Test of Hadronic Scaling at Cosmic-Ray Energies

P. M. Fishbane, J. L. Newmeyer, and J. S. Trefil
Department of Physics, University of Virginia, Charlottesville, Virginia 22901
(Received 27 June 1972)

The use of data from cosmic-ray interactions with dense targets is discussed from the point of view of extracting information on scaling at high energies. We present a result which relates single-particle distributions on nuclear targets to single-particle distributions on nucleons. Contrary to the conclusion first drawn from these data, we find no serious conflict between present data and a flat pionization region.

Inclusive reactions such as $\pi+p\rightarrow\pi+$anything continue to be of great interest. Their Feynman scaling[1] properties have by now been explored at accelerator energies, but this aspect of their behavior at still higher energies requires further work. Because much of this behavior reveals itself only on a logarithmic energy scale, energies which are presently attained only in cosmic rays are important. Since the flux of primary cosmic rays falls rapidly with increasing energy, dense materials (such as nuclear emulsions) are very attractive as experimental targets. The question which we study here concerns the possibility of extracting information on (for example) the reaction $\pi+p\rightarrow\pi+$anything (which we shall call I below) when we have data on the reaction $\pi+A\rightarrow\pi+$anything, where A is a nucleus (we shall refer to this reaction as II below).

The statement of Feynman scaling for the Reaction I is that in the limit of high energies

$$d\sigma \xrightarrow[s\to\infty]{} d^2q\,dx\,x^{-1}f(x,q), \tag{1}$$

where $x=2p_{\parallel}^{\text{c.m.}}/s^{1/2}$ and q is the transverse momentum of the detected pion. Equation (1) contains the (experimental) observation that q is limited in magnitude with a scale of ~200 MeV.

Experimental questions of interest are these: (i) Does Feynman scaling as expressed by (1) or its equivalent in terms of the rapidity r $[r\sim\ln(s)^{1/2}\times x/m]$ hold true? (ii) If so, then what is the shape of the scaling function f for (1)? In general, it has been hypothesized that f is split into three regions, according to whether r is near the projectile or target rapidity (projectile or target fragmentation region), or in a large region in between whose width in r space grows like $\ln s$ (pionization region).

It has further been hypothesized that f will be flat in the pionization region.[2] A recent analysis of cosmic-ray interactions in emulsions has been advanced as evidence that the scaling function at 1 TeV is markedly different from this form in the pionization region.[3,4] For future reference, we note that the behavior of f in the pionization region is related to the multiplicity of a reaction of type I, since

$$\langle n\rangle=\frac{1}{\sigma_{\text{inel}}}\int\frac{d\sigma}{dr\,dq^2}\,dr\,dq^2,$$

so that a flat pionization region implies $\langle n\rangle=n_0\ln s$.

It should be clear from the above discussion that if we wish to test scaling with nuclear emulsion data, we must be able to calculate the effect of the nucleus on measured inclusive cross sections. This is not a simple procedure, since it is possible that an intranuclear cascade (see Fig. 1) can develop. We shall devote the remainder of this note to showing (a) how single-particle distributions for Reaction I are related to the distribution for Reaction II, (b) that the nuclear effects are most important in the pionization region, and least important at high rapidities, and (c) that when cascading is taken into account, current data are consistent with a flat pionization region.

We begin by assuming a scaling function $f_0(r_0;$

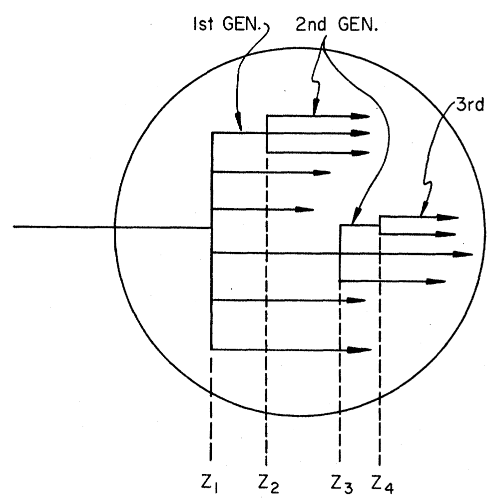

FIG. 1. A typical intranuclear cascade.

r, q) for I which is of a basic multiperipheral model (MPM) shape.[5] $f_0(r_0; r, q)$ is strongly peaked for small q (the results are essentially independent of the precise manner), and is a constant (=1) in r from 0 to r_0 (we always work in the laboratory frame) and zero otherwise. We assume a simple factorized form, $f_0(r_0; r, q_0) = g(r_0, r)|h(\vec{q})|^2$, where $|h(\vec{q})|^2 = (\sigma_T^2/16\pi^2)\exp(-aq^2)$, h pure imaginary. r_0 is simply equal to the multiplicity φ; we ignore that spread in the multiplicity distribution and assume asymptotically constant cross sections. This choice of shape evidently allows us to concentrate on the important pionization region. We assume that this f describes every reaction in the cascade, and ignore the possibility that energy degradation in the cascade may cause some of the downstream reactions to deviate from scaling simply because they are not kinematically in the scaling region.

In order to fold in the nuclear physics, we make use of the Glauber theory,[6,7] which is well known to describe the interactions of particles and nuclei at high energies. We are dealing with a process in which an arbitrary number, N, of inelastic collisions occurs on a nucleus, and in which the final state of the nucleus is not measured. The theoretical expression for the cross section for this process has been worked out,[8] and will be described in detail in a subsequent publication. The description of an anfractuous process of the type in Fig. 1, while straightforward, is somewhat cumbersome to write down in detail, and we shall restrict ourselves to a description of the calculation here.

Since the main contributions to multiparticle production come from incoherent events, we apply closure over final nuclear states, and integrate over the momentum transfer to the nucleus to get production cross sections. We can then proceed as follows: We write the Glauber multiple-scattering series for the incident particle to scatter elastically any number (including zero) of times on nucleons to the left of z_1. This introduces a factor

$$\prod_{z_j<z_1}\{[1-\Gamma(\vec{B}-\vec{s}_j)][1-\Gamma^*(\vec{B}-\vec{s}_j)]\}$$

into the cross section, where the profile function Γ is the Fourier transform of the elastic scattering amplitude, B is the impact parameter, and $\vec{s}_j$ is the projection of the coordinate of the jth nucleon on the impact-parameter plane. At z_1, the incident particle scatters inelastically, producing secondaries (in Fig. 1, $n_1=6$). This is described by a term $\Gamma_{12}^*(B-s_1)\Gamma_{12}(B-s_1)$, where Γ_{12} is the Fourier transform of the inelastic scattering amplitude. We then write the multiple-scattering series for the n_1 particles (we call these the "first generation" particles) to scatter elastically any number of times on the nucleons between z_1 and z_2, which introduces a factor

$$\prod_{z_1<z_l<z_2}\{[1-\Gamma(B-s_l)][1-\Gamma^*(B-s_l)]\}^{n_1}.$$

At z_2, another inelastic collision occurs, which we handle as above, and then another elastic series, this time for n_2 particles to go from z_2 to z_3 ($n_2=8$ in Fig. 1). This introduces a factor

$$\prod_{z_2<z_n<z_3}\{[1-\Gamma(B-s_n)][1-\Gamma^*(B-s_n)]\}^{n_2}.$$

We can break these n_2 particles up into those produced at z_2 (which we call second generation) and those produced at z_1 which move past z_2 (which are still first generation). We can obviously go on in this way to build up any chain of cascades in the nucleus that we desire.

In general, the expressions which result from such a procedure are too complicated to handle simply. We therefore make a number of approximations:

(a) The fact that inelastic collisons occur at different depths z_1 within the nucleus makes the handling of the elastic scattering series very difficult. Therefore we make the "rim" approximation, in which we assume that all of the elastic scatterings of produced particles start at the midpoint of the nucleus, $z=0$, while the inelastic collisions can occur anywhere.[9] Since this approximation errs in opposite directions for inelastic collisions to the left and right of $z=0$, we expect the final result to be insensitive to the approximation. Numerical investigation indicates that this is so.

(b) An exact treatment of our MPM input would require that the multiplicity of a collision depend on the energy of the projectile and the energy of a cascading projectile depends on its position in the previous MPM chain. Labeling the "generation" of a particle as the number of inelastic collisions by which it is removed from the primary projectile, as described above, we make the simplifying assumption that the average multiplicity produced in the inelastic collision of an nth generation secondary with a nucleon is φ/n. Thus, the multiplicity of an inelastic collision of a first generation particle within the nucleus is $\frac{1}{2}\varphi$. This underestimates the output of a first generation particle produced from the top of the chain, but overestimates the output of one from the bottom of the chain. Since we take the production cross section to be independent of energy, it is easy to show that the error involved in this approximation is of order $1/\varphi$.

(c) We ignore the conservation of energy in the production of particles on the MPM chain. At very high energies this is an unimportant effect, but this is not the case where rest masses cannot be ignored. We shall discuss the consequences of this error below.

Once the inclusive cross section for a given sequence of collisions has been written down, we have only to add up all of the cascades. The following general features come out of the derivations:

(1) The problem of counting the number of ways a given final state can be made by different chains is straightforward and does not depend on the Glauber theory. It will be discussed elsewhere.[8] Since the probability of an inelastic collision is proportional to $\sigma_{\rm inel}$, we expect a particular sequence to vanish as $(\sigma_{\rm inel}/\sigma_T)^N$ as N, the number of inelastic collisions, becomes large. In practice, we found that contributions from $N > 5$ could be neglected.

(2) Although our original MPM distribution is flat as a function of r, we find that the distribution expected from a nucleus is skewed toward small r. The reason for this is quite simple, and has to do with the fact that a particle cannot produce secondaries whose energies exceed its own. Thus, particles to the right in the chain in Fig. 1 will produce lower rapidity secondaries, and only the leading particle can produce the highest rapidity.

(3) For any given sequence, there are n final particles which must propagate out of the nucleus by any number of elastic collisions from $z=0$ to $z=+\infty$. The factor $[(1-\Gamma)(1-\Gamma^*)]^n$ discussed above expresses this. We find that this factor gives a strong nonclassical effect for large n, such that these terms are damped far less than expected. In the ordinary Glauber theory, where $n=1$, this effect, due in detail to cross terms in Γ, is 20%, whereas in the results we report below we find this effect decreases the damping terms by factors of 10 or more.[10]

In Fig. 2, we show the particle distributions for a typical emulsion nucleus, taking $\varphi=10$ (which is appropriate in the TeV range), and taking $\sigma_T = 26$ mb and $\sigma_{\rm in} = 22$ mb. These cross sections are appropriate for pions, which constitute most of the produced particles, but if we used numbers corresponding to incident protons, the results

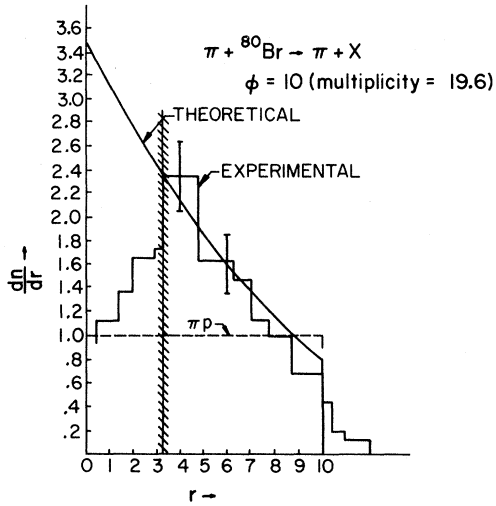

FIG. 2. The solid line in this figure illustrates the theoretical single-particle inclusive buildup at lower rapidities due to intranuclear cascading, with the data of Ref. 3 superimposed. The nucleus is ^{80}Br, and the incident proton's rapidity was taken as $\varphi=10$, corresponding to ~ 20 TeV. The step-function distribution assumed for $\pi + {}^1\mathrm{H} \to \pi + X$ is shown as a dashed line. The disagreement of theory and experiment at low r is discussed in the text.

would be indistinguishable from those shown in the figure. As expected, the distribution is quite different from that for an individual nucleon, and shows a marked buildup at small rapidities.

Before comparing these results with actual emulsion data, one important point should be noted. Although the nucleon and nuclear distributions differ markedly at small r, at large r they are quite similar. Therefore, one way of using these results might be to look only at high-rapidity particles in emulsions. For example, if we wish to work in a region where the effects of the nucleus are less than 10%, we should confine ourselves to the upper 20% of the rapidity plot (note that because of the definition of rapidity, this corresponds to the upper 86% of the longitudinal momentum range). Future experiments could be analyzed in this way, which has the virtue of being largely independent of the nuclear physics.

If, on the other hand, an experimentalist wished to use the entire available r interval, then he must use the entire results of the calculation we are reporting here. Such an experiment, the result of an analysis of nuclear emulsion data at $E \approx 10^4$ GeV, was recently reported[3] and is also shown in Fig. 2. The results were used to argue against a flat pionization region. It is clear from

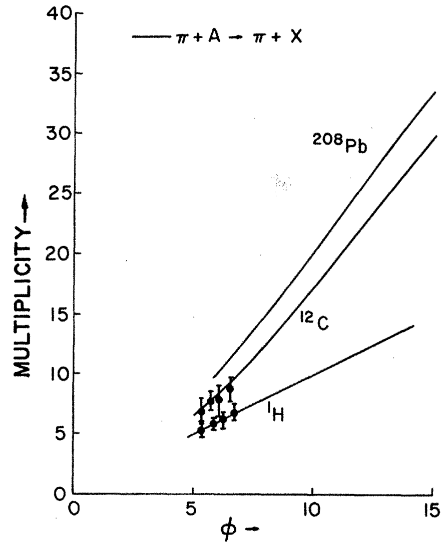

FIG. 3. The solid lines represent pion multiplicities on ^{1}H, ^{12}C, and ^{208}Pb as a function of incident pion rapidity, using the single-particle distribution discussed in the text. The experimental points for ^{1}H and ^{12}C are from Ref. 10, normalized to match the theory at the 200-GeV (lowest) hydrogen point.

Fig. 2 that the experimental results are actually compatible with a flat pionization region if proper care is taken in handling the nuclear physics. It should be noted that the difference between theory and experiment at small rapidities can be attributed to the facts that (i) in Ref. 3, low-rapidity events were excluded by hand, so that the experimental numbers underestimate the true distribution in the regions, and (ii) the assumptions that scaling holds to low energies and that energy conservation can be neglected lead to a theoretical overestimate of small-rapidity events.

In order to check our method, we have explicitly calculated multiplicities for particle-nucleus interactions and compared the results with experiment.[11] In Fig. 3 we show the expected multiplicities from various nuclei, as a function of φ, together with the experimental points of Ref. 10 scaled up by a factor of $\frac{3}{2}$ to take rough account of the production of neutrals and then suitably normalized to the hydrogen data. We see that the agreement is quite good, a check which gives confidence that other features of intranuclear cascades will be correctly explained by our results. It is also interesting that, according to Fig. 3, if the input multiplicity increases as $\ln s$ then the multiplicity on a nuclear target increases faster than $\ln s$. (The larger the nuclear target, the faster the multiplicity increase, although the A dependence is relatively weak.)

In summary, inclusive cross-section experiments on nuclei can provide useful information on the corresponding quantities with nucleon targets. This offers a decided advantage to the experimentalist who wishes to use cosmic rays to study very high energies.

A more detailed version of this report will appear elsewhere.[8]

---

[1] R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969).

[2] J. Beneke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. 188, 2159 (1969).

[3] L. von Lindern, R. S. Panvini, J. Hanlon, and E. O. Salant, Phys. Rev. Lett. 27, 1745 (1971).

[4] For a different treatment of these data, see A. Krzywicki and B. Petersson, Phys. Rev. D 6, 924 (1972).

[5] K. A. Ter-Martirosyan, Nucl. Phys. 68, 591 (1965); see also F. Zachariasen and G. Zweig, Phys. Rev. 160, 1322 (1967).

[6] R. J. Glauber, in *High Energy Physics and Nuclear Structure*, edited by G. Alexander (North-Holland, Amsterdam, 1967), p. 311 *et seq.*

[7] J. S. Trefil and F. von Hippel, Argone Report No. ANL/HEP 458 (to be published), and references contained therein.

[8] P. M. Fishbane, J. L. Newmeyer, and J. S. Trefil, to be published.

[9] J. S. Trefil, Phys. Rev. D 3, 1615 (1971).

[10] P. M. Fishbane, J. L. Newmeyer, and J. S. Trefil, to be published.

[11] L. W. Jones, in *Proceedings of the International Conference for Particle Reactions at the New Accelerators, Madison, Wisconsin, 1970* (Physics Department, University of Wisconsin, Madison, Wis., 1970).