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Superconducting Fountain Effect

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An effect analogous to the fountain effect in superfluid helium has been observed in a superconductor for the first time. A temperature gradient across a Pb-Pb point-contact junction produced a small supercurrent, which was detected by measuring the asymmetry of the critical current.

To fountain effect, or thermomechanical effect, in superfluid liquid helium was discovered by Allen and Jones[1] in 1938. When two vessels containing superfluid helium are joined by a thin capillary and a temperature difference is maintained between them, an osmotic pressure difference is set up because of the different concentrations of normal fluid and superfluid in each. The osmotic pressure difference drives superfluid from the colder vessel to the hotter until a hydrostatic pressure difference is set up which balances the osmotic pressure difference. In the steady state, normal fluid flows hydrodynamically from the hotter vessel to the colder, driven by the hydrodynamic pressure difference, and there is an equal flow of superfluid in the opposite direction. This steady-state counterflow is responsible for the large thermal conductance of superfluid helium. In very thin capillaries the flow of normal fluid is greatly reduced, and the thermal conductance appears much smaller.

The similarities between the superfluid properties of liquid helium and superconductors suggest that an analogous effect should occur in a superconductor.[2] In this Letter, we report the first experimental observation of this analog.

In a superconductor, the analog of the hydrostatic potential difference observed in helium in the presence of a temperature gradient is an electrostatic potential difference. It is important to realize that in any real metal the quasiparticle-lattice scattering rate is much higher than the quasiparticle-quasiparticle scattering rate. This means that hydrodynamic flow of the quasiparticles is strongly inhibited.[2,3] Thus the electrostatic potential difference only gives rise to a small normal-fluid flow, and the hydrodynamic counterflow of normal and superfluids makes only a very small contribution to the thermal conductivity of a superconductor. This is analogous to the case of helium in a *very* thin capillary, where the normal fluid is essentially clamped.

Luttinger[4] has derived a pair of equations describing electrical and thermal transport in a superconductor; his results were subsequently confirmed by a microscopic calculation.[5] His equation for the quasiparticle electric current density $\vec{j}_n$ in a superconductor in which there exists a thermal gradient ∇T may be written in the form[6]

$$\vec{j}_n = \sigma(\vec{E} - \nabla\mu/e) - \sigma Q \nabla T. \quad (1)$$

The Bernoulli term $\frac{1}{2}mv_s^2$ has been included in the chemical potential μ, and σ and Q are the electrical conductivity and thermopower of the quasiparticles. As the temperature is reduced below the transition temperature, σ and Q will drop below their normal-state values because of the decrease in the number of quasiparticles and the effect of the energy gap.

Let us apply Eq. (1) to a rod AB, the ends of which are maintained at temperatures T_A and T_B ($T_A > T_B$). Now if the rod were a normal metal, an electrochemical potential difference would be set up between the ends, so that no net electric current would flow. However, in the case of a superconductor, there can be no electrochemical potential difference in the steady state. An electrostatic field is created which cancels the gradient of the chemical potential, so that the terms in parentheses in Eq. (1) cancel. The electrostatic potential difference between the ends of the rod is given by

$$\varphi_A - \varphi_B = [\mu(T_A) - \mu(T_B)]/e. \quad (2)$$

A temperature difference of 1 K should produce a potential difference of 10^{-8}–10^{-7} V. It must be emphasized that this is an *electrostatic* potential difference, and can only be measured with an electrostatic voltmeter, for example, a vibrating capacitor voltmeter. Equation (1) reduces to $\vec{j}_n = -\sigma Q \nabla T$, and this quasiparticle current density is balanced by a supercurrent density, $\vec{j}_s = -\vec{j}_n$. For small temperature differences ($T_A - T_B \ll T_A, T_B$), we can ignore the temperature dependences of σ and Q, and obtain the result

$$i_s = -i_n = -Q(T_A - T_B)/R, \quad (3)$$

where R is the quasiparticle resistance of the rod.

Now suppose a temperature difference is maintained across a Josephson weak link[7] AB, again with $T_A > T_B$. The thermally produced supercurrent will generate a phase difference $\sin^{-1}(i_s/i_c)$ across the link, where i_c is the critical current. If the critical current of the weak link is measured with an external current source, one should require a smaller external current i_e to exceed the critical current when i_e is in the same direction as i_s than when it is in the opposing direction. If the thermopower is negative (which is the case if it is dominated by diffusion of electrons), the thermally generated supercurrent (flow of positive charge) flows from the hotter (A) to the colder (B) end. Thus one would expect that the critical current which can flow from A to B would be smaller by $2i_s$ than the critical current which can flow from B to A.

The magnitude of this asymmetry is rather small. Consider for example a lead point contact with a 1-K temperature difference across it, close to its transition temperature. If we take Q to be normal-state thermopower, about -0.2 μV K^{-1}, and R to be the normal-state junction resistance, say 1 Ω, we find the asymmetry to be $2i_s \simeq 0.4$ μA.

Our direct observation of the superfluid backflow produced by a temperature gradient was made with a lead-lead point contact (Fig. 1). A lead wire, which was $\frac{1}{16}$ in. in diameter and 99.999% pure, was attached to a differential screw having a pitch of 0.015 cm. The screw could be adjusted from outside the cryostat so the wire could be brought into contact with a lead foil (0.05 cm thick, 99.999% pure). The end of the wire was sharpened to a point with carborundum paper and cleaned with acetic acid. The point contact was mounted in a vacuum can which could be immersed in liquid helium at 4.2 K. The wire and

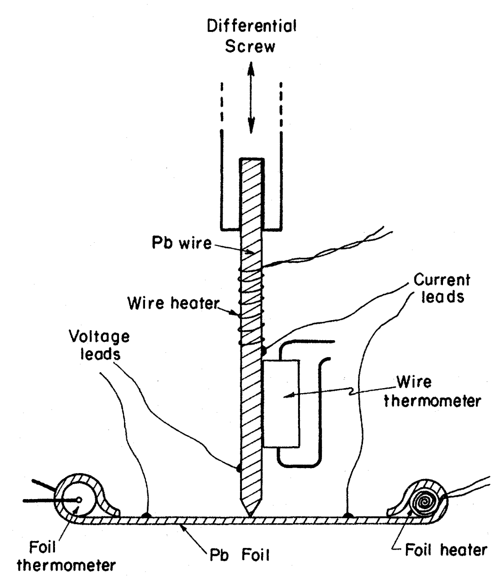

FIG. 1. The Pb–Pb point contact used for observing the superfluid backflow.

foil were independently thermally linked to the vacuum can with copper wires having a conductance of about 3×10^{-3} W K^{-1}. The temperatures of wire and foil were measured with 56-Ω, $\frac{1}{8}$-W, Allen-Bradley carbon resistors using an ac bridge, and heat was supplied to them using direct current through 100-Ω resistance heaters wound noninductively from 0.007-cm-diam Manganin wire. Thermometers and heaters were thermally bonded to the lead with GE7031 varnish. Copper current leads and niobium-tin voltage leads were connected to wire and foil, and the leads were brought through an epoxy seal in the top of the vacuum can. The vacuum can was coated with solder which was superconducting at 4.2 K, and two cylindrical concentric Mumetal cans outside the Dewar reduced the ambient magnetic field to less than 3 mG. The experiment was performed in a screened room, with double copper walls and filtered mains supply, to further reduce electrical interference.

The point contact was adjusted to obtain a critical current of less than 1 mA at 4.2 K. Two methods for determining the asymmetry were used. The first measured the critical current in each direction using an ac modulation technique. A small sinusoidal current (0.2 μA peak to peak, 280 Hz) was superimposed on a direct current. The direct current was slowly increased until an alternating voltage across the junction was detected with a lock-in detector. The corresponding value of the direct current was taken as the critical current. The second method used a symmetrical current ramp and allowed the sum and difference of the positive- and negative-going critical currents to be plotted directly on an x-y recorder.

Because of the small size of the asymmetry, measurements were only made close to the transition temperature of the junction. Sufficient heat was supplied to one side of the junction to reduce the critical current to about 10 μA, and the asymmetry was then measured as a function of critical current as the latter was reduced by further increasing the heat supplied. Close to the critical temperature of the junction, the critical current varied very rapidly with temperature, so that the temperature difference across the junction was approximately constant for critical currents between 0 and 10 μA. The asymmetry measurements were made with the temperature gradient in both directions, and also with no gradient by heating both sides simultaneously.

In Fig. 2(a) we show a result obtained using the point-by-point ac modulation technique. The asymmetry is defined as $\delta I_c = I_{c(wf)} - I_{c(fw)}$, the difference between the critical current measured from wire to foil, $I_{c(wf)}$, and the critical current measured from foil to wire, $I_{c(fw)}$. An asymmetry due to the temperature gradient is clearly present. Figure 2(b) shows a result obtained with the automatic asymmetry measuring circuit, and again the thermally produced asymmetry can be seen. The difference between asymmetries for the two opposing directions of temperature gradient is practically independent of critical current. This is as expected, since the asymmetry should depend on the temperature difference only, which changes very little in the region of the measurements.

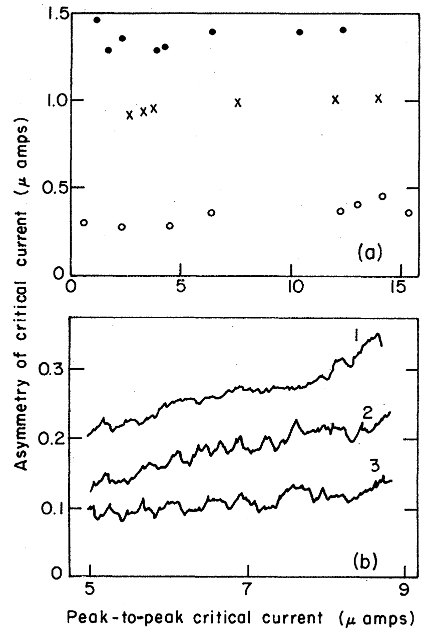

FIG. 2. The dependence of the asymmetry of the critical current on the peak-to-peak critical current (i.e., sum of critical currents in opposite directions). (a) Results from ac modulation method: closed circles, $T_f = 7.1$ K, $T_w = 5.3$ K; crosses, $T_f = T_w = 6.9$ K; open circles, $T_f = 5.3$ K, $T_w = 7.0$ K. (b) Results from automatic measurement circuit: curve 1, $T_f = 7.1$ K, $T_w = 5.7$ K; curve 2, $T_f = T_w = 7.1$ K; curve 3, $T_f = 4.4$ K, $T_w = 7.1$ K.

The thermally produced asymmetry is in the sense predicted. Lead has a negative thermopower, and one would therefore expect (see earlier discussion) that $I_{c(wf)} > I_{c(fw)}$ when the foil was heated, and that $I_{c(wf)} < I_{c(fw)}$ when the wire was heated. According to our definition, we should expect a positive asymmetry in the former case, a negative asymmetry in the latter, and of course zero asymmetry if both foil and wire were heated to the same temperature. The results shown in Figs. 2(a) and 2(b) illustrate these results, although there is a small additional asymmetry which is independent of temperature gradient.

Successful measurements of the asymmetry were frequently frustrated by the mechanical and electrical instability of the point-contact junctions. Small mechanical vibrations or electrical switching transients caused discontinuous and irreversible changes in the I-V characteristics of the junction. However, seventeen different asymmetry measurements were made. The magnitude of the measured asymmetry varied from 8 times less than that predicted by Eq. (3) to almost 7 times greater. This spread can be attributed to the widely different structures obtained each time the junction was remade. Thus, the effective length and area of the junction, the amount of oxide between the two surfaces, and the straining of the metal were all likely to vary enormously. In particular, the region across which the temperature gradient existed was probably very ill-defined, so that the effective quasiparticle resistance R was also correspondingly badly defined, and not necessarily given by the resistance of the I-V characteristic. With these factors in mind, we feel that the spread in our data is acceptable. It is important to note that although the magnitude of the asymmetry was very irreproducible, the sense was always as expected, a fact which gives us confidence that we were indeed observing the superfluid backflow.

In summary, we have observed an asymmetry in the critical current of a Pb-Pb point contact induced by a temperature difference across the junction. We ascribe this asymmetry to the counterflow of supercurrent analogous to the counterflow of superfluid helium observed in the fountain effect. The asymmetry was always in the expected direction, and of approximately the expected magnitude.

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Feynman-Graph Expansion for the Equation of State near the Critical Point (Ising-like Case)*

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The scaling equation of state of an Ising-like ferromagnet is derived by an expansion in $\epsilon = 4 - d$, where d is the dimension of space. The result is compared with numerical calculations on the three-dimensional Ising model. It is also established that the "linear model" is exact up to order ϵ^2.

The Feynman-graph method used previously for calculating critical exponents[1] is used here to obtain the equation of state near the critical point. It is calculated in an expansion in $\epsilon = 4 - d$, where d is the dimension of space. The calculation is performed for an Ising-like ferromagnet, but the result may be applied to liquid-gas transitions and other critical points by relabeling the variables.[2] The equation of state is obtained in the scaling form predicted by Widom and others. In terms of the magnetic field H, the magnetization M, and the reduced temperature $t = (T - T_c)/T_c$, a