New Magnetoelastic Interaction

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(Received 20 July 1972)

We propose a mechanism for the mixing of acoustic magnons and optic phonons propagating along the c axis in terbium observed at 79 K. The interaction involves an intermediate state in which an electron-hole pair is exicted virtually by the phonon and subsequently recombines into a magnon. This interaction becomes possible when the spin-orbit coupling is taken into account, and it also explains the mixing of optic magnons and acoustic phonons as seen in terbium at 200 K and dysprosium at 4.7 K.

The magnon and phonon spectra of the heavy rare earths Tb and Dy have been carefully measured along the high symmetry directions in the momentum space.[1-3] The spectra for Tb along the c axis at 79 K is reproduced in Fig. 1.[1] There are two regions of mode mixing, i.e., between transverse acoustic phonons (TA) and acoustic magnons (MA) with a gap Δ_1, and between transverse optic phonons (TO) and MA with a gap Δ_2. Jensen[1] showed that the gap Δ_1 may be explained in terms of the magnetoelastic interaction which gives rise to static magnetostriction, but the gap Δ_2 cannot be understood in this way. At 200 K the mode mixing occurs between optic magnons (MO) and TA.[2] In Dy the mixing of MO and TA modes was seen at 4.7 K.[3]

The conventional theories of magnon-phonon mode mixing are based on the following mechanisms: (1) the magnetostrictive interactions; and (2) the modification of crystal field levels by the strains caused by the phonon.[4] That these mechanisms fail to explain the Δ_2 gap may be qualitatively understood from symmetry considerations. When a phonon is excited along the c axis of a hcp crystal, all atoms in the same basal plane vibrate in phase, so the phonon may be described as rigid layers of atoms vibrating against each other. Then the arrangements of the atoms in the layers play no role in the translational symmetry of the phonon and the effective periodicity of the lattice is $\frac{1}{2}c$ instead of c. This means that the phonon spectra along the c axis may be described by the double-zone scheme. The same is true with the magnons. Thus, if the magnon and phonon spectra are plotted in the double-zone representation, the TO and MA crossing no longer occurs, and so they are not expected to mix. In order to explain the Δ_2 gap we must invoke a mode-mixing mechanism which causes a breakdown of the double-zone representation. To this end none of the above-mentioned interactions is adequate.

A symmetry-breaking mechanism of the required nature was found in the study of the electron band structure of hcp metals. Herring showed by symmetry arguments that without the spin-orbit coupling all energy levels are doubly degenerate (or quadruply degenerate with spin) across the hexagonal zone surface AHL.[5] This again implies that the electron bands may be described by the double-zone scheme. Spin-orbit coupling lifts most of the double degeneracy on the AHL plane except at the points A and L and along the lines joining them.[6,7] One can see in the calculated nonrelativistic bands of Gd that there are many accidental crossings of the 5d bands without mix-

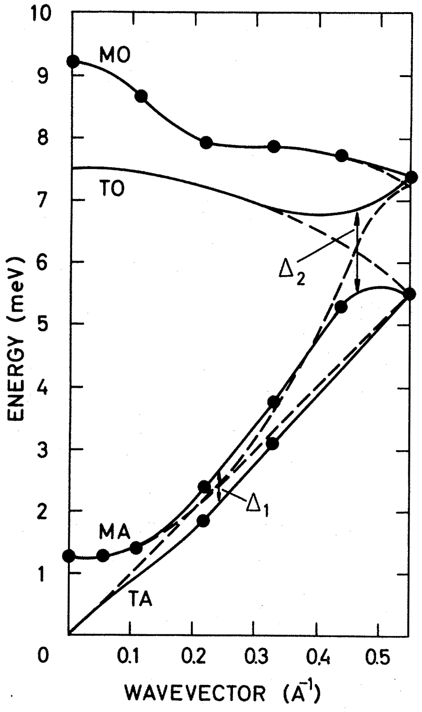

FIG. 1. Magnon and transverse phonon (TA, TO) dispersion relations for Tb in the c direction at 79 K. The magnon-phonon interaction causes a mixing of the modes and energy gaps Δ_1 and Δ_2 at the crossing points of the unperturbed dispersion relations, indicated by dashed lines.

ing,[8] while in the relativistic bands most of the crossings are removed by mixing so that the double-zone scheme is no longer valid.[9]

Electron spin-orbit terms may affect the phonon-magnon mode mixing provided that the interaction involves the conduction electrons. A phonon may be absorbed by the electron system through the excitation of an electron-hole pair. In absence of the spin-orbit interaction the pair must have the same spin. Similarly a magnon may break up into an electron-hole pair of opposite spins. So, as long as the spin is a good quantum number the two modes cannot mix through a pair intermediate state. However, when the spin states are mixed by the spin-orbit coupling, it is possible for a spin flip to occur in either the electron or the hole state so that the phonon can turn into a magnon and vice versa. At the same time the double-zone representation is broken so that the two modes can mix wherever they cross in the single-zone representation.

The application of symmetry arguments to phonons and magnons requires a rather subtle distinction. Both phonons and magnons are Bose excitations, i.e., they transform like particles with integral spin (single representations). They in themselves do not transform according to the double representations of the spin-$\frac{1}{2}$ particles, and therefore *all modes* should remain doubly degenerate on the *AHL* plane. Existing data show this effect clearly at the point *A* of the Brillouin zone.[1-3]

Along the Γ*A* line, however, two spinless modes may have their "accidental" crossover degeneracy removed, a process which involves electrons in the intermediate state and invokes the spin-orbit interaction. In this fashion we obtain a picture in which the *single-zone scheme* is the only acceptable one, but with a sticking together of bands at the *AHL* plane, characteristic of spinless modes, present.

Another salient feature of the data is that the longitudinal phonons are not mixed with the magnons. This can also be explained on the basis of symmetry alone. A necessary requirement for two modes to mix is that they must transform according to the same irreducible representation under symmetry transformations of the crystal. A longitudinal phonon propagating along the *c* axis is invariant under any 60° rotation about the *c* axis while a magnon is not because the rotation changes the direction of the magnetization. However, a transverse phonon has its polarization vector transformed the same way as the magnetization vector, so it has the same transformation property as the magnon. Then a simple consideration of the direction of the charge-density fluctuation of a phonon and the spin-density fluctuation of a magnon reveals that only the transverse phonon mode polarized perpendicular to the magnetization may mix with the magnon mode.

We will now show that the proposed interaction has the right size to explain the Δ_2 gap. It is quite straightforward to write down the expression for the mode-mixing matrix element:

$$A_{\vec{q}} = \frac{(2S)^{1/2}}{N} \sum_{n,n',\vec{k}} \frac{f_{n\vec{k}}(1-f_{n',\vec{k}+\vec{q}})}{E_{n',\vec{k}+\vec{q}} - E_{n\vec{k}}} M(n,n',\vec{q}) \times I(n,n',\vec{q})[\lambda(n,\vec{k}) + \lambda(n',\vec{k}+\vec{q})],$$

where the indices n, n' sum over the electron bands, $\vec{k}$ sums over wave vectors in the first Brillouin zone, $E_{n\vec{k}}$ is the band energy, $M(n,n',\vec{q})$ is the electron-phonon interaction matrix element, $I(n,n',\vec{q})$ is the *s-f* exchange matrix element, $\lambda(n,\vec{k})$ is the spin-mixing parameter due to the spin-orbit coupling, and S is the total angular momentum per ion. The various matrix elements are dependent on the wave vector $\vec{k}$. To a very crude approximation we replace them by constants that represent their average values. Then we find

$$A_{\vec{q}} \cong \frac{\langle M\rangle\langle\lambda\rangle}{\langle I\rangle}(2S)^{1/2} J(\vec{q}),$$

where $J(\vec{q})$ is the Fourier transform of the indirect exchange. From experiments we find $|J(\vec{q})| \cong 1$ meV for terbium[10] and 0.5 meV for dysprosium.[3] The ratio $\langle M\rangle/S\langle I\rangle$ is roughly equal to 1 because at room temperature the phonon contribution to the electrical resistivity is roughly the same as the spin disorder contribution. The spin-mixing parameter may be estimated as the ratio of one half of the spin-orbit band splitting over the *d* bandwidth. From band calculations we find $\langle\lambda\rangle = 0.03$.[9] Putting these results together we obtain $|A_q| \cong 0.7$ meV for Tb and 0.4 meV for Dy, or a total mode splitting of 1.4 and 0.8 meV, respectively. The observed values are 1.7 and 0.5 meV, respectively.

It is also easy to see that the mode-mixing matrix element is insensitive to the temperature. By contrast, the magnetostrictive interaction is at least proportional to the magnetization. Indeed at 200 and 210 K the gap Δ_1 is not observed but the mixing between MO and TA is just as strong as Δ_2 at 79 K (the Curie temperature of Tb is

215 K). We may therefore conclude that the magnetostriction is mainly responsible for the Δ_1 gap but the pair excitation mechanism proposed here is responsible for the Δ_2 gap.

It may be worth pointing out that, if our interpretation of the MA and TO mode mixing is correct, the phenomenon is the first experimental confirmation that the electron bands of heavy rare earths are strongly influenced by the spin-orbit coupling.

A part of this work was done at the Danish Atomic Research Establishment at Risø. The author would like to thank Professor H. Højgaard Jensen at the Ørsted Institute and Professor A. R. Mackintosh at Risø for their hospitality. He is also indebted to Professor L. M. Falicov and Professor Mackintosh for numerous helpful discussions, and to Professor Falicov for a critical reading of the manuscript.

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Spin Assignments to Highly Excited States in ^{24}Mg from Triple-Correlation Measurements*

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(Received 24 July 1972)

Triple correlation measurements using the reaction ^{16}O(^{12}C,α_1)^{24}Mg* $\rightarrow \alpha_2 +$ ^{20}Ne(2^+) $\rightarrow \gamma +$ ^{20}Ne(g.s.) have been made to determine spins and parities of several highly excited states in ^{24}Mg. Our results allow unambiguous assignments of spin and parity 8^+ to the states at 16.30 and 16.84 MeV. The group at 16.55 MeV is a doublet with one member having spin and parity 8^+, while the other member is most likely a 9^- state.

The observation[1] of three narrow states at energies 16.30, 16.55, and 16.84 MeV in ^{24}Mg, which are strongly populated in the reaction ^{16}O(^{12}C, α)^{24}Mg, has generated considerable speculation about their structure.[2] In particular, it has been suggested that one of these states might be the 10^+ member of the ground-state rotational band, predicted[3] to lie near 17 MeV. Initial attempts[4,5] to determine the spins of these three states were unsuccessful, principally because these states decay predominantly by α-particle emission to excited states of ^{20}Ne with nonzero spin. The theoretical angular correlation of the two α particles thus contains a sum over the unobserved spin projections in the residual state in ^{29}Ne; this summation washes out the pronounced structure of the individual transitions to the various magnetic substates which can be observed, for example, in transitions leading to spin-0 states. More recently the measurements of the α-α angular correlations were extended to extreme backward angles,[6] where the theoretical correlation still displays some sensitivity to the spin. This measurement showed that the three states all possess high spin. Subject to the assumption that the α decay proceeds via the lowest allowed L value consistent with the conservation of angular momentum and parity, the spins were suggested to be 8, 9, or 10. However, these results are sensitive to small admixtures of higher L values in the α decay.

In the present Letter we report definite assignments of 8^+ to both the 16.30- and 16.84-MeV states. The 16.55-MeV group appears to be a doublet with one of the states being 8^+ and the other one probably being 9^-. The experiment con-