Beat Heating of a Plasma*

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If two laser beams have a difference frequency nearly equal to the plasma frequency, nonlinear interaction resonantly excites longitudinal plasma oscillations. These then induce transitions to other transverse modes. Nonlinear damping of the longitudinal mode heats the plasma. The process is optimized by having parallel beams, equal laser intensities, and damping equal to the frequency mismatch.

We propose a new method of heating a plasma, utilizing the excitation of longitudinal plasma waves by resonance with the difference frequencies of a set of transverse waves. The energy is provided by two (nearly) parallel laser beams, with frequencies ω_L, ω_{L-1} differing by approximately the plasma frequency: $\omega_L - \omega_{L-1} \equiv \omega_p + \Delta_L$, with the mismatch Δ_L small (say, $10^{-2}\omega_p$), and $\omega_L \gg \omega_p$. The nonlinear interaction of transverse and longitudinal waves [see Eq. (2) and the insets of Fig. 1] excites a longitudinal wave, with wave vector $\vec{k}_p = \vec{k}_L - \vec{k}_{L-1}$, which is nonlinearly damped, if its amplitude is sufficiently large. (There is no Landau damping, since $\omega_p/k_p \approx c$; collisional damping is too weak for our purposes.) This wave in turn interacts with each of the two transverse waves $(L, L-1)$ to produce two more at $\vec{k}_{L-2} = \vec{k}_{L-1} - \vec{k}_p$ and $\vec{k}_{L+1} = \vec{k}_L + \vec{k}_p$. When $\vec{k}_L$ and $\vec{k}_{L-1}$ are nearly parallel, the new mismatches $\Delta_l \equiv \omega_l - \omega_{l-1} - \omega_p$, determined by the dispersion relation $\omega_l^2 = k_l^2c^2 + \omega_p^2$, are also small; otherwise they are not.

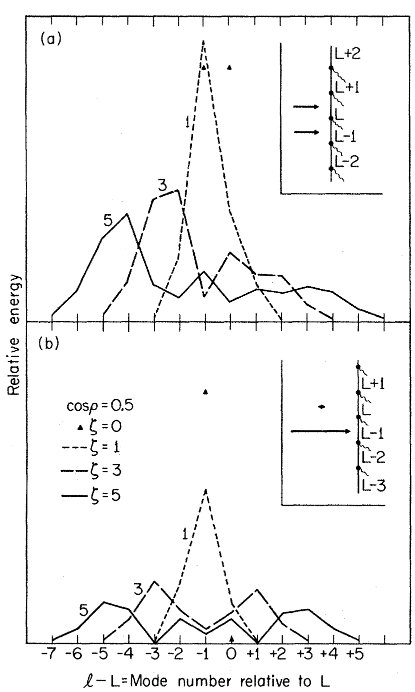

FIG. 1. Mode energy as a function of mode number $l \equiv \omega_l/\omega_p$, at several positions ζ [defined below Eq. (3)]. In case (a), the laser intensities are equal ($\alpha = 1$). In case (b), they are very unequal ($\alpha = 10$). The damping rate is comparable to the mismatch ($\cos\rho = 0.5$).

In quantum language, a coherent set of photons L undergoes induced (by $L-1$) decay into photons $L-1$ and plasmons. The damping of the plasmons deposits energy irreversibly into the plasma. Some of the plasmons, before they are absorbed, engage in further three-wave interactions, inducing the decay of the photons $L-1$ into photons $L-2$, and so on, coherently cascading the photon frequency downward. Others induce transitions upward in frequency, by converting L into $L+1$, and so on. Because energy is conserved in these interactions, and also the number of photons is conserved, the process must be preferentially downward, to allow for the plasma heating. [See curve 5 of Fig. 1(a) for an example.] For maximum efficiency, the downward rate should be maximized relative to the upward spreading. This is accomplished if the two laser intensities are roughly equal, and if the damping rate approximates the mismatch [see Eq. (5)].

The resonant interaction between two transverse waves and one longitudinal mode has been studied by Kroll, Ron, and Rostoker,[1] Tsytovich,[2] and Wolff,[3] among others. The fundamental equations for the interaction of the scalar potential $\varphi(z,t)$ of the longitudinal wave and the vector potential $A_x(z,t)$ of a set of parallel, linearly x-polarized transverse waves are

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\varphi(z,t) = \frac{e\omega_p^2}{2mc^2}A_x^2, \tag{1a}$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2 - c^2\frac{\partial^2}{\partial z^2}\right)A_x(z,t) = \frac{e}{m}\frac{\partial^2\varphi}{\partial z^2}A_x, \tag{1b}$$

where we treat the unperturbed electron plasma as cold, uniform, and stationary, and ignore dissipation for the time being. To derive these equations, we use the invariance of the canonical x momentum[3] to obtain $v_x = -eA_x/mc$. The Lorentz force along z is thus

$$(e/c)v_x B_y = -(e^2/2mc^2)\partial A_x^2/\partial z,$$

whence

$$4\pi n_0 e\dot{v}_z = -(\partial/\partial z)\omega_p^2[\varphi + (e/2mc^2)A_x^2].$$

This is then inserted into the last term of

$$(\partial^2/\partial t^2)\partial^2\varphi/\partial z^2 = -(\partial^2/\partial t^2)4\pi n e = 4\pi e(\partial^2/\partial t\partial z)nv_z \cong (\partial/\partial z)4\pi n_0 e\dot{v}_z;$$

We have used the Poisson and continuity equations, and have dropped harmonic-producing[3] terms. Integration with respect to z then yields Eq. (1). For Eq. (1b), we use the wave equation $\Box A_x = -4\pi j_x/c = (4\pi e/c)(n_0 + \delta n)eA_x/mc$, and replace δn by $-(4\pi e)^{-1}\partial^2\varphi/\partial z^2$.

The coupled equations for the wave amplitudes are obtained from (1) by setting

$$A_x(z,t) = \sum_l A_l(z,t)\exp i(k_l z - \omega_l t) + \text{c.c.},\quad \varphi(z,t) = -i\varphi_1(z,t)\exp i(k_p z - \omega_p t) + \text{c.c.}$$

Assuming that the amplitudes vary slowly, we obtain

$$(\partial/\partial t + \gamma)\varphi_1 = \kappa\sum_l A_l A_{l-1}^* \exp(-i\Delta_l t), \tag{2a}$$

$$(\partial/\partial t + c_l\partial/\partial z)A_l = (\kappa/l)[A_{l+1}\varphi_1^*\exp(-i\Delta_{l+1}t) - A_{l-1}\varphi_1\exp(i\Delta_l t)], \tag{2b}$$

where $\kappa \equiv e\omega_p/2mc^2$, and $c_l \equiv k_l c^2/\omega_l \approx c$ is the group velocity of mode l; we have set $\omega_l \approx l\omega_p$ in the coefficient of (2b), and have introduced a phenomenological damping coefficient γ in (2a). The corresponding wave-energy densities are

$$W_p \equiv (\omega_p\partial\epsilon/\partial\omega)k_p^2|\varphi_1|^2/4\pi = \omega_p^2|\varphi_1|^2/2\pi c^2,\quad W_l \equiv \omega_l^{-1}[\partial(\omega^2\epsilon)/\partial\omega](\omega_l/c)^2|A_l|^2/4\pi = \omega_l^2|A_l|^2/2\pi c^2,$$

where $\epsilon(\omega) = 1 - \omega_p^2/\omega^2$. We note that Eq. (2b) conserves transverse action (photon number),

$$(\partial/\partial t)\sum_l W_l/\omega_l = -(\partial/\partial z)\sum_l c_l W_l/\omega_l,$$

while the set (2a), (2b) conserves energy (with dissipation),

$$(\partial/\partial t)(W_p + \sum_l W_l) = -(\partial/\partial z)\sum_l c_l W_l - 2\gamma W_p.$$

(Note that the longitudinal mode has zero group velocity.)

We study here (a) the boundary-value problem, in which two laser beams with steady intensities cW_L^0 and cW_{L-1}^0 are incident on a semi-infinite ($z>0$) plasma, and we look for quasi-steady-state solutions as functions of z; and (b) the initial-value problem, in which two laser modes are present at $t=0$, uniform in space, with energy densities W_L^0 and W_{L-1}^0, and we look for the evolution as a function only of time. The full space-time problem will be studied in a later publication, together with the important effect of plasma nonuniformity.

In the steady-state problem, the dissipation rate is $Q = 2\gamma W_p = -(d/dz)\sum_l c_l W_l$, from the energy conservation law. Dividing both sides by the constant action flux density $\sum_l c_l W_l/\omega_l \equiv J$, we find that $Q/J = -\omega_p d\langle l\rangle/dz$, where $\langle l\rangle(z)$ is the action-weighted mean mode number. Thus, it is desired to have $\langle l\rangle$ decrease as rapidly as possible. Its derivative is given by Eq. (5). Similar considerations apply to the initial-value problem, where $-d\langle l\rangle/dt$ is to be maximized.

The set (2) has the characteristic rate[4] $\Gamma_0 = \kappa A_L^0 = [W_L^0/(8nmc^2)]^{1/2}\omega_p/L$ but this is not the actual rate, which is found below. In dimensionless variables [$t' \equiv \Gamma_0 t$, $z' \equiv \Gamma_0 z/c$, $\Delta' \equiv \Delta/\Gamma_0$, $\gamma' \equiv \gamma/\Gamma_0$, $\varphi_1' \equiv \varphi_1/A_1^0$, $A_l' \equiv A_l/A_L^0$], Eqs. (2) retain the same from [denoted (2′) below], but with κ deleted. The boundary (or initial) conditions are $A_L'(0) = 1$, $A_{L-1}'(0) \equiv \alpha e^{i\theta}$ (α positive).

An explicit solution for the boundary-value problem may be found, if all the mismatches are set equal ($\Delta_l \to \Delta$); this is equivalent to ignoring dispersion, which is not too bad if $l \gg 1$. To be consistent, we then set $c_l \to c$ and $l \to L$ in the coefficient of (2b). For time-independent A_l, the steady-state solution of (2a′) is $\varphi_1'(z',t') = (\gamma' - i\Delta')^{-1}B\exp(-i\Delta' t')$, where $B \equiv \sum_l A_l'(z')A_{l-1}'^*(z')$. Thus, $\varphi(z,t)$ is driven at the (common) beat frequency $\omega_l - \omega_{l-1} = \omega_p + \Delta$, not at its natural frequency ω_p. Substituting φ_1' into (2b′), we obtain

$$L\,dA_l'/dz' = (\gamma' + i\Delta')^{-1}B^*A_{l+1}' - (\gamma' - i\Delta')^{-1}BA_{l-1}'. \qquad (3)$$

We use (3) to show that dB/dz' vanishes, i.e., $B(z') = B(0) = \alpha e^{-i\theta}$. Introducing $\rho \equiv \tan^{-1}(\Delta/\gamma)$ and $\zeta \equiv 2\alpha L^{-1}(\gamma'^2 + \Delta'^2)^{-1/2}z' \equiv 2\alpha L^{-1}c^{-1}(\gamma^2+\Delta^2)^{-1/2} \times \Gamma_0^{\,2}z$, and setting $A_l'' \equiv A_l' \exp il(\theta - \rho)$, we can write (3) as $2dA_l''/d\zeta = A_{l+1}'' - A_{l-1}''$. This is the recursion relation for Bessel functions, except for sign. Hence, the solution of (3) satisfying the boundary conditions yields

$$\begin{aligned} |A_{L+n}'|^2 &= J_n^{\,2} + \alpha^2 J_{n+1}^{\,2} - 2\alpha(\cos\rho)J_nJ_{n+1}, \\ |A_{L-1-n}'|^2 &= \alpha^2 J_n^{\,2} + J_{n+1}^{\,2} + 2\alpha(\cos\rho)J_nJ_{n+1}, \end{aligned} \qquad (4)$$

where the argument of the Bessel functions is ζ.

From Eq. (3), we may directly calculate the evolution of mean[5] mode number $\langle l\rangle(z) \equiv \sum_l l|A_l|^2(z)/\sum_l|A_l|^2(z)$. In dimensional variables, we find the cascade rate

$$\Gamma \equiv -c\frac{d\langle l\rangle}{dz} = \frac{1}{4L^3}\frac{\gamma\omega_p^{\,2}}{\gamma^2+\Delta^2}\frac{\alpha^2}{(1+\alpha^2)^2}\frac{W^0}{nmc^2}, \qquad (5)$$

where $W^0 \equiv W_L^{\,0} + W_{L-1}^{\,0}$ is the total input energy density. We recall that Γ represents the rate of plasma heating, and is to be maximized. (Note that in contrast to Γ_0, it varies linearly with W^0.) For given W^0, it vanishes as $\alpha^2 \to 0$ or ∞, and is maximized at $\alpha = 1$. This is illustrated in Fig. 1, which presents $|A_l'|^2$ versus l, at several ζ, for the two cases $\alpha = 1$ ($W_{L-1}^{\,0} = W_L^{\,0}$) and $\alpha = 10$ ($W_{L-1}^{\,0} = 100\ W_L^{\,0}$). The latter case evolves almost symmetrically about $L-1$, and little heating results. The former case is quite asymmetric, which is desired. The dependence of Γ on the damping rate γ is similar, Γ being maximized when $\gamma = \Delta$. This is evident from Eqs. (4), where the asymmetry between higher and lower modes is seen to be proportional to $\cos\rho = \gamma/(\gamma^2+\Delta^2)^{1/2}$.

Formula (5) leads to an estimate of required laser intensity, for a criterion that $\langle l\rangle$ change by unity in one centimeter, say. Taking $L \sim 10$, $\gamma \sim \Delta \sim 10^{-2}\omega_p$, $\omega_p \sim 2\times10^{13}$ sec^{-1}, $\alpha \sim 1$, $n \sim 10^{17}$ cm^{-3}, we obtain $W^0c \sim 10^{14}$ W cm^{-2}. The longitudinal field produced is then sufficient to produce damping by parametric instability,[6] with γ of the order assumed.[7]

To study the effects of variable mismatch Δ_l (caused by dispersion), we have numerically integrated Eqs. (2) for the uniform case ($\partial/\partial z \equiv 0$). Of the several cases studied, we report only the following: ω_L was chosen to be 1.8×10^{14} sec^{-1} (CO_2 laser), and $L = 10$. The initial power density P (expressed in W/cm^2) was in modes 10 and 9, with $\alpha^2 = 0.1$. The damping coefficient was chosen to be $\gamma = 10^4P^{1/2}$ sec^{-1}, corresponding to $\gamma' = 0.3$, and $\gamma/\omega_p = 0.005$ at 10^{14} W/cm^2.

The mismatch Δ_L was adjusted for optimum energy transfer. Because the mismatch Δ_l decreases algebraically with l, as a result of the plasma dispersion, it is desirable to choose Δ_L positive. Then for higher modes, the mismatch increases, and thus coupling to those modes is inhibited; while for lower modes, the coupling is enhanced as Δ_l decreases and passes through zero to negative values. The best choice of Δ_L was in the range $0.01 < \Delta_L/\omega_p < 0.03$. With this choice, energy transferred to higher modes was blocked, and eventually made to cascade back down, with littler energy remaining in those modes. Figure 2 shows the fractional energy transfer after a time interval $t = 5\gamma^{-1}$. (For later times, the transfer rate becomes relatively slow.) We note that the effective threshold power density is 10^{14} W/cm^2; for weaker power, the mismatch prevents appreciable energy transfer.

When the calculation was repeated with Δ_l held constant (i.e., neglecting dispersion), typically 0.3 to 0.4 of the energy remained in the higher modes $l > L$, and did not cascade down. In this and other respects, there was agreement between the time-dependent uniform case and the space-dependent steady-state case.

When the initial laser beams are not (nearly) parallel, spontaneous frequency conversion to other transverse modes will not occur because of the large mismatches. The longitudinal mode then catalyzes the complete transfer of action from L to $L-1$. For the antiparallel case, for example, the longitudinal mode has $\vec{k}_p = \vec{k}_L - \vec{k}_{L-1}$

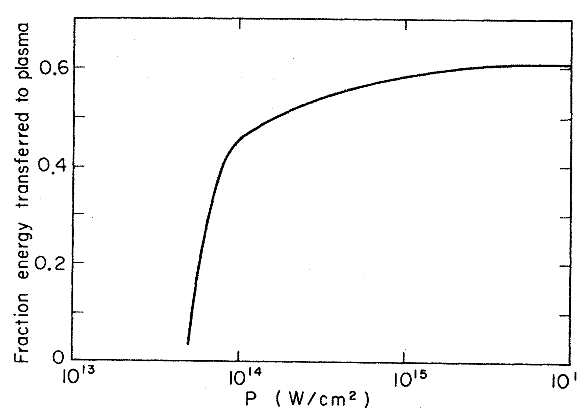

FIG. 2. Fractional energy transfer as a function of laser power density.

as before, but now $|\vec{k}_p| \approx 2k_L \approx 2\omega_L/c$, whereas in the parallel case $k_p \approx \omega_p/c \ll 2\omega_L/c$. As a result, the wave coupling on the right-hand side of Eq. (1b), proportional to k_p^2, is greatly enhanced. To balance the advantage of enhanced coupling are two disadvantages and one further advantage. First, the damping rate may be greatly enhanced by Landau damping (since now $\omega_p/k_p \ll c$), beyond the optimum $\gamma \sim \Delta$. Secondly, the further transition to $L-2$ cannot be induced by the longitudinal mode $\vec{k}_p$, since this would require $\vec{k}_{L-2} = \vec{k}_{L-1} - \vec{k}_p = 2\vec{k}_{L-1} - \vec{k}_L$, or $k_{L-2} \approx 3k_L$, violating the dispersion relation. This means that the further decay must be induced instead by a third laser beam $L-2$ in any desired direction, and the corresponding longitudinal wave excited, $\vec{k}_p' = \vec{k}_{L-1} - \vec{k}_{L-2}$, is not the same as $\vec{k}_p$. Thirdly, no energy is lost on up-conversion, since each transition must be seeded by its own laser beam.

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[4] We may take A_L^0 real and positive.

[5] The weighting has been changed, because the replacement $l \to L$ changes the invariant from J to $\sum_l |A_l|^2$.

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Effect of Superconducting Fluctuations on the Spin Relaxation of Quasi–One-Dimensional Compounds

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An investigation is made of the effect of order-parameter fluctuations on the nuclear relaxation time of superconducting $A15$ compounds. It is found that because of the one-dimensional chain structure of these compounds the fluctuations make an important contribution to the NMR.

The nuclear relaxation time T_1 in superconducting compounds with the composition V_3X (X = Si, Ge, etc.) was observed[1] to display an anomaly above the transition temperature T_0; $1/T_1T$ was observed to increase by about 20% at temperatures a few degrees above T_c [$(T-T_c)/T_c \approx 0.2$]. At the time at which these observations were made, it was not clear to what cause this anomaly should be attributed; V_3Si undergoes a martensitic transformation[2] at about 21°K, and this transformation causes anomalies in the Knight-shift and quadrupolar interactions,[3] and therefore an anomaly in $1/T_1T$ is not too surprising; V_3Ga is rarely pure and usually some Ga atoms occupy V sites; in addition, all these compounds display anomalies in their electronic properties which may be attributed to a sharp peak in the density of states function,[4] and conceivably an anomaly in $1/T_1T$ may be attributed to this peak. Recently,[5] experiments on Nb_3Al indicated a similar anomaly