the observed average final energies. On the basis of these initial conditions we further calculated by the method of Boneh, Fraenkel, and Nebenzahl[5] the average energy of the ^{5}He at the time of its decay taking into consideration its exponential decay with a half-life of 8×10^{-22} sec. This curve is also shown in Fig. 3. The measured average energy of the ^{5}He at the time of breakup (6.3 ± 0.8 MeV) yields a unique graphical solution to the initial conditions, i.e., $E_F(0)=40\pm11$ MeV and $E_5(0)=3.9\pm0.9$ MeV. This solution applies only to neutrons which are coincident with α particles above 9 MeV. An approximate correction for the missing events gives $E_F(0)=31\pm11$ MeV and $E_5(0)=3.2\pm0.9$ MeV.

The average initial conditions obtained here for the emission of ^{5}He are similar to those obtained in Ref. 5 from trajectory calculations which were used to reproduce the properties of the long-range α particles in spontaneous fission of ^{252}Cf and in particular their angular distribution relative to the fragments. Our results are also in good agreement with a similar trajectory calculation performed by Musgrove[6] who fitted his calculation to the experimental angular distribution of Raisbeck and Thomas[7] which is narrower than the experimental angular distribution[8] used by Boneh, Fraenkel, and Nabenzahl. Rajagopalan and Thomas[9] recently remeasured this angular distribution and found it to be in substantial agreement with the results of Raisbeck and Thomas.[10] We have also performed trajectory calculations in which $E_F(0)$ is a Gaussian distribution with $\sigma=12$ MeV and a mean $\bar{E}_F(0)=28$ MeV, $E_5(0)$ has a Maxwellian distribution with $\bar{E}_5(0)=3.0$ MeV, and the initial emission of the ^{5}He is isotropic between 30° and 150° with respect to the fragment direction. The calculated α spectrum (which takes into account the lifetime of ^{5}He and the backward recoil of the α in the ^{5}He decay) is shown in Fig. 2 to be in good agreement with the experimental results.

One of us (E.C.) would like to thank J. Pederson for his valuable comments.

[1]I. Halpern, Annu. Rev. Nucl. Sci. 21, 245 (1971).
[2]F. Ajzenberg-Selove and T. Lauritsen, *Energy Levels of Light Nuclei VI* (North-Holland, Amsterdam, 1959).
[3]V. N. Nefedov, O. I. Ivanov, V. P. Karin, and A. S. Tishin, Yad. Phys. 3, 465 (1966) [Sov. J. Nucl. Phys. 3, 337 (1966)].
[4]H. R. Bowman, S. G. Thompson, J. C. D. Milton, and W. J. Swiatecki, Phys. Rev. 126, 2120 (1962).
[5]Y. Boneh, Z. Fraenkel, and I. Nebenzahl, Phys. Rev. 156, 1305 (1967).
[6]A. R. de L. Musgrove, Austr. J. Phys. 24, 129 (1971).
[7]G. M. Raisbeck and T. D. Thomas, Phys. Rev. 172, 1272 (1968).
[8]Z. Fraenkel, Phys. Rev. 156, 1283 (1967).
[9]M. Rajagopalan and T. D. Thomas, Phys. Rev. C 5, 1402 (1972).
[10]Raisbeck and Thomas (Ref. 7) and Rajagopalan and Thomas (Ref. 9) also made trajectory calculations to fit their experimental results. They state (Ref. 9) that their results are in substantial agreement with the results of Musgrove (Ref. 6) despite the fact that they obtain a much lower fragment kinetic energy at scission (7.5 MeV versus 25 MeV of Ref. 6).

(^{6}Li, d) on ^{58}Ni and ^{64}Ni

Hans H. Gutbrod* and Roger G. Markham
Nuclear Structure Research Laboratory,† University of Rochester, Rochester, New York 14627
(Received 17 July 1972)

The (^{6}Li,d) reaction has been performed on ^{58}Ni and ^{64}Ni at 38 MeV. For ^{58}Ni angular distributions were obtained and compared to distorted-wave Born-approximation calculations. One spectrum was taken for ^{64}Ni. A reduction in cross section is observed and is compared with the results of (^{16}O, ^{12}C), (p,t), and (^{3}He,n) experiments.

The investigation of four-particle configurations in nuclei has recently been extended from nuclei of the *sd* shell into the Ni region[1-3] via the (^{16}O,^{12}C) reaction. However, extraction of reliable spectroscopic information is difficult because of the strong Q-value dependence of the cross section[4] and the uncertainties in knowing the various configurations of the transferred four-nucleon cluster.[5] A more attractive approach is the (^{6}Li,d) reaction which extensive studies[6-7] on light nuclei have shown to be a good α-transfer reaction. Furthermore, distorted-wave Born-approximation (DWBA) calculations[7] indicate that the Q-value dependence of the cross section is

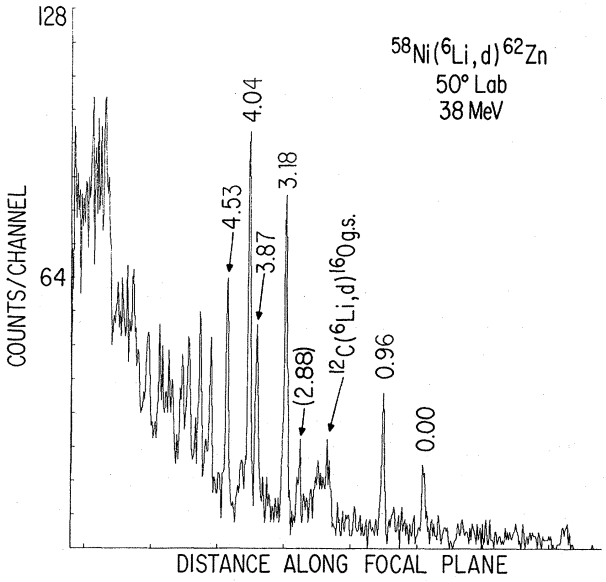

FIG. 1. Deuteron spectrum taken with spark counter in the focal plane of the split-pole spectrograph. Energy resolution, about 80 keV (full width at half-maximum).

much weaker. Since Li^{3+} beams with energies up to 40 MeV—where compound contributions in (^{6}Li, d) are expected to be very small—are now available with intensities of several microamperes, the expected low (^{6}Li, d) cross sections no longer pose a severe experimental problem. Furthermore, the higher energy resolution obtainable in a (^{6}Li, d) experiment is expected to provide more definitive spectroscopic information. Thus it is desirable to study the (^{6}Li, d) α-transfer reaction and to compare the results with the previous (^{16}O, ^{12}C) work.[2,3]

Self-supporting targets of ^{58}Ni and ^{64}Ni have been bombarded with a high-intensity 38-MeV $^{6}Li^{3+}$ beam from the University of Rochester MP tandem Van de Graaff accelerator. The target thickness was ~100 μg/cm^2 and the beam intensity was ~2 μA. The deuterons were analyzed in a split-pole spectrograph and detected by a sonic spark counter[8] mounted in the focal plane. Absolute cross sections were measured relative to the elastic scattered Li^{3+} at 10°, where $\sigma_{el}/\sigma_{Ruth} = 1.0 \pm 0.05$. The position-momentum calibration of the spark counter was made by observing the positions of known states populated in the reaction ^{12}C(^{6}Li,d)^{16}O and by observing the position of elastic scattered Li as a function of field strength. With this calibration, excitation energies could be determined to better than ± 20 keV.

Shown in Fig. 1 is a deuteron spectrum taken at 40° in the ^{58}Ni(^{6}Li,d)^{62}Zn experiment. The known states at 1.80, 2.17, 2.34, 2.74, and 2.88 MeV, excited via the reaction ^{64}Zn(p,t)^{62}Zn,[9] are weakly excited in comparison to the stronger transitions to states at 3.18 and 4.04 MeV. Strong states were seen in the (^{16}O,^{12}C) experiment[2,3] at 3.20 and 3.80 MeV. The higher energy resolution obtained here allows the complete separation of two states at 3.87 and 4.04 MeV, which probably appeared as one broad peak in the ^{12}C spectrum.[2,3] States seen at 4.95 and 5.20 MeV in the present experiment were not observed in the (^{16}O,^{12}C) experiment. For the ^{64}Ni target, a deuteron spectrum was taken only at 50°. The spectrum showed that the 0^+ ground state and the 2^+ states at 1.08 and 2.82 MeV were populated.

TABLE I. ^{58,64}Ni(^{6}Li,d)^{62,68}Zn.

	E_x (MeV)	J^π	$(d\sigma/d\omega)_{50°}$ (μb/sr)	$(d\sigma/d\omega)_{20°}$ Relative
^{62}Zn	0.0	0^+	1.0 ± 0.2	1.0 ± 0.0
	0.96 ± 0.02	2^+	1.3 ± 0.2	1.4 ± 0.2
	3.18 ± 0.02	$(4^+)(3^-)$	4.2 ± 0.3	4.7 ± 0.3
	3.87 ± 0.02		2.7 ± 0.2	0.8 ± 0.2
	4.04 ± 0.02		5.0 ± 0.3	2.6 ± 0.2
	4.53 ± 0.02		2.6 ± 0.2	2.5 ± 0.2
^{68}Zn	0.0	0^+	0.43 ± 0.16	
	1.08	2^+	0.43 ± 0.16	
	2.82	2^+	0.26 ± 0.16	

In both cases, it appears that the (^{6}Li,d) reaction is very selective. The energies of the excited states and the measured cross sections at 50° and 20° are shown in Table I. The population of the ground state and the first excited state in ^{62}Zn and ^{68}Zn differ by a factor of 2–3 at 50°, whereas in (^{16}O,^{12}C) they differ by a factor of 10.[4]

How much of this decrease in cross section is due to kinematics and how much is due to the nuclear structure can be estimated from a DWBA analysis.[7] The DWBA calculations were performed with the code DWUCK.[10] As in Ref. 7 the reaction was assumed to proceed by the transfer of a spin-0, mass-four particle into a bound state of a Woods-Saxon well; the parameters $V = 105$–110 MeV, $r_0 = 1.2$ F, $a = 0.5$ F were used. The number of radial nodes was chosen to correspond to the center-of-mass motion of four particles in the fp shell for $L = 0, 2, 4$ and in the fp,g shell for $L = 3$. The optical-model parameters for the outgoing channel were taken from Mairle *et al.*[11] For ^{6}Li, a strong absorption parameter set was used, which has been extracted from heavy-ion

TABLE II. Optical-model parameters.

	U (MeV)	r_{0r} (F)	a_r (F)	W_D (MeV)	W_S (MeV)	r_{0i} (F)	a_i (F)	$V_{s.o.}$ (MeV)	r_c (F)
d[a]	70.4	1.25	0.742	•••	14.0	1.24	0.714	7.0	1.3
Li[b]	100.0	1.19	0.488	17.0	•••	1.26	0.26	•••	1.3

[a]Ref. 11. [b]Ref. 12.

elastic-scattering data[12] and which has been used in DWBA calculations for the $(d,{}^6\mathrm{Li})$ reaction.[13] Both parameter sets are presented in Table II. The calculated angular distributions (Fig. 2) do not show a strong diffraction structure. Since early work suggested that the spin and parity of the 3.18-MeV state in ^{62}Zn is 3^- and since quartet-model calculations predict a 4^+ state at this energy,[2] the calculations were done for $L=3$ and $L=4$ transfers. The results differ only in the fine structure of the angular distribution, therefore precluding a determination of the angular momentum transferred. The calculations show that the shape of the predicted angular distributions is not very dependent on the mass number within the Ni isotopes. This fact suggests that even without a complete angular distribution for the reaction on ^{64}Ni, a comparison of the cross sections at one angle only for the reactions on the two Ni isotopes will reflect the dependence on the nuclear structure.

Table III shows the extracted relative enhancement factors, $\xi \equiv \sigma(\mathrm{expt})/\sigma(\mathrm{DWBA})$, for the α transfer. Disregarding the strongly excited states at 3.85 and 4.04 in ^{62}Zn, which have unknown spin and parity, the ground state has the largest α-particle enhancement factor. The reduction in the cross section for ^{68}Zn relative to ^{62}Zn cannot be explained by the kinematics alone, as the DWBA calculations predict a decrease in cross section of only (20–30)%. More refined DWBA calculations, which take into account finite-range effects and the strong cluster structure of ^{6}Li, are not expected to exhibit a stronger Q-value dependence.[13] The strong decrease, therefore, suggests the presence of a blocking effect which is less pronounced than as reported in the $(^{16}\mathrm{O},{}^{12}\mathrm{C})$ experiments,[4] but larger than observed in (p,t) experiments.[14] Recent $(^3\mathrm{He},n)$ work[15] on the Ni isotopes shows a strong decrease in the cross section with neutron number similar to the present result.

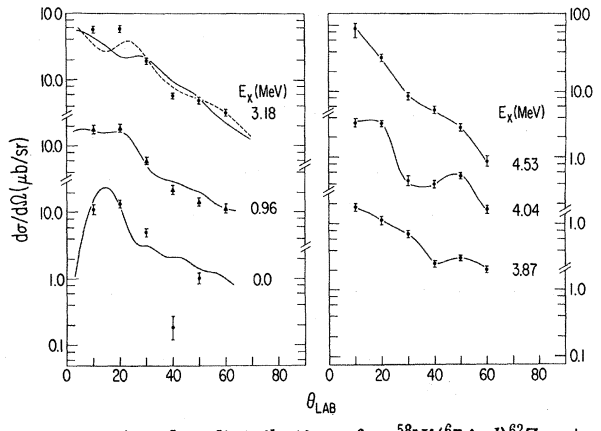

FIG. 2. Angular distributions for ^{58}Ni$(^6\mathrm{Li},d)^{62}$Zn at $E_{\mathrm{lab}}=38$ MeV. For the transitions to the states at 0.0, 0.96, and 3.18 MeV, the solid lines in the picture represent DWBA calculations with $L=0,2,4$. The dashed curve in the distribution of the 3.18-MeV state corresponds to an $L=3$ transfer. The lines in the right part of the figure are only to connect the experimental points.

In conclusion, it is now possible to investigate α-particle configurations in the fp shell via the $(^6\mathrm{Li},d)$ reaction. With the small Q-value dependence in the cross section and the good energy resolution, the $(^6\mathrm{Li},d)$ reaction should provide more spectroscopic information than is possible with the $(^{16}\mathrm{O},{}^{12}\mathrm{C})$ reaction. For the reactions studied, a blocking effect seems to decrease the α-particle enhancement factor of ^{68}Zn versus that of ^{62}Zn by a factor of ~2. With the assumption of a simple α-particle transfer, the zero-range DWBA calculations are able to describe the experimental data surprisingly well. Encouraged by these results, we have planned further investigations of this reaction in the fp shell and heavier mass

TABLE III. ^{58}Ni$(^6\mathrm{Li},d)^{62}$Zn.

E_x (MeV)	J^π	L	DWBA[a]	Expt[a]	Expt/DWBA	ξ_{rel}
0.00	$0_1{}^+$	0	61	28	0.46	1.0
0.96	$2_1{}^+$	2	455	44	0.10	0.21
3.18	(3^-)	3	724	137	0.19	0.41
	(4^+)	4	681	137	0.20	0.44

[a]$\sum_i d\sigma(\theta_i)/d\omega$ in μb/sr.

regions.

*Work supported by the U. S. Atomic Energy Commission.

†Work supported by a grant from the National Science Foundation.

[1]A. M. Friedman, H. J. Fortune, G. C. Morrison, and R. H. Siemssen, in *Proceedings of the International Conference on Nuclear Reactions Induced by Heavy Ions, Heidelberg, July 1969,* edited by R. Bock and W. R. Hering (North-Holland, Amsterdam, 1970), p. 171.

[2]H. Faraggi, A. Jaffrin, M. C. Mermaz, J. C. Faivre, J. Gastebois, B. G. Harvey, J. M. Loiseaux, and A. Papineau, Ann. Phys. (New York) 66, 905 (1971).

[3]H. Faraggi, M. C. Lemaire, J. M. Loiseaux, M. C. Mermaz, and A. Papineau, Phys. Rev. C 4, 1375 (1971).

[4]A. Cunsolo *et al.*, J. Phys. (Paris), Colloq. 6, 171 (1971).

[5]I. Rotter, Nucl. Phys. A122, 567 (1968), and A135, 378 (1969), and Fortschr. Phys. 16, 195 (1968); D. Robson, Comments Nucl. Particle Phys. 5, 16 (1972).

[6]K. Bethge, Annu. Rev. Nucl. Sci. 20, 255 (1970), and references therein.

[7]L. J. Denes, W. W. Daehnick, and R. M. Drisko, Phys. Rev. 148, 1097 (1966).

[8]H. W. Fulbright and J. A. Robbin, University of Rochester Report No. UR-NSRL-9, 1969 (unpublished), and Nucl. Instrum. Methods 71, 237 (1969).

[9]L. C. Farwell, J. J. Kraushaar, and W. W. Baer, Nucl. Phys. A186, 545 (1972).

[10]P. D. Kunz, Code DWUCK, Boulder, Colo.

[11]C. Mairle *et al.*, Nucl. Phys. A134, 180 (1969).

[12]U. C. Voos, W. von Oertzern, and R. Bock, Nucl. Phys. A135, 207 (1969).

[13]H. H. Gutbrod, H. Yoshida, and R. Bock, Nucl. Phys. A165, 240 (1971).

[14]G. Bassani, N. M. Hintz, and C. P. Kavaloski, Phys. Rev. 136, B1006 (1969).

[15]W. P. Alford, R. Lindgren, and D. Elmore, private communication.

Continued-Fraction Method for Perturbation Theory

D. F. Scofield*

Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio 45433

(Received 24 July 1972)

A direct operational approach to the solution of the Schrödinger equation has led to an iterative nonperturbative method for its solution. The method, when applied to the Mathieu equation and the anharmonic-oscillator equation, is superior to the perturbation-iteration method and to Rayleigh-Schrödinger perturbation theory both in terms of rate of convergence and range of coupling constant allowed.

In the application of the linked-cluster many-body perturbation theory (LCMBPT) to atoms[1] one must evaluate ever more topologically complicated diagrams. Moreover, the theory is not especially rapidly convergent if precise results are desired. In addition, special techniques must be employed to sum to infinite order certain classes of diagrams. Thus, it would be of advantage for practical calculations to develop a formalism which eliminates many of the difficulties with LCMBPT, but which retains as many of the virtues of LCMBPT as possible. This Letter presents a summary of a contribution to this endeavor. The method reported has as its asymptotic expansion the Rayleigh-Schrödinger perturbation theory (RSPT), but converges for the case of the anharmonic oscillator in three iterations where RSPT diverges. The method provides more rapidly convergent results for the Mathieu equation than perturbative approaches.

We begin by considering the motivation which led to the continued-fraction method (CFM) which has provided these results. Consider the time-independent Schrödinger equation $H|\alpha\rangle = E_\alpha|\alpha\rangle$ for any state $|\alpha\rangle$, assumed for simplicity to be nondegenerate and not necessarily the ground state. A splitting of the Hamiltonian is assumed of the form $H = H_0 + \lambda V$ with $H_0|\alpha\rangle_0 = E_\alpha{}^0|\alpha\rangle_0$. From here on we will suppress the α subscripts and the explicit indication of the dependence of the perturbation (λV) on the coupling constant λ. Let $P_0 = |\alpha\rangle_{00}\langle\alpha|$ and $Q_0 = 1 - P_0$ with $P_0{}^2 = P_0$ and $P_0{}^\dagger = P_0$ be projection operators onto and out of the space spanned by $|\alpha\rangle_0$. Then with the introduction of the reduced resolvent $T_0(E) = Q_0/(E - H_0)$, we can obtain the reaction operator (or transition operator) $t(E)$ for the state $|\alpha\rangle$, whose matrix elements give the level shifts

$$E - E^0 = {}_0\langle\alpha|t(E)|\alpha\rangle_0, \tag{1}$$

where

$$t(E) = V + VT_0(E)t(E). \tag{2}$$

The Lippman-Schwinger equation (2)[2] may be