ocally that A is *positive* for the $5^2D_{3/2}$ states of Rb^{87} and Rb^{85}, *negative* for the $5^2D_{5/2}$ states, and *positive* for the $7^2S_{1/2}$ states.

The anomalous D-state hyperfine structure of rubidium is reminiscent of the anomalous P-state hyperfine structure of lithium.[7] Since core polarization[8] is believed to be responsible for the anomalous P-state hyperfine structure of lithium, core polarization may also be at least partially responsible for the D-state hyperfine-structure anomaly in rubidium. Also, it is probably not coincidental that the fine-structure interval (2.96 cm^{-1}) for the $5D$ state of rubidium is about an order of magnitude smaller than one would expect from the Landé formula.[9]

These experiments demonstrate that some very interesting physics is to be found in the non-P excited states of the alkali atoms. They also demonstrate that cascade decoupling and cascade radio-frequency spectroscopy can be used to measure the properties of these states with the same sort of precision and convenience that characterized the optical double resonance and level-crossing experiments on excited P states.[3] We are presently extending these experiments to other S and D states and perhaps to F states, and we shall improve our resolution by operating at higher frequencies. We shall also set limits on the quadrupole coupling constants of these states and measure their radiative lifetimes by analyzing the widths of the resonances.

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Neoclassical Transport in Tokamaks in Banana/Plateau Regimes*

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The predictions of the full two-regime neoclassical transport theory have been obtained by numerical computation; they are compared with observations on the ST Tokamak.

Several publications[1-4] have presented computations of radial transport in Tokamak discharges, based upon incomplete or approximate sets of neoclassical transport coefficients. In Ref. 4 a code was used which incorporated the complete neoclassical transport theory, but limited itself to the lowest-collision-frequency ("banana") regime.[5] There were two important deficiencies in this treatment: First, the "banana" equations are singular at the axis where the high-collision-frequency ("plateau") regime is always entered in the complete theory; second, present-day

Tokamak experiments in fact fall into the plateau regime over much of their radial profiles, especially if the collision frequencies are enhanced by the presence of impurities.

Recently, a complete neoclassical transport theory including both banana and plateau regimes (and the transition between them) has been worked out[6]; the purpose of this Letter is to present predictions of this theory applied to the Tokamak, as obtained with two computer codes of somewhat different structure. The Princeton code solves implicitly the finite-difference equations obtained by exact space and time centering; all nonlinear difference terms are linearized, including those arising from the most significant dependences of the transport coefficients on the dependent variables. The Texas code is also centered and implicit, but uses a predictor-corrector technique which avoids linearizing the transport coefficients, so that more complicated dependences may be readily handled. Results with the two codes are in substantial agreement.

The neoclassical transport equations, incorporating a simple fit to the banana/plateau transition, are

$$\frac{\partial n}{\partial t}=-\frac{1}{r}\frac{\partial}{\partial r}(rnv), \quad \frac{\partial B_\theta}{\partial t}=c\,\frac{\partial E_z}{\partial r},$$

$$\frac{3}{2}\frac{\partial(nT_e)}{\partial t}=-\frac{1}{r}\frac{\partial(rQ_e)}{\partial r}-nevE_r+\frac{cE_z}{4\pi r}\frac{\partial(rB_\theta)}{\partial r}-\frac{3m_e}{m_i}\frac{n}{\tau_e}(T_e-T_i),$$

$$\frac{3}{2}\frac{\partial(nT_i)}{\partial t}=-\frac{1}{r}\frac{\partial(rQ_i)}{\partial r}+nevE_r+\frac{3m_e}{m_i}\frac{n}{\tau_e}(T_e-T_i),$$

$$v=-\frac{\rho_{e\theta}^2}{\tau_e}\left(\frac{r}{R}\right)^{1/2}\left\{\frac{1.12}{1+1.78\nu_e^*}\left[\left(1+\frac{T_i}{T_e}\right)\frac{n'}{n}-\frac{3}{2}\frac{T_e'}{T_e}-(\tfrac{3}{2}-y)\frac{T_i'}{T_e}\right]+\frac{1.25}{1+0.66\nu_e^*}\frac{T_e'}{T_e}\right\}-\frac{2.44}{1+0.85\nu_e^*}\frac{E_z}{B_\theta}\left(\frac{r}{R}\right)^{1/2},$$

$$Q_e=-\frac{nT_e\rho_{e\theta}^2}{\tau_e}\left(\frac{r}{R}\right)^{1/2}\left\{\frac{1.25}{1+0.66\nu_e^*}\left[\left(1+\frac{T_i}{T_e}\right)\frac{n'}{n}-\frac{3}{2}\frac{T_e'}{T_e}-(\tfrac{3}{2}-y)\frac{T_i'}{T_e}\right]+\frac{2.64}{1+0.35\nu_e^*}\frac{T_e'}{T_e}\right\}-\frac{4.35}{1+0.40\nu_e^*}\frac{nT_eE_z}{B_\theta}\left(\frac{r}{R}\right)^{1/2},$$

$$Q_i=-\frac{0.68}{1+0.36\nu_i^*}\frac{n\rho_{i\theta}^2}{\tau_i}\left(\frac{r}{R}\right)^{1/2}T_i'+ynT_iv,$$

$$E_z=\frac{\eta_\parallel}{1-1.9(r/R)^{1/2}/(1+\nu_e^*)}\left(\frac{c}{4\pi r}\frac{\partial(rB_\theta)}{\partial r}+\frac{cnT_e}{B_\theta}\left(\frac{r}{R}\right)^{1/2}\times\left\{\frac{2.44}{1+0.85\nu_e^*}\left[\left(1+\frac{T_i}{T_e}\right)\frac{n'}{n}-\frac{3}{2}\frac{T_e'}{T_e}-(\tfrac{3}{2}-y)\frac{T_i'}{T_e}\right]+\frac{4.35}{1+0.40\nu_e^*}\frac{T_e'}{T_e}\right\}\right),$$

$$E_r=\frac{T_i}{ne}\frac{\partial n}{\partial r}-\frac{1}{e}(\tfrac{3}{2}-y)\frac{\partial T_i}{\partial r}, \quad Y=\frac{1.33+3\nu_i^*}{1+\nu_i^*},$$

$$\nu_i^*=\frac{R^{3/2}B_z}{\tau_i r^{1/2}B_\theta(T_i/m_i)^{1/2}}, \quad \nu_e^*=\frac{R^{3/2}B_z}{\tau_e r^{1/2}B_\theta(T_e/m_e)^{1/2}},$$

where $\rho_{e,i\theta}=c(2m_{e,i}T_{e,i})^{1/2}/eB_\theta$, $\tau_e=3m_e^{1/2}T_e^{3/2}/4(2\pi)^{1/2}ne^4\ln\Lambda$, and $\tau_i=3m_i^{1/2}T_i^{3/2}/4\pi^{1/2}ne^4\ln\Lambda$, The other notation is standard or self-explanatory. The banana regime has $\nu_{e,i}^*\ll 1$, and the plateau regime has $\nu_{e,i}^*\gg 1$. Both codes have used the above equations; the Texas code has also investigated the effect of more exact fitting to the detailed structure of the banana/plateau transition.[6]

All results given here are for standard parameters of the ST Tokamak with hydrogen plasma ($a=14$ cm, $R=109$ cm, $I=40$ kA, $B=30$ kG, $Z=1$). The initial density profile is parabolic, $n(r)=n_c[0.8(1-r^2/a^2)+0.2]$, with $n_c=10^{13}$ cm^{-3}; $n(a)$ is held fixed at 2×10^{12} cm^{-3}. The initial electron temperature profile is also parabolic, $T_e(r)=T_{ec}\times[0.8(1-r^2/a^2)+0.2]$, with $T_{ec}=200$ eV; $T_e(a)$ is fixed at 40 eV. Initially, $T_i(r)$ is uniform at 20 eV; $T_i(a)$ is fixed at 20 eV. The skin phase of Tokamaks is circumvented by initially distributing the full current within the plasma; in Figs. 1 and 2 it is initially parabolically distributed, $j_z(r)=j_{zc}(1-r^2/a^2)$.

Profiles at 60 msec of n, T_e, T_i, E_z, and $\iota=2\pi RB_\theta/rB$ are shown in Fig. 1. Of principal in-

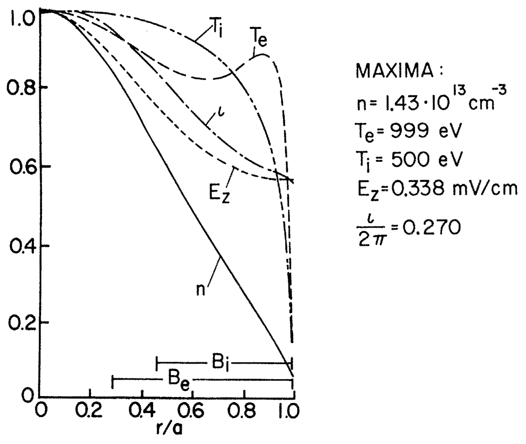

FIG. 1. Neoclassical "standard ST" case in the constant-current phase. Profiles are given at 60 msec for density n, temperatures $T_{e,i}$, electric field E_z, and rotational transform ι. Initial current distribution is parabolic: $j_z = j_{zc}(1 - r^2/a^2)$. Banana regime zones ($\nu_{e,i}{}^* < 1$) are indicated by $B_{e,i}$ (Princeton code).

terest is the time evolution of the n and T_e profiles: These are shown in Fig. 2. It may be observed that the trapped-particle pinch effect has produced a significant increase in time of the central plasma density; as expected, this is less pronounced than the increase obtained with pure banana transport.[4] The T_e profile does not become centrally peaked, but rather broadens in time qualitatively as found before. The same case has been computed with the more exact fit to the detailed banana/plateau transition. The only significant difference is a slightly more pronounced outward peak in T_e at late times.

The effect on the T_e profile of different initial j_z profiles is dramatic. In Fig. 3 we show the result of two initial j_z profiles: "peaked," $j_z(r) = j_{zc}(1 - r^2/a^2)^3$; or "flat," $j_z(r) = j_{zc}(1 - r^6/a^6)$. It is observed that these lead to T_e profiles that are strongly centrally peaked or outwardly peaked, respectively; this peaking persists to late times (>100 msec). A simple neoclassical model, in which the particle diffusion is neglected, was previously treated analytically.[7] Small T_e perturbations about an equilibrium were found to grow unstably; this instability was found to be quenched by a mild (anomalous) enhancement of electron thermal transport. Results of our present computations show that different initial conditions indeed lead to sharply different late-time T_e profiles; whereas when the electron thermal transport is enhanced somewhat, a unique T_e profile is obtained at late times, independent of initial conditions.

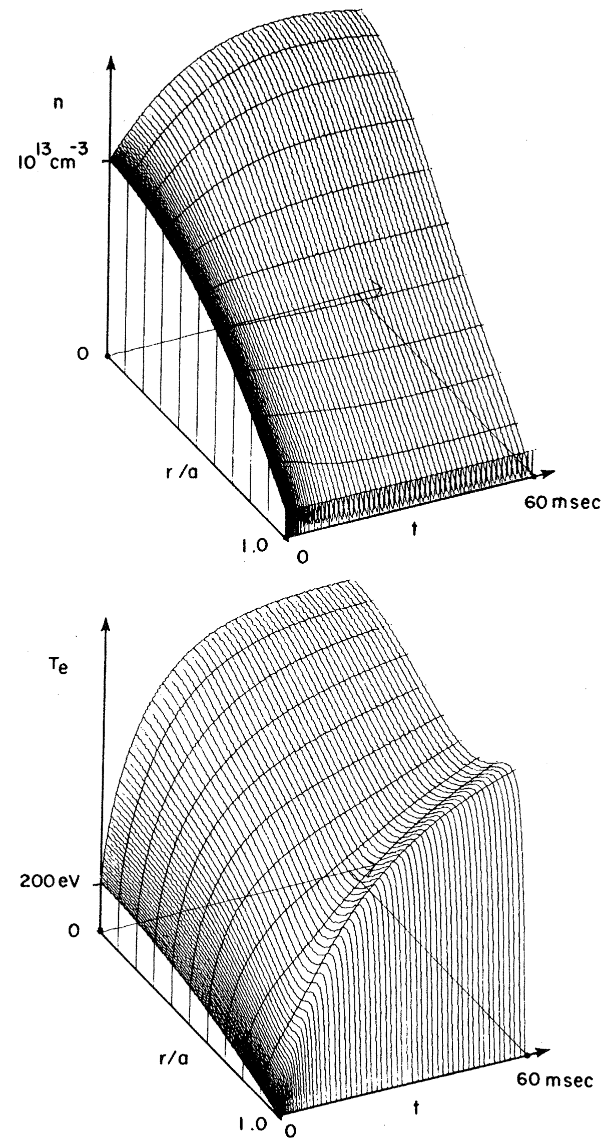

FIG. 2. As Fig. 1, showing the time evolution of the n and T_e profiles (Texas code).

Turning to the ST experiments,[8,9] we note the usual good agreement between the neoclassical value of $\beta_{\theta e} = 16\pi\int_0^a nT_e r\,dr/a^2 B_{\theta a}{}^2$, which is 0.64 in the case of Figs. 1 and 2 at 60 msec, and the typical experimental values. The experimental profiles of T_e and n, however, are in striking disagreement: The typical T_e profile is found to be strongly peaked, while the n profile is quite flat—roughly the inverse of the situation depicted in Figs. 1 and 2. The excitation of the central-peaking mode of Fig. 3 by neutral-gas edge cooling[4] may be responsible.

Though experimental values of maximum T_e roughly agree with Fig. 1, the measured E_z is

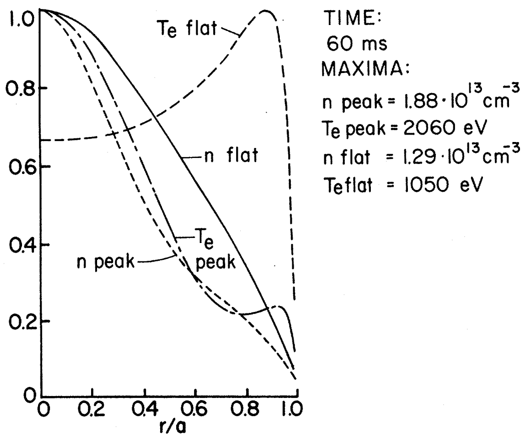

FIG. 3. Profiles of n and T_e when the initial current distribution is flat: $j_z = j_{zc}(1-r^6/a^6)$; or peaked: $j_z = j_{zc}(1-r^2/a^2)^3$ (Princeton code).

typically 10–20 times higher. This is partly because of the difference in T_e profile, and partly because of a resistivity enhancement factor of 3–6, apparently accounted for by high-Z impurities.[8,9]

A large, fundamental discrepancy is found in the particle and energy confinement times. For the case of Fig. 1 at 60 msec, we have $\tau_p = \int_0^a nr\,dr/(nrv)_a = 2.9$ sec, $\tau_{Ee}' = 1.5\int_0^a nT_e r\,dr/(rQ_e)_a = 0.88$ sec, and $\tau_{Ei}' = 1.5\int_0^a nT_i r\,dr/(rQ_i)_a = 0.90$ sec. These exceed the typical experimental values by almost 2 orders of magnitude. Part of the discrepancy is explained classically by the enhanced effective Z; a sizable anomaly remains, especially for particle transport.

In conclusion, we note that the discrepancies between neoclassical theory and experiment, previously recognized, have become even more striking as a result of the present calculations. The inclusion of trapped-particle pinching has raised the theoretical τ_p; and the proper rounding of the banana/plateau transition has raised τ_{Ee}' and τ_{Ei}'—especially the latter. The present determination of the neoclassical T_e and n profile has also established a marked profile anomaly.

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Excitation of Plasma Waves by Two Laser Beams*

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We analyze the effects of (i) the nonlinearity of the large-amplitude plasma wave, and (ii) the inhomogeneity of the plasma, on the excitation of the plasma wave by beating two laser beams.

Recently, there has been considerable interest in the nonlinear excitation of plasma waves by beating two electromagnetic waves,[1-3] both as a plasma heating mechanism for laboratory fusion devices, where the frequencies of presently available high-power lasers are too great to interact with typical confined plasmas, and as a means for studying and controlling the ionosphere. Here we wish to report the analysis of the following two important effects on this process: (1) the nonlinear behavior of the large-amplitude plasma wave; (2) the effect of an inhomogeneous plasma. First we study the growth and saturation of the large-amplitude plasma wave in a cold homogeneous collisionless plasma due to the beating of two laser beams with frequencies much above the