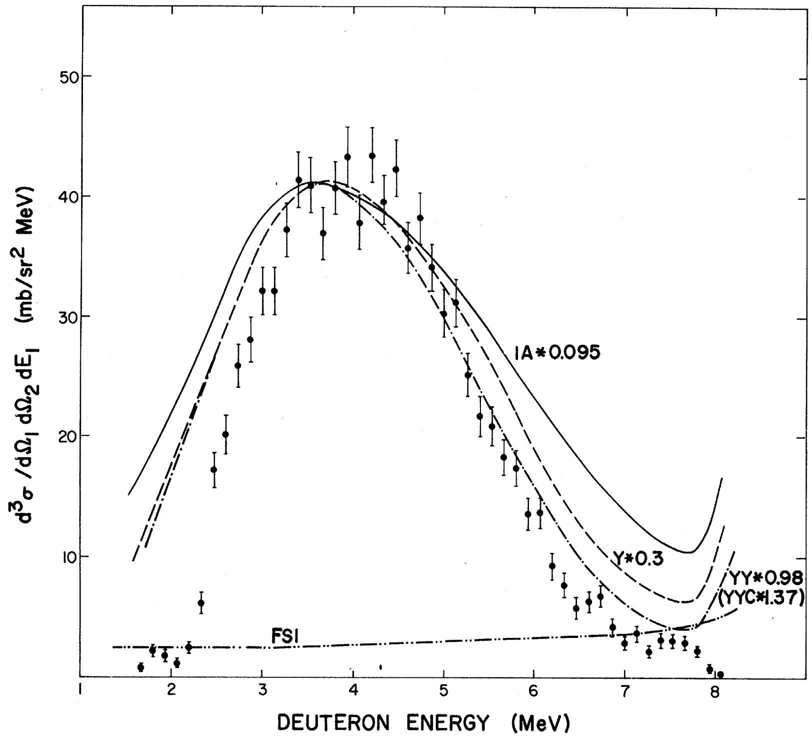

FIG. 1. Experimental d-p coincidence spectrum measured at $E_d = 12.5$ MeV with $\theta_d = 20°$ and $\theta_p = 18°$ as the projection on the deuteron energy axis. The details of the measurement are described in Ref. 6. The solid curve marked *IA* is the normalized one-nucleon-exchange quasifree N-d scattering prediction with on—energy-shell nucleon-deuteron amplitude (normalization factor is 0.095). The dashed curve marked *Y* includes an off—energy-shell correction of the form given by Eq. (3) (normalization factor is 0.3). The dashed-dotted curve marked *YY* includes the off—energy-shell correction given by Eq. (2) (normalization factor is 0.98). Angular and energy smearings are not considered and no effort to obtain best fit was made. The dash—double-dotted curve marked *FSI* represents the nucleon-deuteron final-state interaction contribution for this geometry.

produced extremely well with the normalization constant unchanged (data are from Ref. 7, Table 2). The inclusion of the Coulomb amplitude in the two graphs corresponding to p-d scattering only changes the normalization factor to 1.37. Fits to the experimental data in Figs. 2 and 3 are worse with the form factor of Eq. (3), and the normalization factor is reduced to 0.25. It should be emphasized that the point here *is not* the detailed choice of form factor or the precise value of the range parameter; the point *is* that one-nucleon-exchange quasifree N-d scattering with a *large off-shell* dependence of these amplitudes provides a detailed fit to the $d+d \rightarrow d+n+p$ spectra both in shape and in magnitude, as has been discussed in greater detail above.

It is possible to demonstrate from a simple calculation that such a rapid off-shell dependence of the N-d scattering amplitude should be expected. A Born-approximation amplitude with no recoil and employing a separable nucleon-nucleon

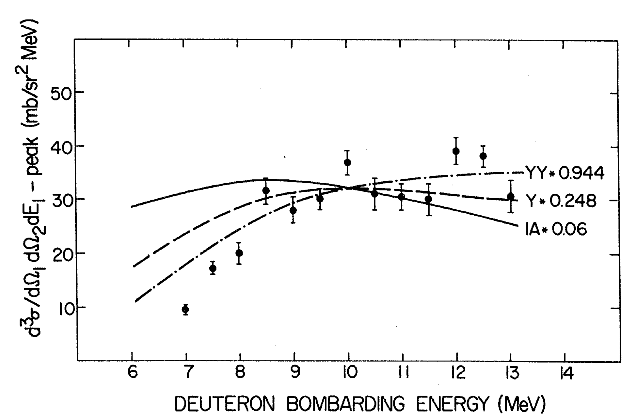

FIG. 2. Peak cross section for the d-p coincidence spectra from the reaction $d+d \rightarrow d+p+n$ for $\theta_d = -\theta_p = 20°$. Data are from Ref. 6, Table 1. The prediction calculated with on—energy-shell nucleon-deuteron amplitude (*IA*) and predictions including off—energy-shell corrections of Eq. (3) (curve *Y*) and that of Eq. (2) (curve *YY*) are shown together with the normalization factors.

potential is

$$t_{Nd}(E, P_f, P_i) = \int \psi_f^*(r_1, r_2) V(r_1 - r_2) V(r_1' - r_2') \psi_i(r_1', r_2')\, d^3r_1\, d^3r_2\, d^3r_1'\, d^3r_2'. \tag{4}$$

The form factor carrying the dependence of the scattering amplitude on the relative momentum P is

$$v(P^2) = \int \exp(-i\vec{p}\cdot\vec{r}_1)\psi_d(r_2) V(r_1 - r_2)\, d^3r_1\, d^3r_2, \tag{5}$$

which reduces for a very short-range NN potential to

$$v(P^2) \sim \int \frac{\sin Pr}{Pr} \psi_d(r) r^2\, dr. \tag{6}$$

If this were evaluated directly, it would give a dominant term $(P^2+\alpha^2)^{-1}$ which demonstrates the desired relation for the range parameter, $\beta=\alpha$, but has the Yamaguchi form arising from an emphasis of the short range in the N-d interaction. Judging from the empirical success of the form factor $(P^2+\alpha^2)^{-2}$, the short-range part of the deuteron must be excised and just the periphery of the deuteron emphasized in the nucleon-deuteron interaction. This could be an exclusion-principle effect operative only in the dominant quartet scattering amplitudes. The excision of the center of the deuteron wave function has been used as an empirical coordinate-space cutoff on calculated matrix elements,[9,10] being dependent on both bombarding-energy and detector-angle settings. The choice $(P^2+\alpha^2)^{-2}$ corresponds to a linear cutoff in the integral for $v(P^2)$.

It is necessary to point out that off-shell N-d amplitudes have been parametrized before in the triton wave function used by Barbour and Phillips[11] as the first stage in their photodisintegration calculations. Their wave function implies a rapid off-shell dependence of the triton pole term in the N-d scattering amplitude with a form factor falling off like P^{-6} which is still less than the P^{-8} dependence expected for the three-nucleon form factor.[12] The difference between the present work and that of Barbour and Phillips is principally in choosing the range parameter α^2 equal to the deuteron binding energy rather than their ν^2 equal to the triton binding energy. The result is that the form factors become important at a much smaller momentum. The present calculation is not necessarily in conflict with the Barbour-Phillips triton wave function since there is no reason to suppose that the nucleon-deuteron amplitude is dominated by the triton pole term, and nonpole terms could have different off-shell dependence. It is clear that the nucleon-deuteron interaction changes its peripheral nature and becomes a close, strong interaction in the compact triton.

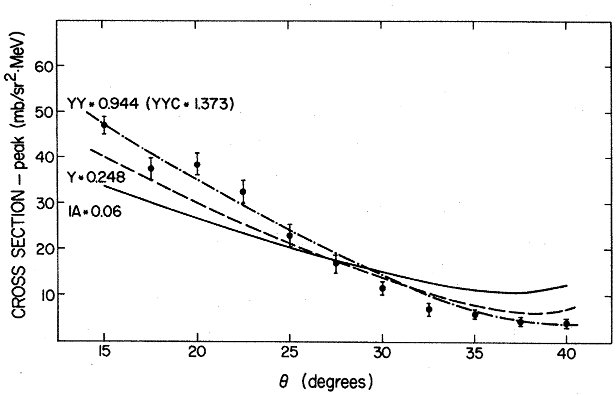

FIG. 3. Peak cross section for the d-p coincidence spectra from the reaction $d+d\to d+p+n$ for $E_d=12.5$ MeV and symmetric geometry, $\theta_d=-\theta_p=\theta$. Data are from Ref. 7, Table 2. For an explanation of the curves see text and captions for Figs. 1 and 2.

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