Second-Sound Velocity and Superfluid Density Near the Tricritical Point in He^3-He^4 Mixtures

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(Received 7 August 1972)

Second-sound velocity measurements were made in He II near the tricritical point (T_t, x_t). The corresponding superfluid density along the coexistence curve can be described by $\rho_s/\rho = 2.03\epsilon_t$ ($\epsilon_t \equiv 1 - T/T_t$). This agrees with recent theories. At constant concentration x and near the superfluid transition temperature T_λ, the results are consistent with $\rho_s/\rho \sim k_\lambda \epsilon_\lambda^{2/3}$ ($\epsilon_\lambda \equiv 1 - T/T_\lambda$) which is expected on the basis of universality arguments. The amplitude k_λ becomes small for small $x_t - x$, but does not appear to vanish at x_t.

The behavior of a system near a tricritical point may be described in terms of two order parameters and their conjugate fields.[1,2] In the case of He^3-He^4 mixtures, one convenient order parameter is the departure $1 - x/x_t$ of the molar He^3 concentration x from its value x_t at the tricritical point. The other, describing the superfluid ordering, is related to the density $|\Psi|^2$ of He^4 atoms in the zero-momentum state. Several recent experiments[3-6] have yielded considerable information about the singularities associated with the concentration ordering. Although none of these extend sufficiently close to the tricritical point to yield the asymptotic behavior in the form of exponents with high accuracy, all of the results are consistent with recent theories.[1,2,7,8] There have been essentially no measurements, however, which yield information about the critical behavior associated with the superfluid ordering, and $|\Psi|^2$ is not even approximately known from experiment. Although it has not been possible so far to devise an experimental method for the direct measurements of $|\Psi|^2$, a closely related[9] parameter is the superfluid density ρ_s. We wish to report results for ρ_s/ρ which were derived from measurements of the second-sound velocity u_2 near the tricritical point. The measurements were made in the superfluid phase along the coexistence curve, and along three lines of constant concentration between the phase-separation temperature T_σ and the superfluid transition temperature T_λ. Along the phase-separation curve they are consistent with $\rho_s/\rho = k_\sigma \epsilon_t$, where $\epsilon_t \equiv 1 - T/T_t$ (T_t is the tricritical temperature). This behavior agrees with recent predictions by Riedel and Wegner.[7,8] Along lines of constant x and near T_λ we find, consistent with the observed behavior of the pure system[10] and the concept of universality,[11] that the data permit $\rho_s/\rho \sim \epsilon_\lambda^{2/3}$ ($\epsilon_\lambda \equiv 1 - T/T_\lambda$). The amplitude k_λ of ρ_s/ρ at T_λ decreases rapidly as $1 - x/x_t$ becomes small, but k_λ seems to remain finite at x_t.

Much of the apparatus used in this work was the same as that employed for measurements on pure He^4.[10] However, the large effect of the gravitational field near the tricritical point, the small effective thermal conductivity of He^3-He^4 mixtures,[12] and the large concentration gradients which result in the superfluid even from rather small temperature gradients made it necessary to redesign the sample cell and second-sound cavity. A schematic diagram is shown in the insert of Fig. 1. Most of the sample (about 2 cm^3 of liquid) is contained in volume A, where the liquid depth is about 0.1 cm. The fluid used for the second-sound measurements, however, is contained in the resonant cavity B. The top of volume B is level with the bottom of volume A. B has a height of 0.3 cm and a diameter of 0.6 cm. When a mixture of average concentration x_t is employed, the He-II–He-I interface is in volume A, and the resonant cavity B contains only superfluid. The measured velocity, therefore, is that appropriate to the coexistence curve (except for gravity effects). All solid walls adjacent to the fluid were made of copper in order to assure a uniform background sample temperature. The measurements were made at frequencies between 8 and 20 kHz, and the quality factor Q of the resonances was between 300 and 1000 over the range of the experiment.

When second sound was generated with the superleak condenser transducers,[10] there was power dissipation in the sample proportional to the square of the voltage used to drive the transducer. Near the tricritical point, the smallest possible voltage (0.5 V rms) which resulted in a reasonable signal was used, and we estimate the corresponding power dissipation to be 10^{-2} erg sec^{-1}. All data were corrected to zero power with the aid of the measured power dependence[13]

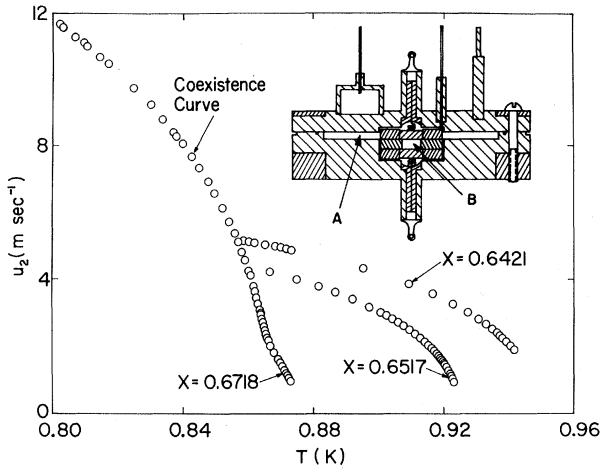

FIG. 1. Second-sound velocity as a function of temperature, and a schematic diagram of the sample cell and resonance cavity.

of u_2. Near T_t, long relaxation times were encountered,[14] and required waiting several hours after changing the temperature before making a velocity measurement. During this time the sample temperature was kept constant to $\pm 10^{-6}$ K. The final velocities are within 0.1% of the values corresponding to the equilibrium system in the gravitational field. The results for $T \gtrsim 0.8$ K are shown in Fig. 1.

In order to obtain the superfluid density from the second-sound velocity, we combined thermodynamic measurements[4-6,15] with our own data, and used an exact relation between ρ_s and u_2, which is given by linear two-fluid hydrodynamics,[16] and which may be written in the form

$$u_2^2 = \frac{\rho_s}{\rho_n}\left\{\frac{3}{xM_3}\frac{S^2T}{C_{xp}} + \frac{M_3}{M_4^2}\frac{x^3}{c}\left(\frac{\partial\Phi}{\partial x}\right)_{TP}\right\}D^{-1};$$
$$D = 1 + \frac{\rho_s}{\rho_n}\left[\frac{x}{\rho}\left(\frac{\partial\rho}{\partial x}\right)_{TP}\right]^2\left(\frac{M_3 x}{M_4 c}\right)^2. \tag{1}$$

Here c and x are the mass and molar concentration of He^3, respectively. The entropy S, specific heat C_{xp}, and chemical potential difference $\Phi = \mu_3 - \mu_4$ are per mole of solution; and M_3 and M_4 are the molar masses of He^3 and He^4.[17] In order to compare the measurements with theory, it is necessary to estimate the exponent which describes the asymptotic behavior of ρ_s/ρ from the results at nonzero ϵ_t or ϵ_λ. Even in the pure system,[10] where an independent precise experimental estimate of T_λ is available, and where measurements have been made for ϵ_λ as small as 10^{-5}, this could not be done as accurately as is desirable. The problem in that case was attributable to the existence of singular contributions to ρ_s which are of higher order than the asymptotic term. In the mixtures near the tricritical point we must also expect such contributions. Here the conclusions about the leading exponent are even less precise because in this case T_λ or T_t cannot be determined independently and must be treated as an adjustable parameter in a least-squares fit to the data. In addition, the amplitude of ρ_s near T_λ is so small that measurements have not yet been possible for $\epsilon_\lambda \lesssim 10^{-3}$. Nonetheless, we fitted the results along the coexistence curve and along the line with $x = 0.6517$ by the pure power laws

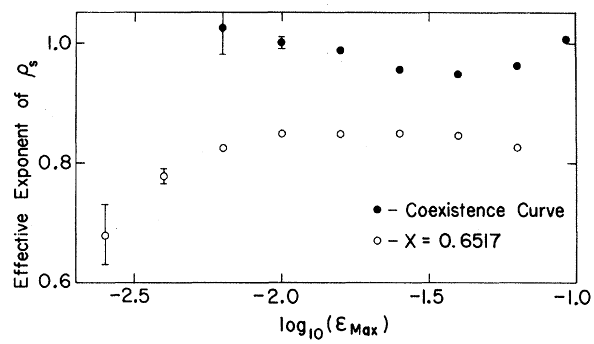

FIG. 2. Effective exponent of ρ_s obtained by fitting to a pure power law all data with $\epsilon \leq \epsilon_{max}$.

$$\rho_s/\rho = k_\sigma \epsilon_t^{\zeta_\sigma}, \quad \rho_s/\rho = k_\lambda \epsilon_\lambda^{\zeta_\lambda}, \tag{2}$$

using only data for which ϵ_t or ϵ_λ were less than some ϵ_{max}. The resulting values of ζ are shown in Fig. 2 as a function of ϵ_{max}. Where the statistical errors are sufficiently large, they are shown explicitly; but they are based upon the assumption that the functional form Eq. (2) is valid. Clearly Eq. (2) is not adequate, especially for the results near T_λ at $x = 0.6517$, since ζ_λ depends upon ϵ_{max} by more than the statistical errors. We may conclude only that the results along the coexistence curve are consistent with $\zeta_\sigma = 1$, and that at $x = 0.6517$ the value $\zeta_\lambda = \frac{2}{3}$ would be in agreement with the data. However, considerably different behavior for smaller ϵ cannot be excluded by the experiment. In particular, we cannot make any statement about the absence or presence of recently suggested[7,8] logarithmic corrections to pure power-law behavior along the coexistence curve. For the coexistence curve the parameters

$$T_t = 0.86720\ \text{K}, \quad k_\sigma = 2.034, \quad \zeta_\sigma = 1.000, \tag{3}$$

and Eq. (2) give a good representation of the data

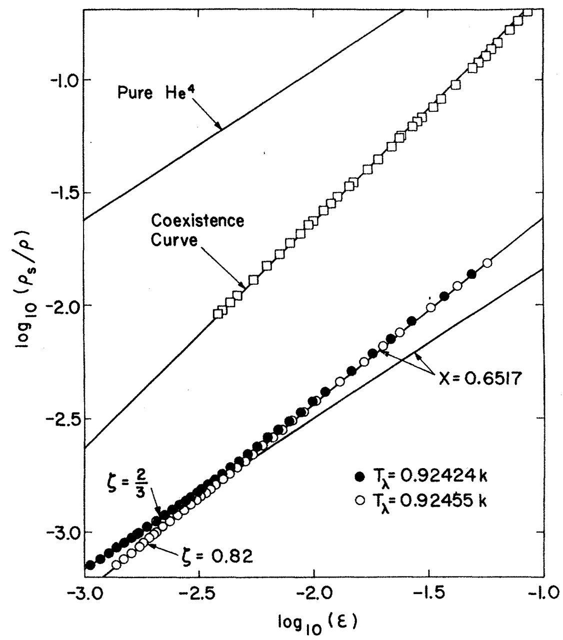

FIG. 3. Superfluid density as a function of ϵ_λ or ϵ_t on logarithmic scales.

for $\epsilon_t \lesssim 2\times 10^{-2}$.

In Fig. 3 we show ρ_s/ρ as a function of ϵ_t or ϵ_λ on logarithmic scales. For ϵ_t, we used the T_t given by Eq. (3); but for $x = 0.6517$ we show the data for two choices of T_λ. One choice results in pure power-law behavior over a wide range of ϵ, but yields an exponent $\zeta_\lambda = 0.82$. The other gives $\zeta_\lambda = \frac{2}{3}$, but is consistent with power-law behavior only for $\epsilon_\lambda \lesssim 2\times 10^{-3}$. The large deviations for $\epsilon_\lambda \gtrsim 5\times 10^{-3}$ from the line representing Eq. (2) with $\zeta_\lambda = \frac{2}{3}$ demonstrates that $\zeta_\lambda = \frac{2}{3}$ is possible only if there are additional higher-order contributions which are much larger than of order ϵ. This is similar to the behavior observed in pure He^4 under pressure.[10] As in the pure system, we have attempted to compare the data with the function

$$\rho_s/\rho = k_\lambda \epsilon_\lambda^{\zeta_\lambda}[1 + A\epsilon_\lambda^{y}]; \tag{4}$$

but in the present case there was not enough experimental information to permit all the parameters to be least-squares adjusted. Therefore we fitted the data for $x = 0.6517$ and $x = 0.6412$ with $\epsilon_\lambda \lesssim 3\times 10^{-2}$ by Eq. (4); but consistent with the results in the pure system and universality arguments, we fixed the exponents at $\zeta_\lambda = \frac{2}{3}$ and $y = \frac{1}{3}$. We obtained a statistically satisfactory fit, indicating that our choice of exponents was permitted by the data. For $x = 0.6718$, data were available only for $10^{-3} \lesssim \epsilon_\lambda \lesssim 4\times 10^{-3}$, and we were able to

TABLE I. Least-squares parameters of Eq. (4), and the phase-separation temperatures.

x	T_λ	K_λ	A	T_σ
0.6718	0.87492	0.0317	5.00[a]	0.8661
	0.87487	0.0352	4.00[a]	
0.6517	0.92446	0.03917	4.09	0.8599
0.6421	0.94812	0.05062	3.06	0.8570

[a]Held constant.

estimate k_λ and T_λ only by making an *a priori* choice of A as well. We list all resulting parameters in Table I. For completeness, we also give T_σ for the three concentrations.[18]

It is apparent from Fig. 3 and Table I that the amplitudes k_λ are $1\frac{1}{2}$ orders of magnitude smaller than in pure He^4; and one might expect k_λ to vanish at x_t. However, the results at $x = 0.6718$ do not appear consistent with a vanishing k_λ. For $x = 0.6718$, the amplitude is only slightly smaller than for $x = 0.6517$, although $x_t - x$ differs by about a factor of 7[18] between the two concentrations. The data seem to indicate that k_λ saturates at a nonzero value as $x_t - x$ decreases towards zero, but it is not entirely clear at this time to what extent this behavior might be attributable to the gravitational inhomogeneity. Although our sample height was kept to a minimum, we have seen explicitly[14] the effect of gravity upon this system in the form of "rounding" of u_2 at the junction between the phase-separation curve and the lines of constant x. We can therefore estimate from experiment the importance of this effect. At $x = 0.6517$ and 0.6421 we know that our conclusions pertain to the gravity-free system. At $x = 0.6718$ we do not believe gravity to be of overwhelming importance either; but a more detailed analysis should be carried out.

We are grateful to Dr. R. B. Griffiths, Dr. B. I. Halperin, Dr. P. C. Hohenberg, and Dr. E. Riedel for discussions on the theory pertinent to this work; to Dr. M. Barmatz for his advice on the acoustic aspects of the experiment; and to Dr. G. Goellner and Dr. H. Meyer for communicating to us their results for $(\partial\Phi/\partial x)_T$ prior to publication.

[1]R. B. Griffiths, Phys. Rev. Lett. 24, 715 (1970).
[2]E. K. Riedel, Phys. Rev. Lett. 28, 675 (1972).
[3]E. H. Graf, D. M. Lee, and J. D. Reppy, Phys. Rev. Lett. 19, 417 (1967).
[4]T. A. Alvesalo, P. M. Berglund, S. T. Islander,

G. R. Pickett, and W. Zimmermann, Jr., Phys. Rev. Lett. 22, 1281 (1969), and Phys. Rev. A 4, 2354 (1971).

[5]S. T. Islander and W. Zimmermann, Jr., Phys. Rev. A (to be published).

[6]G. Goellner and H. Meyer, Phys. Rev. Lett. 26, 1534 (1971), and private communication.

[7]E. K. Riedel and F. J. Wegner, to be published.

[8]E. K. Riedel, private communication.

[9]B. D. Josephson, Phys. Lett. 21, 608 (1966).

[10]D. S. Greywall and G. Ahlers, Phys. Rev. Lett. 28, 1251 (1972), and to be published.

[11]See, for instance, L. P. Kadanoff, in "Critical Phenoma, Proceedings of the International School of Physics 'Enrico Fermi,'" edited by M. S. Green (Academic, to be published); R. B. Griffiths, Phys. Rev. Lett. 24, 1479 (1970).

[12]G. Ahlers, Phys. Rev. Lett. 24, 1333 (1970).

[13]Along the coexistence curve, for instance, we found that $u_2(V) - u_2(0) = aV^2$, with $a = 2.8 \times 10^{-6} \epsilon_t^{-1}$ cm V^{-2} sec^{-1}.

[14]G. Ahlers and D. S. Greywall, in Proceedings of the Thirteenth International Conference on Low Temperature Physics, Boulder, Colorado, 1972 (to be published).

[15]E. C. Kerr, Phys. Rev. Lett. 12, 185 (1964).

[16]I. M. Khalatnikov, *Introduction to the Theory of Superfluidity* (Benjamin, New York, 1965).

[17]For $\epsilon_t \leq 0.1$, the quantity D in Eq. (1) does not differ from unity by more than 0.2%, and the contribution to u_2 in Eq. (1) which depends upon $(\partial\Phi/\partial x)_{TP}$ is less than 1%.

[18]Linear extrapolations in x of T_σ and T_λ yield the estimates $T_t = 0.8671$ K, in agreement with Eq. (3), and $x_t = 0.6750$.

Transport of rf Energy to the Lower Hybrid Resonance in an Inhomogeneous Plasma*

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(Received 7 March 1972; revised manuscript received 10 July 1972)

Localized coupling structures, such as open-ended wave guides located close to the plasma boundary, radiate a spectrum of electrostatic waves that are accessible to the lower hybrid layer in an inhomogeneous plasma. The energy flow is confined within narrow channels, similar to the resonance cone effect discussed by Fisher and Gould. Radical changes in the energy-flow picture result if the constant-density contours are aligned at a small angle $[\sim (m_e/m_i)^{1/2}]$ to the magnetic field.

The possibility of exploiting the lower hybrid resonance for heating plasmas has received considerable attention.[1-5] Recently, interest has been stimulated because of the potential application to rf heating of tokamaks. One aspect that has not received the attention it deserves is the method of coupling the rf power into the plasma in a manner that is conveniently extendable to fusion-scale devices. In the plasma regime for tokamaks ($n \gtrsim 10^{14}$ cm^{-3} and $\omega_{pe}^2 < \omega_{ce}^2$), the lower hybrid frequency is close to the ion plasma frequency (ω_{pi}) which lies in the lower end of the microwave band (1–5 GHz). A particularly convenient coupling structure is then an open-ended rectangular wave guide flush mounted in the metal vacuum-chamber wall. The wave guide propagates energy in the TE_{10} mode and the short side of the guide (electric field direction) is oriented parallel to the toroidal magnetic field. It can be shown that the incident polarization is appropriate for accessibility[6] and that most of the spectral energy can propagate to the resonance layer if $d \ll c/\omega$ where d is the short-side dimension of the guide.[7] In the rest of this Letter we discuss the mechanism by which energy is transported to the hybrid resonance.

We consider the slab geometry illustrated in Fig. 1 in which we assume no variation in the y direction. At the plasma boundary $x = 0$, the exciting structure defines an imposed tangential electric field (E_z) or potential $\varphi(x=0, z) = \varphi_0(z)$. We assume that the majority of the spectral energy of $\varphi_0(z)$ is in the regime $|k_z| \gg \omega/c$, so that an electrostatic approximation $E = -\nabla\varphi$ is valid. From cold-plasma theory, φ is determined as the solution to

$$\frac{\partial}{\partial x} K_\perp \frac{\partial \varphi}{\partial x} + \frac{\partial}{\partial z} K_\parallel \frac{\partial \varphi}{\partial z} = 0,$$

where for the regime of interest $\omega_{ce}^2 \gg \omega_{pe}^2$, $K_\perp \simeq 1 - \omega_{pi}^2/\omega^2$, and $K_\parallel = 1 - \omega_{pe}^2/\omega^2$. By representing φ_0 by a Fourier integral, we can obtain the solution of the transform of φ by WKB methods. In the region $0 \leq x < x_H$, where $K_\perp(x_H) = 0$, the solution is in the form of an outward-propagating