as before, but now $|\vec{k}_p| \approx 2k_L \approx 2\omega_L/c$, whereas in the parallel case $k_p \approx \omega_p/c \ll 2\omega_L/c$. As a result, the wave coupling on the right-hand side of Eq. (1b), proportional to k_p^2, is greatly enhanced. To balance the advantage of enhanced coupling are two disadvantages and one further advantage. First, the damping rate may be greatly enhanced by Landau damping (since now $\omega_p/k_p \ll c$), beyond the optimum $\gamma \sim \Delta$. Secondly, the further transition to $L-2$ cannot be induced by the longitudinal mode $\vec{k}_p$, since this would require $\vec{k}_{L-2} = \vec{k}_{L-1} - \vec{k}_p = 2\vec{k}_{L-1} - \vec{k}_L$, or $k_{L-2} \approx 3k_L$, violating the dispersion relation. This means that the further decay must be induced instead by a third laser beam $L-2$ in any desired direction, and the corresponding longitudinal wave excited, $\vec{k}_p' = \vec{k}_{L-1} - \vec{k}_{L-2}$, is not the same as $\vec{k}_p$. Thirdly, no energy is lost on up-conversion, since each transition must be seeded by its own laser beam.

The idea of using laser beats to heat a plasma was suggested to us by M. Rosenbluth and N. Kroll. Discussions with Y.-R. Shen and C. Townes were helpful for orientation. S. Bodner, W. Kunkel, D. Nicholson, and R. Riddell provided us with good advice and encouragement.

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Effect of Superconducting Fluctuations on the Spin Relaxation of Quasi–One-Dimensional Compounds

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An investigation is made of the effect of order-parameter fluctuations on the nuclear relaxation time of superconducting $A15$ compounds. It is found that because of the one-dimensional chain structure of these compounds the fluctuations make an important contribution to the NMR.

The nuclear relaxation time T_1 in superconducting compounds with the composition V_3X (X = Si, Ge, etc.) was observed[1] to display an anomaly above the transition temperature T_0; $1/T_1T$ was observed to increase by about 20% at temperatures a few degrees above T_c $[(T-T_c)/T_c \approx 0.2]$. At the time at which these observations were made, it was not clear to what cause this anomaly should be attributed; V_3Si undergoes a martensitic transformation[2] at about 21°K, and this transformation causes anomalies in the Knight-shift and quadrupolar interactions,[3] and therefore an anomaly in $1/T_1T$ is not too surprising; V_3Ga is rarely pure and usually some Ga atoms occupy V sites; in addition, all these compounds display anomalies in their electronic properties which may be attributed to a sharp peak in the density of states function,[4] and conceivably an anomaly in $1/T_1T$ may be attributed to this peak. Recently,[5] experiments on Nb_3Al indicated a similar anomaly

in $1/T_1T$ of Al^{27} several degrees above T_c. Nb_2Al does not undergo a martensitic transformation, nor does it possess large anomalies in the electronic properties in the normal state[6] (as large as in Nb_3Sn, say). Therefore, it appears that the anomaly in $1/T_1T$ is of a more general nature than the martensitic transformation in these compounds, or even the existence of a sharp peak in the density of states.

All these compounds possess the $A15$ (β-W) crystal structure in which the V (or Nb) atoms are arranged in three interpenetrating families of linear chains. If the coupling between the chains is not very large, the system may possess some quasi-one-dimensional properties; namely, some sub-bands of the $3d$ (or $4d$) band may possess planar constant-energy surfaces in some parts of the Brillouin zone, the appropriate wave functions being localized on the chains perpendicular to these surfaces,[7] describing electrons moving along these chains. If the Fermi surface happens to be planar and near a $K_x = 0$ (or, sometimes, a zone boundary) plane, a sharp peak in the density of states may result[8]; but for a general value of K_F, where no peak in the density of states need occur, quasi-one-dimensional properties are retained. Therefore, it may prove instructive to attempt to estimate the anomaly in $1/T_1T$ of "pure" one-dimensional systems, caused by their enhanced critical fluctuations, although it is clear that such a model is an idealization and cannot be regarded as a quantitative theory for these materials.

The effect of order-parameter fluctuations on the properties of superconductors above T_c has been studied recently by many authors.[9,10] It was shown that the effects on transport properties like the electrical resistivity, for example, are the more pronounced the lower the dimensionality of the system. Therefore, the $A15$ compounds seem to present good conditions for studies of the superconducting order-parameter fluctuations.

In this note we present a semimicroscopic calculation of the nuclear relaxation time T_1 in superconductors above the transition temperature. We limit ourselves to the so-called classical critical region of fluctuations. We extend the Aslamazov-Larkin[9] (AL) theory to calculate T_1 by using a simple BCS model for superconductivity. The crucial step, taking into account the linear chain structure of the transition-metal atoms in the $A15$ crystals, is made only in the final part of the calculation, when the fluctuation propagator of the order-parameter field is assumed to be one-dimensional.

The nuclear spin relaxation time can be written in terms of the electron spin-spin correlation function $\chi(\vec{k}, \omega)$ as

$$(T_1T)^{-1} \propto \mathrm{Im}\textstyle\sum_{\vec{k}} \chi(\vec{k}, \omega). \quad (1)$$

The contribution to T_1^{-1} of the superconducting order-parameter fluctuations is then given by

$$\left(\frac{1}{T_1T}\right)_{\mathrm{fluct}} = \left(\frac{1}{T_1T}\right)_0 \lim_{\omega \to 0} \frac{1}{\pi\omega N_0^2} \mathrm{Im}L(\omega), \quad (2)$$

where $(1/T_1T)_0$ results if one neglects order-parameter fluctuations, N_0 is the electron density of states at the Fermi energy, and

$$L(\omega) = \textstyle\sum_{\vec{k}} \chi_{\mathrm{fluct}}(\vec{k}, \omega) = \sum_{\vec{k}} \langle s_{\vec{k}}^{-} s_{-\vec{k}}^{+} \rangle_{\omega}^{\mathrm{fluct}}. \quad (3)$$

Here $s_{\vec{k}}$ is the electron spin density operator. To calculate the electron spin correlation function we employ the standard thermal Green's-function diagram technique.[9] As shown in Figs. 1(a) and 1(b), each diagram contributing to $\chi(\vec{k}, \omega_n)$ starts and ends with an "external" vertex which carries $\vec{k}, \omega_n$ and *flips* the spin of the electrons, thus creating an electron-hole pair with *opposite* spins. This is in contrast to diagrams which appear for

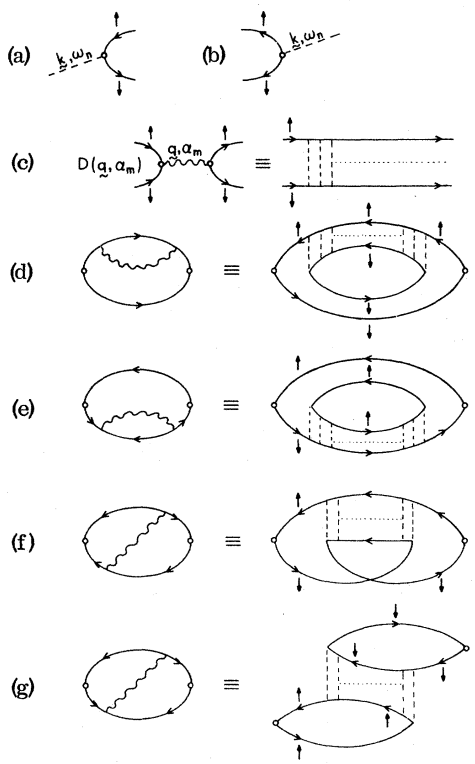

FIG. 1. Graphical illustrations of the various contributions to $\chi(k,\omega)$. (g) contributes only to electrical resistivity (Ref. 9).

the electrical resistivity calculation,[9] where the electron-hole pair is created with the same spin. This fact is of considerable importance in comparing the effect of fluctuations on the resistivity and $1/T_1$. To obtain the normal-state spin-correlation function $L_0(\omega)$, which neglects Cooper-pair fluctuations, we connect diagrams 1(a) and 1(b) and find $\mathrm{Im}L_0(\omega) = \pi\omega N_0^2$.[2]

To lowest order in the Cooper-pair fluctuations, we insert between diagrams 1(a) and 1(b) in all possible ways the Cooper-pair fluctuation propagator $D(\vec{q}, \omega_m)$ shown in Fig. 1(c). Here $\vec{q}$ and ω_m are the "momentum" and "energy" of the particle-particle ladder of opposite spins, and the dashed lines correspond to the BCS electron-electron interaction with coupling strength g. The leading contributions to $\chi(\vec{k}, \omega_n)$ due to fluctuations are depicted in Figs. 1(d), 1(e), and 1(f).

To indicate the sign of the contribution to χ of each diagram, we have given also the details of each diagram. We notice that the diagrams 1(d) and 1(f) are encountered also in the resistivity calculation, and they carry the minus sign since they involve a closed fermion loop. The diagram 1(f), however, has a plus sign; this is opposite to the sign of the corresponding AL diagram which is shown in Fig. 1(g). We also notice that the leading diagram of AL involving two fluctuation propagators[9] does not contribute to $1/T_1$. If we neglect higher-order diagrams involving either self-energy or vertex corrections, and limit ourselves to the lowest-order contribution to χ due to Cooper-pair fluctuations, then we obtain

$$L_{\mathrm{fluct}}(\omega_n) = \sum_{\vec{q}, \omega_m} D(\vec{q}, \omega_m) F_{\vec{q}, \omega_m}(\omega_n), \tag{4}$$

where, using the usual solutions,

$$F_{\vec{q}, \omega_1}(\omega) = T^2 \sum_{\omega_2} \sum_{\vec{p}_1 \vec{p}_2} G_{\vec{p}}(\omega_2) G_{\vec{q}-\vec{p}_1}(\omega_1 - \omega_2) \times \{G_{\vec{p}_2}(\omega_1 - \omega) G_{\vec{q}-\vec{p}_2}(\omega_1 + \omega - \omega_2) - G_{\vec{p}_2}(\omega_2)[G_{\vec{p}_2}(\omega_2 + \omega) + G_{\vec{p}_2}(\omega_2 - \omega)]\}. \tag{5}$$

We notice that the most divergent contribution to Eq. (4) comes from $\omega_m = 0$, and that $D(\vec{q}, 0)$ is significant only for $q < \xi_0^{-1}$, where ξ_0 is the coherence length at $T = 0$. Thus, we set $q = 0$ and $\omega_m = 0$ in F, evaluate it in the BCS model, and obtain after analytical continuation

$$\mathrm{Im}F(\omega) = (\pi\omega N_0^2) 7\zeta(3)/\pi^2 T, \tag{6}$$

where ζ denotes the zeta function. If one uses the d-electron Green's function obtained from a tight-binding calculation, a similar F is found but with different values for the constants. Now we return to Eq. (2) define the enhancement factor $\delta = (1/T_1)/(1/T_1)_0$, and obtain

$$\delta = A \textstyle\sum_{\vec{q}} D(\vec{q}, 0), \tag{7}$$

where $A = 7\zeta(3)/\pi^2 T$ in the BCS model.

We next turn to the calculation of $\sum_{\vec{q}} D(\vec{q}, 0)$. First we consider an idealized system with three perpendicular families of *noninteracting* linear chains of cross section a^2 (a few angstroms in diameter). Using the classical Ornstein-Zernike fluctuation propagator $D(q) \sim [q^2 + \xi^{-2}(T)]^{-1}$, where $\xi(T) = \xi_0[T_c/(T - T_c)]^{1/2}$, we write

$$\sum_{\vec{q}} D(\vec{q}, 0) = \frac{3}{N_0 \xi_0^2 a^2} \int \frac{dq}{2\pi} \frac{1}{q^2 + \xi^{-2}(T)}, \tag{8}$$

and obtain the one-dimensional BCS enhancement factor

$$\delta_{\mathrm{BCS}} = \frac{3}{2} \frac{7\zeta(3)}{\pi^2 T} \frac{1}{N_0 a^2 \zeta_0} \left(\frac{T_c}{T - T_c}\right)^{1/2}. \tag{9}$$

If we assume $T_c \sim 10°\mathrm{K}$, $N_0 = 2$ states/eV atom, $a \sim 5$ Å, and $\xi_0 \sim 50$ Å, we obtain

$$\delta_{\mathrm{BCS}} \sim 0.7[T_c/(T - T_c)]^{1/2}, \tag{10}$$

which is a good order-of-magnitude estimate for δ.[10a]

A more realistic model should take into account the coupling between the families of chains. We assume that the generalized Landau-Ginzburg free-energy function has the form[11]

$$F = \alpha(T) \sum_i |\psi_i|^2 + \alpha(0)\eta(T) \sum_{i \neq j} \psi_i \psi_j + \gamma \sum_i |\psi_i|^4 + \mu \sum_i |\partial\psi_i/\partial x_i|^2, \quad i = x, y, z, \tag{11}$$

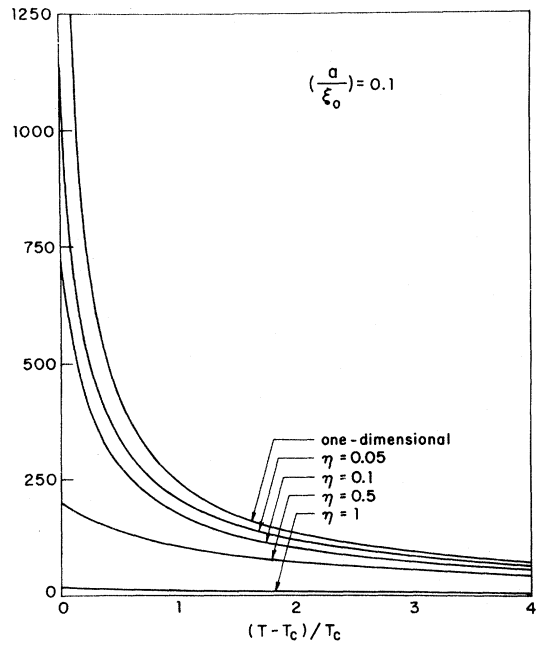

FIG. 2. Enhancement of the fluctuations of a coupled-chain system over those of an isotropic three-dimensional system. η is the ratio of interchain to intrachain coupling.

where ψ_i is the order parameter of the ith chain, and η plays the role of coupling between chains. De Gennes and Barisic[11] applied this free-energy function to predict an anisotropy in H_{c2}, which, however, was not observed in V_3Si[12]; but for Nb_3Sn a similar approach appears to account very well[13] for the anisotropy in tunneling experiments.[14]

Using the fluctuation propagator resulting from Eq. (11), we obtain

$$\sum_{\vec{q}} D(\vec{q}, 0) = \sum_{\vec{q}} \sum_i 1/E_i, \qquad (12)$$

where E_i are the eigenvalues of the 3×3 matrix with diagonal elements $a^2[q_i^2 + \xi^{-2}(T)] + 2\eta(a/\xi_0)^2$ and off-diagonal elements $-\eta(a/\xi_0)^2$. To evaluate Eq. (12) a cutoff is introduced at $q_c = \xi_0^{-1}$. For $T \sim T_c$ the results are not sensitive to the value of the cutoff.

We now return to Eq. (7) and calculate the enhancement factor δ for various values of η and $a/\xi_0 = 0.1$ as a function of temperature. The results of δ/δ^0, where δ^0 is the isotropic three-dimensional factor, are shown in Fig. 2. Values of $\eta \sim 0.1$ appear to be reasonable, yielding a three-order-of-magnitude enhancement of the effect of fluctuations at $T \approx T_c$. There is no divergence at T_c for the coupled-chain system, thus this approach is self-consistent (unlike the "pure" one-dimensional system, which does not possess a discontinuous superconducting transition at all). At $(T - T_c)/T = 0.2$, the effect of the fluctuations is about half of that of a "pure" one-dimensional system. It is seen that even for rather strong coupling between the chains ($\eta = 1$), the fluctuations are enhanced by about an order of magnitude over the isotropic case. This property of strongly coupled cross-linked chains to retain their one-dimensional properties has been pointed out and discussed in Ref. 7. Direct coupling between parallel chains quenches the one-dimensional properties much more effectively, as shown by Barisic and Marcelja.[15]

One does not expect a comparable effect on the electrical resistance of the $A15$ compounds, since the electrical current is mostly due to the conduction electrons, and since the heavy d electrons, located on the linear chains, contribute very little to the electrical current.

The Knight shift and $1/T_1T$ of V and Nb consist of several contributions: the direct s-electron contact term, the d-electron orbital contribution, and the d-electron spin core-polarization contribution. This calculation applies only to the last part, which gives the temperature-dependent part of $1/T_1T$ in V_3Si and V_3Ga (roughly, 50% of the total contribution). When we consider the Al^{27} resonance in Nb_3Al, the Nb $4d$ electrons influence T_1 probably by hybridization with the Al $3s$ and $3p$ electrons, and these in turn relax the Al^{27} nuclei through direct contact ($3s$) or core polarization ($3p$). The d electrons may also relax the Al nuclei through core polarization. According to the work of Clogston and Jaccarino,[4] the effect of the $3d$ electrons on the NMR of the X atoms is very large.

One of us (M.W.) benefitted considerably from discussions with P. G. de Gennes, A. C. Gossard, E. Ehrenfreund, B. G. Silbernagel, H. Guttfreund, and S. Alexander.

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[10a]The "classical" Ornstein-Zernike propagator is well known to be a crude but useful approximation for superconductors (see, e.g., Ref. 9). This approximation is reasonably accurate near the transition temperature, but fails in the immediate vicinity of T_c. An estimate of the "failure" temperature T_f is given, e.g., by J. P. Hurault and K. Maki, Phys. Rev. B 2, 2560 (1970). They find that for a one-dimensional system, if it is clean enough, i.e., the mean free path is of the order of ξ_0 (or larger) $\eta_f = (T_f - T_c)/T_c < (p_0 a)^{4/3}$, where p_0 is the Fermi momentum. In our case $\eta_f \sim 0.05$. Thus below T_f the divergence $(T - T_c)^{1/2}$ disappears, and we can replace the factor $(T - T_c)/T_c$ in Eq. (9) by $(p_0 a)^{4/3}$. However, the order of magnitude of the enhancement factor does not change.

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Superconducting Fountain Effect

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An effect analogous to the fountain effect in superfluid helium has been observed in a superconductor for the first time. A temperature gradient across a Pb-Pb point-contact junction produced a small supercurrent, which was detected by measuring the asymmetry of the critical current.

The fountain effect, or thermomechanical effect, in superfluid liquid helium was discovered by Allen and Jones[1] in 1938. When two vessels containing superfluid helium are joined by a thin capillary and a temperature difference is maintained between them, an osmotic pressure difference is set up because of the different concentrations of normal fluid and superfluid in each. The osmotic pressure difference drives superfluid from the colder vessel to the hotter until a hydrostatic pressure difference is set up which balances the osmotic pressure difference. In the steady state, normal fluid flows hydrodynamically from the hotter vessel to the colder, driven by the hydrodynamic pressure difference, and there is an equal flow of superfluid in the opposite direction. This steady-state counterflow is responsible for the large thermal conductance of superfluid helium. In very thin capillaries the flow of normal fluid is greatly reduced, and the thermal conductance appears much smaller.

The similarities between the superfluid properties of liquid helium and superconductors suggest that an analogous effect should occur in a superconductor.[2] In this Letter, we report the first experimental observation of this analog.

In a superconductor, the analog of the hydrostatic potential difference observed in helium in the presence of a temperature gradient is an electrostatic potential difference. It is important to realize that in any real metal the quasiparticle-lattice scattering rate is much higher than the quasiparticle-quasiparticle scattering rate. This means that hydrodynamic flow of the quasiparticles is strongly inhibited.[2,3] Thus the electrostatic potential difference only gives rise to a small normal-fluid flow, and the hydrodynamic counterflow of normal and superfluids makes only a very small contribution to the thermal conductivity of a superconductor. This is analogous to the case of helium in a *very* thin capillary, where the normal fluid is essentially clamped.