ω-φ Mixing and Photon Interactions from an Extension of Odorico's Bootstrap*

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The bootstrap condition applied by Odorico to the scattering of pseudoscalar mesons is extended to processes involving vector mesons and photons. The ω-φ mixing angle is predicted to be the quark-model value. A photon interacting as a U-spin scalar is one of a small number of solutions.

Recently, Odorico has proposed a strong bootstrap condition for VPP and TPP interactions, where P denotes a pseudoscalar meson, and V and T Reggeized vector and tensor mesons.[1] The condition is such that the zeros in $PP\to PP$ amplitudes are straight lines in the Mandelstam plane. This requires not only an internal symmetry group, but, if the group is SU(3), it also requires the η-X mixing angle to be $\tan^{-1}(1/\sqrt{2})$.[1]

In this paper we extend Odorico's hypothesis to the invariant amplitudes for the processes $PP\to PV$ and $PP\to P\gamma$, where γ is a photon (defined to be any weakly interacting vector particle). Predictions are obtained for the ω-φ mixing angle, and various ratios of VVP, TVP, $V\gamma P$, and $T\gamma P$ interaction constants.

The $PP\to PP$ process.—In order to show where the results come from, we consider first PP scattering, and describe the relation between Odorico's hypothesis and an earlier bootstrap hypothesis. Mass differences between P mesons are neglected. The s-, t-, and u-channel amplitudes are taken to be $ab\to cd$, $a\bar{c}\to\bar{b}d$, and $\bar{c}b\to\bar{a}d$, respectively. The various V-meson trajectory residues are

$$V_{st}=f_{rdc}f_{rab},\quad V_{su}=f_{rdc}f_{rba},$$
$$V_{ts}=f_{rd\bar{b}}f_{ra\bar{c}},\quad V_{tu}=f_{rd\bar{b}}f_{r\bar{c}a},\qquad(1)$$
$$V_{ut}=f_{rd\bar{a}}f_{r\bar{c}b},\quad V_{us}=f_{rd\bar{a}}f_{rb\bar{c}},$$

where f_{rij} is the VPP interaction constant, and a summation over the V mesons r is implied. The first index of V_{ij} is the channel of the residue. The T-trajectory residues are defined by analogous formulas, with the TPP interaction constants denoted by d_{rij}. Thus, $T_{st}=d_{rdc}d_{rab}$, etc.[2] Since f and d are antisymmetric and symmetric, respectively, in the last two indices, it follows that

$$V_{ij}=-V_{ik},\quad T_{ij}=T_{ik},\qquad(2)$$

if i, j, and k represent the three Mandelstam channels in any order.

In previous references, the author has used various dynamical assumptions to write the following bootstrap condition for each pair of Mandelstam channels[3]:

$$T_{ij}-V_{ij}=T_{ji}-V_{ji}.\qquad(3)$$

One could include a positive proportionality constant in front of the T terms of this equation. Our normalization convention for d/f is that this constant may be omitted.

It can be shown that Odorico's bootstrap on the residues, which follows from this assumption of linear zeros, is equivalent to Eq. (3) and the additional condition

$$T_{ij}V_{ij}(T_{ij}^2-V_{ij}^2)=0,\qquad(4)$$

for each of the six sets of values for i and j.[4,5] This condition is unusual in that a linear combination of solutions for any of the external mesons may not be a solution.

We first summarize briefly some well-known implications of the regular bootstrap equation, Eq. (3). This condition requires a Lie-group symmetry.[3] If the conditions were extended so that all particles may be external, the particles must correspond to the regular and singlet representations of one of the groups SU(n).[3] We take the group to be SU(3). The VPP interactions involve only octets; the overall interaction constant is denoted by f_{888}. The TPP interaction constants are d_{888}, d_{188}, d_{818}, and d_{111}, where the first subscript is the representation of the T trajectory. The constants f_{888}^2, d_{888}^2, and d_{188}^2 are normalized to be equal to the sum over all PP states coupled to a particular V or T, while d_{818} is equal to $d(T_jP_1P_j)$, where j is any octet state and P_1 is the P singlet. The condition of Eq. (3), applied only to PP scattering, determines the ratios of these constants in terms of one real parameter r, i.e.,[6]

$$f_{888}=(\tfrac{9}{5})^{1/2}d_{888},\quad d_{188}=-(\tfrac{16}{5})^{1/2}d_{888},$$
$$d_{818}=rd_{888},\quad d_{111}=-(\tfrac{5}{2})^{1/2}r^2 d_{888}. \tag{5}$$

In all solutions listed in this paper, sign changes that do not change the magnitude of any interaction constant are ignored. No limitation of the η-X mixing angle is implied by Eq. (3).

The additional restrictions, that result from the Odorico condition of Eq. (4), are obtained most easily by considering first the amplitudes $K^+\pi^0\to K^+(\eta, X)$, and then checking the results with other amplitudes. This procedure shows that r must be either $(\frac{1}{10})^{1/2}$ or $-(\frac{2}{5})^{1/2}$, and that the mixing angle is $\theta=\tan^{-1}(1/\sqrt{2})$, where $\eta=(\cos\theta)\eta_8-(\sin\theta)\eta_1$. The solution $r=(\frac{1}{10})^{1/2}$ was discussed by Odorico; several experimental arguments are given in support of this solution in Ref. 1. The solution $r=-(\frac{2}{5})^{1/2}$ was overlooked in Ref. 1; it corresponds to the usual quark model in which the η interacts as if made of a strange quark and a strange antiquark.

The $PP\to PV$ process.—We label the particles a, b, c, and d, as before, identifying d with the V meson. The VPP and TPP interaction constants are taken to be one of the two solutions to the $PP\to PP$ equations. The new, VPP and TVP, interaction constants are denoted by D_{rdi} and F_{rdi}, respectively. The three subscripts correspond to the SU(3) indices of the T or V on the trajectory, and the external V and P, respectively. The various residues may be obtained by replacing the first f or d by D or F, respectively, in the $PP\to PP$ residues of Eq. (1), i.e.,

$$V_{st}=D_{rdc}f_{rab},\quad T_{st}=F_{rdc}d_{rab},\ \text{etc.} \tag{6}$$

These satisfy the symmetry conditions of Eq. (2).

The $PP\to PV$ scattering amplitude τ may be written in the form $\tau=\epsilon_{\alpha\beta\gamma\delta}p_\alpha{}^a p_\beta{}^b p_\gamma{}^c E_\delta A(s,t,u)$, where p^i is the four-momentum of the P meson i, and E is the polarization vector of the V meson. We assume that the invariant amplitude $A(s,t,u)$ satisfies the same restrictions imposed on the $PP\to PP$ amplitudes in Ref. 1. It can be shown that this condition is equivalent to the special condition of Eq. (4), together with the following bootstrap equation:

$$T_{ij}-V_{ij}=-(T_{ji}-V_{ji}), \tag{7}$$

where the T and V are those of Eq. (6). The difference in sign between Eq. (3) and Eq. (7) occurs because the factor $\epsilon_{\alpha\beta\gamma\delta}p_\alpha{}^a p_\beta{}^b p_\gamma{}^c$ is antisymmetric in the exchange of any two P mesons.

The F and D may be written in terms of the constants F_{888}, D_{888}, D_{881}, and D_{818}, where the second and third indices are the representations of the external V and P mesons, respectively. It can be shown that the regular bootstrap condition, Eq. (7), requires that these satisfy the condition[5]

$$F_{888}=(\tfrac{9}{5})^{1/2}D_{888},\quad D_{881}=rD_{888},\quad D_{818}=RD_{888}, \tag{8}$$

where r is the parameter of Eq. (5) and R is a new real constant, as yet arbitrary. The D and F are normalized as are the d and f.

One may find the restrictions imposed by the Odorico condition of Eq. (4) by considering first the amplitudes $K^+\pi^0\to K^+(\omega,\varphi)$, and then checking the other $PP\to PV$ amplitudes. Although the algebra is different from that of the PP scattering case, the results are remarkably similar. We neglect one special, nonphysical solution, in which D_{818} is finite, but F_{888}, D_{888}, and D_{881} are zero. For each of the two allowed values of r $[(\frac{1}{10})^{1/2}$ and $-(\frac{2}{5})^{1/2}]$, the ratio R is restricted to one of the two values $(\frac{1}{10})^{1/2}$ and $-(\frac{2}{5})^{1/2}$, and the ω-φ mixing angle is $\alpha=\tan^{-1}(1/\sqrt{2})$, where $\omega=(\sin\alpha)\omega_8+(\cos\alpha)\omega_1$. The solution $R=-(\frac{2}{5})^{1/2}$ is the quark-model solution suggested by experiment; in it the $\varphi\rho\pi$ coupling constant vanishes.[7] This prediction for the mixing angle has been made before from a duality condition, together with the assumption that the ω-φ mass difference is so great that the trajectories cannot be considered degenerate.[8] In contrast, our prediction makes no use of the ω-φ mass difference.

The photon interactions.—We now turn to the $PP\to P\gamma$ process, where the photon (γ) is defined, *a priori*, to be any vector particle interacting sufficiently weakly that intermediate γ trajectories may be neglected. The bootstrap conditions are the same as for the $PP\to PV$ case. The new interactions are of the types $T\gamma P$ and $V\gamma P$. It is clear that a solution to the regular bootstrap equation of Eq. (7) is obtained if the γ interactions are proportional to those of any linear combination of vector mesons. In a previous work by the author, it has been shown that this is essentially the only solution to photon bootstrap equations.[9] Specifically, the result is

$$F(T_i\gamma P_j)=\mu F_{ixj}, \tag{9a}$$
$$D(V_i\gamma P_j)=\mu[D_{ixj}+SD_{888}(20)^{-1/2}\delta_{ij}], \tag{9b}$$

where x is any linear combination of the states of the V octet, and μ and S are real proportionality constants. The indices i and j range over the singlet and octet states in Eq. (9b). The constant $D_{888}(20)^{-1/2}$ is included for convenience.

If there is only one photon, the octet state x must be self-conjugate, i.e., $x=(\sin\beta)\rho^0+(\cos\beta)\omega_8$. We apply the Odorico condition of Eq. (4) to attempt to determine the angle β and the SU(3) singlet interaction parameter S. The procedure used is to consider first the amplitude $K^0\pi^+\to K^+\gamma$, and then check with other $PP\to P\gamma$ amplitudes. The resulting solutions are independent of whether the ratio r of Eq. (5) is $(\frac{1}{10})^{1/2}$ or $-(\frac{2}{5})^{1/2}$. We neglect one special solution, in which the photon interacts as a pure SU(3) singlet. It is clear from the $PP\to PV$ results that the angles $\beta=0$ and 90°, corresponding to pure isoscalar and isovector photons, must lead to solutions. Our result is that the only other solutions are obtained by rotating these by ±60° in the SU(3) (hypercharge-I_z) plane. The photon must interact either as the middle component of an I-, U-, or V-spin triplet, or an I-, U-, or V-spin singlet. The U-spin singlet solution, $\tan\beta=\sqrt{3}$, agrees with experiment. As far as we know, this is the most stringent limitation on photon interactions that has been obtained from any bootstrap principle.

In the U-spin singlet solution, the singlet-octet interaction ratio S of Eq. (9b) cannot be zero, but is restricted to one of the values

$$S=1,\ -2,\ \text{or}\ 4. \qquad (10)$$

This "singlet part" of the photon would not contribute to antisymmetric (f-type) interactions, such as the charge of hadrons in an octet. It would be detectable in the $P\gamma$ decays of vector mesons. In Table I, we have listed the predicted coupling constants for various $V\to P\gamma$ decays, in terms of r, S, and the η-X mixing angle θ. We have set R and the ω-φ mixing angle equal to their values in the quark-model solution, $-(\frac{2}{5})^{1/2}$ and $\tan^{-1}(1\sqrt{2})$. The predicted constants for η and X interactions may be obtained by setting $\tan\theta$ equal to $1/\sqrt{2}$ and $-\sqrt{2}$, respectively.[10]

TABLE I. Coupling constants for $V\to P\gamma$ decays in units of $\mu D_{888}/(20)^{1/2}$.

Decay	Constant
$\omega\to\pi^0\gamma$	-3
$\varphi\to\pi^0\gamma$	0
$K^{*+}\to K^+\gamma$	$-1+S$
$K^{*0}\to K^0\gamma$	$2+S$
$\rho^0\to\pi^0\gamma$	$-1+S$
$\rho^0\to\eta\gamma$	$-\sqrt{3}\,[\cos\theta+(5)^{1/2}r\sin\theta]$
$\omega\to\eta\gamma$	$(1/\sqrt{3})(-1+S)[\cos\theta+(5)^{1/2}r\sin\theta]$
$\varphi\to\eta\gamma$	$(\frac{8}{3})^{1/2}(1+\frac{1}{2}S)[\cos\theta-\frac{1}{2}(5)^{1/2}r\sin\theta]$

We take the $\omega\to\pi^0\gamma$ decay rate as a convenient standard. The present experimental upper limit on the $\rho^0\to\pi^0\gamma$ decay is about twice that predicted with no singlet interaction ($S=0$).[11] Thus, these data favor $S=1$ over the other solutions of Eq. (10). However, if $S=1$, our prediction is that the $\rho^0\to\pi^0\gamma$, $\omega\to\eta\gamma$, $X\to\omega\gamma$, and $K^{*+}\to K^+\gamma$ rates all are zero. Future experiments that lower the present upper limits on some of these rates will distinguish between our predictions and the usual ones involving no SU(3) singlet part of the photon.[12]

Present measurements of the $\varphi\to\eta\gamma$ decay rate are not all in agreement.[12,13] Basile *et al.* measure a rate consistent with a pure octet η and the quark model [$S=0$, $r=-(\frac{2}{5})^{1/2}$].[13] This measurement is also consistent with Odorico's solution for the η [$r=(\frac{1}{10})^{1/2}$, $\tan\theta=1/\sqrt{2}$], if $S=1$.

In conclusion, our extension of Odorico's bootstrap hypothesis predicts an ω-φ mixing angle of $\tan^{-1}(1/\sqrt{2})$, and leads to two possible solutions for VVP interactions, one of which agrees with experiment. A photon interacting as a U-spin scalar is one of a small number of solutions. In this solution, the predicted $P\gamma$ decays of vector mesons contain a significant term corresponding to an SU(3) singlet part of the photon.

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[1]R. Odorico, Phys. Lett. 38B, 37 (1972).

[2]We are assuming a representation of the mesons in which the coupling constants are all real. If more general representations are used, Eq. (1) should be modified, as shown by R. H. Capps, Phys. Rev. D 3, 3059 (1971).

[3]Capps, Ref. 2; this paper lists the bootstrap condition and gives references for the derivation.

[4]The different patterns of zeros discussed in Ref. 1 correspond to different possibilities concerning which factors in Eq. (4) are equal to zero.

[5]A more detailed discussion of our extension of Odorico's bootstrap condition will be published later.

[6]R. H. Capps, Phys. Rev. 165, 1899 (1968). The constant $G_{8,18}$ of this reference is equal to $\sqrt{2}\,d_{818}$.

[7]If one assumes that the $\pi^+\pi^-\pi^0$ decay rate of the φ is all $\rho\pi$, and uses the decay rate listed by the P. Söding *et al.* Phys. Lett. 39B, 1 (1972), and the decay formula of R. H. Capps, Phys. Rev. 144, 1182 (1966), Eq. (16), the result is a value of the ω-φ mixing angle differing from $\tan^{-1}(1/\sqrt{2})$ by about 3°.

[8]C. B. Chiu and J. Finkelstein, Phys. Lett. 27B, 510 (1968).

[9]R. H. Capps, Phys. Rev. D 4, 2997 (1971). The proof assumes a set of bootstrap equations that are

complete in the sense that all intermediate hadrons may also be external hadrons.

[10]The kinematic factors involved in the decay rates are calculated by C. Becchi and G. Morpurgo, Phys. Rev. 140, B687 (1965).

[11]See H. Harari, in *Proceedings of the Fifth International Symposium on Electron and Photon Interactions at High Energies, Cornell University, Ithaca, New York, 1971*, edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N.Y., 1972), p. 305.

[12]For a recent review of some data concerning various $V \rightarrow P\gamma$ decays, see J. Le Francois, in *Proceedings of the Fifth International Symposium on Electron and Photon Interactions at High Energies*, (Ref. 11), p. 55.

[13]M. Basile *et al.*, Phys. Lett. 38B, 117 (1972).

ERRATA

CONDENSED π^- PHASE IN NEUTRON STAR MATTER. Raymond F. Sawyer [Phys. Rev. Lett. 29, 382 (1972)].

On p. 384, the last sentence, which reads, "However this state has an immense current flowing in the magnetic energy will preclude its formation. magnetic energy will preclude its formation," should be replaced by "However this state has an immense current flowing in the z direction, and in a homogeneous system the magnetic energy will preclude its formation."

MAGNETIC HYPERFINE MODULATION OF DYE-SENSITIZED DELAYED FLUORESCENCE IN AN ORGANIC CRYSTAL. R. P. Groff, R. E. Merrifield, A. Suna, and P. Avakian [Phys. Rev. Lett. 29, 429 (1972)].

Our magnetic field units, mT (1 millitesla = 10 oersteds), were incorrectly expressed by the journal as mTorr.

INDUCED NEUTRAL CURRENT EFFECTS IN UNIFIED MODELS OF WEAK AND ELECTROMAGNETIC INTERACTIONS. Kazuo Fukikawa, Benjamin W. Lee, A. I. Sanda, and S. B. Treiman [Phys. Rev. Lett. 29, 682 (1972)].

C. Bouchiat, J. Iliopoulos, and Ph. Meyer pointed out correctly that the contributions of diagrams (c) and (d) contain a factor

$$-i(G_F\alpha/\sqrt{2}\,\pi)r[\bar{\nu}\gamma^{\alpha}(1+\gamma_5)\nu][\bar{e}\gamma_{\alpha}e], \tag{7}$$

where r is of order $\ln[m_\mu/m(Y^+)]$. More precisely, for $q^2, m^2(Y^+) \ll m_W^2$, r is given by

$$r = 2\int_0^1 d\alpha\,\alpha(1-\alpha)\ln\frac{m_\mu^2 - q^2\alpha(1-\alpha)}{m^2(Y^+) - q^2\alpha(1-\alpha)} + O\left(\left[\frac{m(Y^+)}{m_W}\right]^2, \left[\frac{m_\mu}{m_W}\right]^2, \left[\frac{q^2}{m_W^2}\right]\right)$$

$$\simeq \frac{1}{3}\ln\left[\frac{m_\mu}{m(Y^+)}\right]^2 - \frac{1}{15}\frac{q^2}{m_\mu^2} \text{ for } q^2 \ll m_\mu^2,$$

$$\simeq \frac{1}{3}\ln\frac{q^2}{m(Y^+)^2} \text{ for } m_\mu^2 \ll q^2 \ll m^2(Y^+).$$

Numerically r is of order unity for $m(Y^+) \simeq 1$ GeV. Equation (7) should be added to the right hand sides of Eqs. (3) and (4).

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