Observation of Radio-Frequency Confinement of a Plasma*†

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(Received 2 August 1972)

The breakdown field in a microwave cavity excited in the electric quadrupole mode exhibits a plateau at pressures below the electron mean-free-path limit of diffusion-controlled breakdown. The luminosity of a steady-state plasma in this domain exhibits a maximum at the central nodal point in the cavity field. These results are interpreted as evidence of rf confinement of a low-density plasma.

The purpose of this Letter is to describe two interrelated measurements of rf breakdown field and of plasma luminosity in the electric quadrupole mode of a resonant cavity. The results are interpreted in the light of rf confinement theory as evidence of steady-state confinement of low-density plasma in the Φ quasipotential well in the cavity field at pressures below the electron mean-free-path limit of the diffusion mechanism of rf breakdown.

During the early years of controlled thermonuclear research, there were several proposals[1-4] for the use of nonuniform rf fields to confine plasmas. Most of these proposals were based on an independent-particle model in which electrons were acted on directly by the rf field. Such treatments were therefore limited to plasmas of low density, $n < n_c = \omega^2 m\epsilon_0/e^2$, where n_c is the critical density below which the plasma is transparent to the electromagnetic field, $\omega = 2\pi f$ is the radian frequency of the applied field, ϵ_0 is the permittivity of free space, and e and m are the charge and mass of the electron. One approach to the theory of rf confinement involves the use of Mathieu functions.[1,5]

An alternative approach involves the use of a quasipotential function $\Phi = e^2E^2/4m\omega^2$, where E is the local amplitude of the applied rf field.[3,4] The quasipotential Φ can be considered as the period-averaged energy of the field-driven oscillatory motion of the electron in the absence of collisions. When the amplitude of the oscillatory motion is small compared with the length within which the field changes appreciably, the drift force on an electron in a nonuniform field is $\vec{\mathfrak{F}} = -\text{grad}\Phi$, and hence the electron tends to move in the direction of decreasing rf field. In general, this drift force results from the combined second-order effects due to the nonuniformity of the rf electric field and the interaction of the oscillating electron with the rf magnetic field. The quasipotential is zero at a field node (where $E=0$); and if E^2 and therefore Φ for a particular cavity mode increase in all directions from the node, then the scalar point function Φ has a potential well in the region from the node out to the first local maximum—usually a saddle point. The effective depth of the well is equal to the quasipotential Φ_R at the rim, i.e., of the isopotential Φ that passes through this saddle point. Such potential wells have been calculated for electric quadrupole and higher-order electric multipole fields.[6-8] The total energy of an electron undergoing collisionless motion in a Φ well can be considered as the constant sum of the quasipotential Φ plus a kinetic energy T arising from the drift motion due to the drift force $\vec{\mathfrak{F}}$. The drift motion occurs at a frequency much lower than that of the confining field. Thus, Φ and T represent the energies of coupled modes of oscillation having widely different frequencies.

The principle of rf (ac) confinement has been demonstrated experimentally for the suspension of charged dust particles[5] and for the entrapment of ions.[9,10] The Φ quasipotential theory has been verified quantitatively by electron-beam experiments at low pressure.[11,12] Self and Boot[13] have reported the onset of an rf-confinement-aided breakdown regime at low pressure and have shown that it could be correlated with Φ quasipotential theory. The present work extends their measurements to lower pressure, introduces new luminosity results, and correlates both results with theory.

The Φ well in the present experiment was established by the TM_{011} electric quadrupole mode at 902 MHz in a cylindrical cavity 24 cm high and 36 cm in diameter, fabricated of stainless steel clad on the interior with oxygen-free high-conductivity copper. This field also served to cause breakdown and to maintain a steady-state plasma. The field was measured with a small coupling loop in the cavity wall. This loop was calibrated by comparison with an electric-field

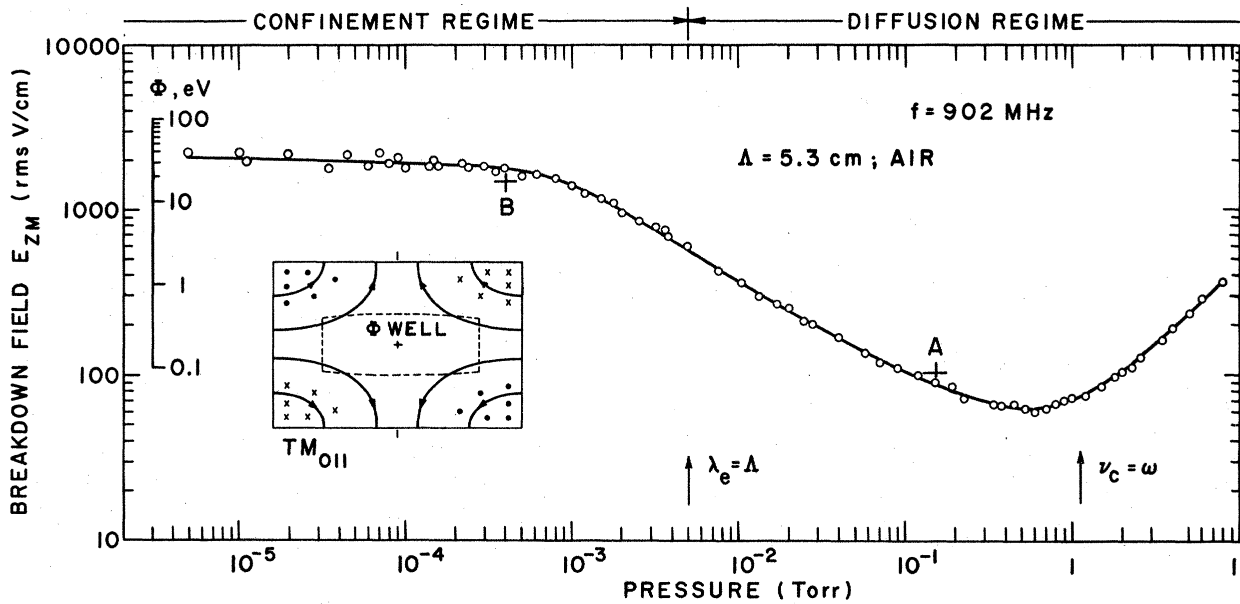

FIG. 1. Breakdown curve for the TM_{011} mode. Note the existence of both a diffusion regime and an rf-confinement–aided regime, with a crossover in the vicinity of the electron mean-free-path limit. The inset shows the field configuration of the TM_{011} mode in an axial cross section of the cylindrical cavity, together with an outline of the rim of the Φ quasipotential well. The collision-frequency transition is shown at $\nu_c = \omega$. A and B indicate the conditions for the luminosity profiles shown in Fig. 2.

probe mounted on the axis of the cavity in a special base plate used with the TM_{010} mode, and the calibration was verified by use of the relation $E^2 = 2PQ/\omega\eta$, where P is the power input to the cavity, Q is the usual quality factor, and η is a geometrical factor.[14] The average error was ~6% in E and ~12% in Φ.

The breakdown curve for this mode is plotted in Fig. 1 together with an inset showing the field configuration and the Φ well.[7,8] In this figure, the diffusion regime (in which breakdown is controlled by electron diffusion) extends down to the pressure at which the electron mean free path λ_e is approximately equal to the characteristic diffusion length Λ over which an electron must (on the average) diffuse to reach the wall of the cavity.[15] This is the lowest pressure at which breakdown can be induced by a nonconfining cavity mode—as we have verified for the TM_{010} dipole mode at 643 MHz, for which the mean-free-path limit was calculated to occur at 5 mTorr and was observed experimentally to occur at ~3 mTorr. However, Self and Boot[13] observed breakdown in the rf confining field of the TM_{011} mode at pressures down to about one decade below the mean-free-path limit. At this lower pressure, their breakdown field corresponded to a Φ well depth approximately equal to the ionization potential V_i. The present work, which extends the measurements to a pressure about three decades below the mean-free-path limit, shows that in the confinement regime the breakdown field levels off on a slowly rising plateau corresponding to a Φ well depth in the range $\Phi_R \approx 15$–30 eV, which is $\gtrsim 1$–$2\frac{1}{2}$ times the ionization potential of the background gas (12.5 eV for the oxygen component of air). Results obtained for hydrogen and helium exhibited a similar dependence on ionization potential. The net input power in the confinement regime was in the range of ~100–400 W.

Profiles of integrated luminosity of steady-state plasmas, obtained by scanning approximately diametrically across the central portion of the cavity axis with a collimated photomultiplier, are shown in Fig. 2. The inset is a diagram of the scanning arrangement used with an X-Y recorder. For these weakly ionized collisional plasmas, the luminosity is approximately proportional to the plasma density. At 902 MHz, the cutoff density is $n_c = 10^{10}$ electrons/cm^3; the density of the plasmas observed in this work was $n \lesssim 10^8$ electrons/cm^3, as estimated from the luminosity.

Profile A was made in the diffusion regime as indicated at A in Fig. 1. The plasma was produced by raising the field to the breakdown value

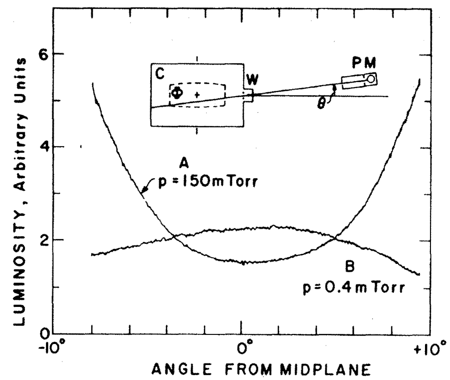

FIG. 2. Luminosity profiles taken at A and B as indicated in Fig. 1. The inset shows the luminosity scanning arrangement, including the collimated photomultiplier PM, the cavity C with viewing window W, and the outline of the Φ well. The sensitivity of the PM and the X-Y recorder was the same for both profiles.

(~ 90 V/cm), whereupon it was adjusted to the sustaining value (~ 10% higher) at which the profile was taken. The potential well depth corresponding to this low field is $\Phi_R \approx 0.09$ eV $\ll V_i$. This low value of Φ is associated mainly with the low sustaining field at A. An additional minor reduction in Φ results from the direct effect of collisions as given by $\Phi_p = e^2E^2/4m(\omega^2 + \nu_c^2)$, where $\nu_c \propto p$ is the collision frequency at pressure p; at A this reduction is only ~ 2%. Profile A is characteristic of a low-density diffusion-dominated plasma in which the luminosity (and therefore the density) is approximately proportional to a positive power of the field E. Thus, in this mode the luminosity is minimum across the midplane of the cavity where $E = 0$ on the axis and increases toward both ends where E has its maximum value.

Profile B was made in the rf-confinement regime as indicated at B in Fig. 1. The sustaining field here was ~ 5% below the breakdown field at the same pressure and corresponded to $\Phi_R \approx 15$ eV $\approx V_i$. The net input power to the cavity at this condition was about 200 W. Profile B obviously differs from profile A in that its maximum occurs in the midplane position where $E = 0$ on the axis, and that its general shape is inverted with respect to that of A. This different shape of profile B, the fact that it was obtained in the regime where breakdown is dominated by rf confinement, and the fact that it was obtained at a sustaining field strength corresponding to $\Phi_R \approx V_i$, all combine to indicate that this profile represents a steady-state rf-confined plasma.

The term "steady-state confinement" as used above is intended only to describe the apparent long-term macroscopic steadiness of the plasma as manifested by the ability to maintain it indefinitely. (Approximately 1 min was required to make each luminosity profile.) However, consideration of the dynamics of individual electrons on a microscopic scale suggests the occurrence of a continuous energy-cycling mechanism in which the confinement time of electrons is only of the order of microseconds—a time which is nevertheless long enough for a steady plasma to be maintained. This energy-cycling mechanism results from stochastic heating,[16] a process in which electrons moving in an rf field gain energy in elastic collisions with neutrals. From an analysis based on a simple theory of stochastic heating and the assumption that energy losses by excitation of atoms in inelastic collisions can be neglected, it can be shown that an average electron moving in a spherically symmetric parabolic Φ well can be heated from an initial energy of ~ 1 eV to an ionizing level of ~ 13–15 eV in ~ 4–5 collision periods.[17] At this energy the relative probabilities of escaping from the Φ well and of giving up its energy in an ionizing collision depend on the relative values of Φ_R and V_i. The above result that the depth of the Φ well in the confinement regime of the breakdown curve is ~ $1-2\frac{1}{2}$ times V_i appears to be compatible with the ~ $\frac{11}{5}$ energy gain[17] in the inelastic (ionizing) collision that terminates a sequence of elastic stochastic-heating collisions. The same result also appears to be the prerequisite for maintenance of the plasma; i.e., the establishment of a large enough population of electrons repetitively undergoing stochastic heating to the energy required to produce adequate excitation and ionization requires this degree of confinement. For the conditions of profile B, this repetition time is ~ 2000–3000 cycles of the field frequency, i.e., ~ 2–3 μsec. Because of their relatively large mass, ions partake of neither the confinement nor the stochastic heating processes. They are evidently confined in the Φ well indirectly by space-charge attraction to the directly confined electrons.

It is a pleasure to thank F. E. Throw for a helpful critical reading of the manuscript.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

†Work based in part on a thesis by C. F. Shelby, submitted in partial fulfillment of the requirements for the

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[17]If one assumes that $\Phi \propto E^2 \propto r^2$ in a spherically symmetrical well about a nodal point of the field, then the mean value of Φ in a sphere of radius r_w can readily be shown to be $\frac{3}{5}\Phi_w$, where Φ_w is the quasipotential at r_w. Since the energy gain per collision can be taken to be twice this mean energy, the ratio of mean energy after a collision to that before is $\frac{11}{5}$. Thus, the number of collisions necessary for an electron to have its energy increased from an initial value u_0 to a value Φ_R is $n = [\ln(\Phi_R/u_0)]/[\ln(\frac{11}{5})]$. For $u_0 = 1$ eV and $\Phi_R = 15$ eV, $n = 3.6 \approx 4$.

Convection-Driven Hydromagnetic Dynamo

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(Received 28 July 1972)

We describe a hydromagnetic dynamo utilizing Bénard convection between rotating parallel planes. The model is based upon an asymptotic expansion in two spatial scales valid for large Taylor number.

It has long been believed that thermal convection provides a plausible thermodynamic basis for a homogeneous dynamo effect, that is, for the spontaneous amplification of magnetic energy within an isolated volume of conducting fluid.[1,2] In this Letter we consider a hydromagnetic dynamo model based upon this hypothesis. The model utilizes a magnetohydrodynamic Boussinesq fluid with constant properties, confined between stress-free, perfectly conducting, isothermal planes $z = 0, L$. The system rotates about the vertical z axis with constant angular velocity. If the magnetic field is zero, this is the classical Bénard problem with rotation. In that case it is well known that for large Taylor number $T = 4\Omega^2 L^4 \nu^{-2}$ and fixed Prandtl number $\nu\kappa^{-1} > 1$, the critical value of the Rayleigh number $R = \alpha\beta L^4 (\kappa\nu)^{-1}$ is of order $T^{2/3}$, the most unstable horizontal wavelengths of the convection pattern being of order $T^{-1/6}L$.[3] Accordingly, we set $\epsilon = l/L = T^{-1/6}$, with l the horizontal scale, and suppose that $\epsilon \ll 1$, $\epsilon^4 R \equiv \tilde{R}(\epsilon) \sim 1$. We are then able to use the multiple-scale theory of spatially periodic kinematic dynamos to study the regenerative effect of the flow.[4] If convection ensues as a pattern of two or more nonparallel systems of rolls, the motion is a periodic dynamo of degree two with respect to z-dependent dynamo waves. Consequently, we require the magnetic Reynolds number of the rolls to be of order $\epsilon^{1/2}$, and take the unit of speed to be