Emission of Coherent Microwave Radiation from a Relativistic Electron Beam Propagating in a Spatially Modulated Field

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(Received 4 February 1972; revised manuscript received 19 May 1972)

Observations of the rf emission from the interaction of a relativistic electron beam with a modulated magnetic field has led to a qualitative theory. This theory predicts emission of high-power radiation at high-frequency harmonics. An improved experiment has confirmed these predictions.

In the last two decades extensive efforts have been devoted to finding new mechanisms for the production of rf radiation using relativistic electron beams.[1,2] Mechanisms have been suggested to explain the highly intense rf emission from celestial objects using coherent synchrotron radiation mechanisms.[3,4]

We recently reported a different mechanism.[5] An intense relativistic electron beam propagating in a spatially modulated magnetic field produced a high level of microwave radiation. The experimental results have some common features with a theory and an experiment, which led to a microwave amplifier known as a Ubitron.[6,7] That theory predicted that the wavelength of the emitted radiation would depend mostly on the period L of the applied magnetic field and not on the magnetic field intensity B. The results we obtained[5] deviated from the theory. We have found that the frequency spectrum can be either very broad or very very narrow. The latter appeared at a critical magnetic field which satisfied the following relations:

$$B_c \approx 2\pi V_{\parallel}\gamma mc/eL, \tag{1}$$

and

$$B_c \approx 2\pi f\gamma mc/e, \tag{2}$$

where f is the frequency, $V_{\parallel}$ is the parallel velocity of the electrons, and $\gamma = (1 - V^2/c^2)^{-1/2}$. On the basis of some qualitative theoretical considerations (to be presented later in this Letter) we were led to the following conclusions: (i) Higher-frequency harmonics should be present in the rf emission with relative power close to the theoretical limits[1]; (ii) at the critical magnetic field of Eqs. (1) and (2), the harmonics should appear as narrow spectral lines; (iii) a wide spectrum is expected to appear at a magnetic field that differs from the critical magnetic field. Although many microwave generators have similar characteristics, the power they generate in the harmonics is very small and never approaches the theoretical limits.[1]

To check the above conclusions, we have used a pulsed annular electron beam of voltage 700 kV and current 18 kA. The pulse duration was 5×10^{-8} sec. The beam propagated in a 5-cm-i.d. cylindrical wave guide embedded in a ripple magnetic field having a period of 3.8 cm. The rippled

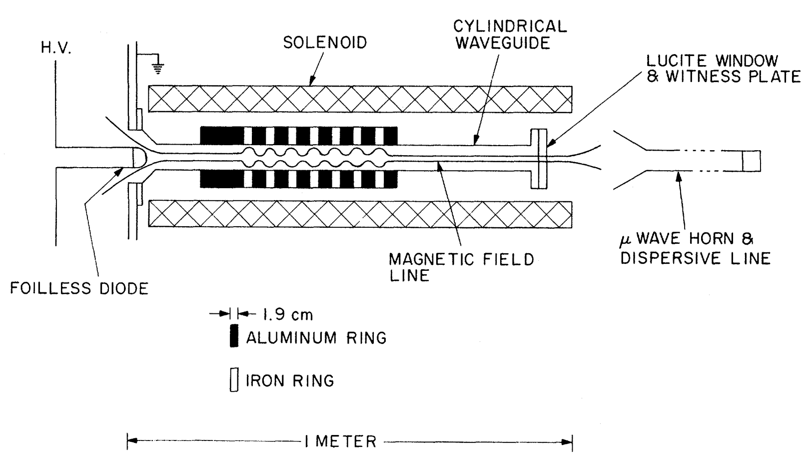

FIG. 1. Experimental schematic.

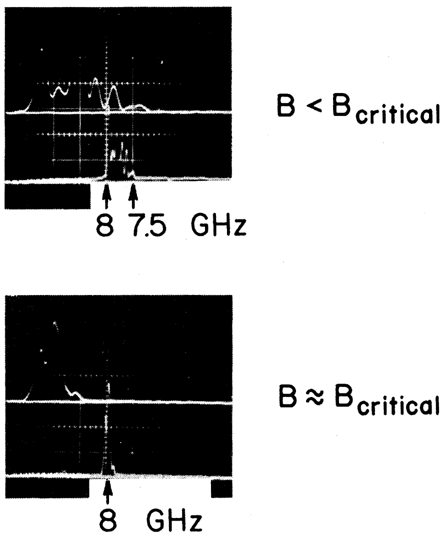

FIG. 2. Dispersed microwave traces in the X band obtained under the following conditions: (top) $B < B_c$ and (bottom) $B \approx B_c$. In each condition the time scale for the top traces is 50 nsec/div and for the bottom traces, 200 nsec/div. In addition, time zero for the bottom traces in each condition was the time the current had started.

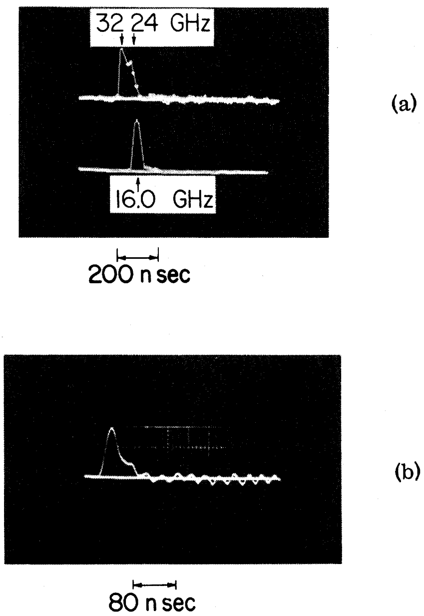

FIG. 3. (a) Top curve, dispersed microwave trace in the K_a band, and bottom curve, dispersed microwave trace in the K_u band. (b) Undispersed microwave trace at the E band. The magnetic field $B \approx B_c$.

field was produced by inserting aluminum and iron rings inside a solenoid. The experimental schematic is shown in Fig. 1. A more detailed description is given elsewhere.[5]

The microwave signals were detected in the X band (6.5–12.0 GHz), K_u band (12.4–18.0 GHz), K_a band (26.5–40 GHz), and E band (60–90 GHz) and were monitored by crystal detectors after propagation in wave guides acting as dispersive mediums.[8,9] Since each frequency has a different propagation velocity in the wave guide, a spectral distribution is thus transformed into a distribution in propagation times.

Figure 2 shows a dispersed microwave signal detected in the X band at two different magnetic field intensities. At the critical magnetic field the spectrum appeared to be very narrow at the different bands; for the first harmonic, $\Delta\lambda/\lambda \lesssim 0.5\%$, and for the second harmonic, $\Delta\lambda/\lambda \lesssim 1\%$. Because the dispersive lines in the K_a band were not long enough, we could not separate the third and the fourth harmonics. However, it can be estimated that $\Delta\lambda/\lambda \approx 1\%$ for these harmonics. The curves in Fig. 3(a), show the dispersed microwave signal at the combination of the third and fourth harmonics and at the second harmonic, respectively. Figure 3(b) shows an undispersed microwave signal in the E band which contains the seventh, eighth, ninth, and tenth harmonics. All curves in Fig. 3 were taken at the critical magnetic field. They confirmed the theory which predicts narrow spectral lines at harmonics of the fundamental frequency when operating close to the critical magnetic field. The intensity of the different harmonics was approximately

$$I_1:I_2:(I_3+I_4):(I_7+I_8+I_9+I_{10}) \approx 1:O(0.1):O(0.1):O(0.01),$$

where $O(N)$ stands for the order of magnitude of the number N. I_1 is $\gtrsim 75$ dB above noise level. The intensity of rf radiation in the harmonics is high, and their ratios are close to the theoretical limits.[1] The frequency ratios were

$$f_1:f_2:\tfrac{1}{2}(f_3+f_4) = 1:2:3.5.$$

As a matter of curiosity, we constructed a rippled magnetic field with random pitches. A very wide spectrum appeared which covered the range from 7-90 GHz with a frequency distribution of the type

$$I \propto f^{-\beta}, \quad \beta \lesssim 1.$$

Figures 4 and 5 show typical traces of dispersed microwave signals in the X and K_u bands, respectively.

A modified picture that may provide some in-

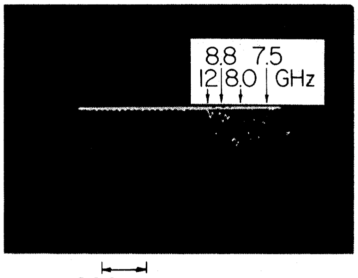

FIG. 4. Dispersed microwave trace in the X band, obtained when a rippled magnetic field with random pitches was used.

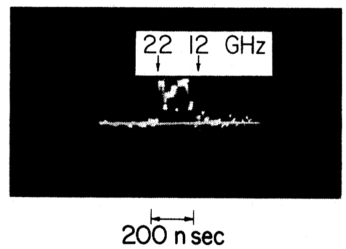

FIG. 5. Dispersed microwave trace in the K_u band, obtained when a rippled magnetic field with random pitches was used.

sights about the mecahnism involved can be obtained if we assume that the medium is a collisionless, relativistic, homogeneous "plasma." Solving the nonlinearized Vlasov equation[10] one can find that two "waves" influence the "plasma" simultaneously: (1) an rf wave with radian frequency ω and wave vector $\vec{k}$, and (2) a fictitious wave representing the rippled magnetic field, with radian frequency $\omega = 0$ and wave vector $k_R = 2\pi/L$. We assume that the magnetic field B, through which the "waves" propagate, is homogeneous. The only waves that may interact with the medium are those that fulfill the relation[10]

$$\omega = (k_\parallel + k_R) V_\parallel - n\omega_{ce} \approx 0, \quad n = -\infty, \cdots, -1, 0, 1, \cdots, \tag{3}$$

where $\omega_{ce} = eB/\gamma mc$ is the electron cyclotron frequency. Several solutions to the dispersion relation are in the literature.[6,9] For example, by choosing $n = 0$ and $k_\perp = 0$, we obtain the theory behind the Ubitron,[6]

$$\omega - (k + k_R) V_\parallel = 0. \tag{4}$$

The necessary condition under which the medium will support growing waves is that[9]

$$\partial f(p_\parallel, p_\perp)/\partial p_\perp > 0 \text{ and/or } \partial f(p_\parallel, p_\perp)/\partial p_\parallel > 0 \tag{5}$$

for some range of particle velocity, where $f(p_\parallel, p_\perp)$ is the distribution function for electrons with momentum components $p_\parallel$ and $p_\perp$. The medium has also to satisfy a sufficient condition[11] which will not be discussed here. It was found[11,12] that when Eq. (5) is valid, relativistic electrons in a homogeneous magnetic field can support growing waves. The radian frequencies of these waves are near $\kappa\omega_{ce}$, where $\kappa < \kappa^*$ and κ^* depends on the distribution function of the electrons. In the present case, the rippled magnetic field acts as a perturbation to the picture and may modify the spectrum of the growing waves. When

$$k_R V_\parallel = \omega_{ce} \tag{6}$$

a resonant force will be exerted on the electrons due to the rippled magnetic field. This condition, Eq. (6), is the well-known relation necessary for a nonadiabatic process to occur.[13] Under this condition, an electron will change its momentum while moving along a magnetic field line. The "efficiency" of this process depends on the number of periods, the parallel and perpendicular velocity components of the electrons, and the average magnetic field.[14] Since for different $V_\parallel$ the process had different efficiencies, a double-hump distribution function can occur, and Eq. (5) will be satisfied.

The resonant conditions [Eq. (6)] will also transform the azimuthal bunchings into longitudinal ones. These in turn will be interacting strongly with the modulated magnetic field, producing a strong rf emission.

A "nonlinear" effect plays an important role in this picture. This effect *does not occur* as a result of processes in warm magnetoplasmas. It originates from the finite perpendicular velocity component of single electrons and from the spatial distribution of the magnetic field. As an example, we shall take the case of a solenoidal magnetic field,

$$B_z(r, z) = B_0[1 + h \sin(k_R z) I_0(k_R r)], \quad B_r(r, z) = -B_0 h k_R \cos(k_R z) I_1(k_R r). \tag{7}$$

The electrons move in the r direction with oscillatory motion. We shall approximate this motion by the equation

$$r = r_1 + r_2 \cos\omega^* t$$

which is substituted in Eq. (7). By expanding the modified Bessel functions $I_0(k_R r)$ and $I_1(k_R r)$ in a series, we find that the magnetic field "seen" by the electrons changes in time as a linear function of $\cos(n\omega^*t)$ and $\sin(n\omega^*t)$, where $n=0, 1, 2, \cdots$. This in turn will appear as a force acting on the electrons with the same time dependence. When Eq. (6) is fulfilled, $\omega^*=\omega_{ce}$; otherwise ω^* represents a spectrum of radian frequencies. Under the resonant condition of Eq. (5), one expects to get radiation having radian frequencies ω_{ce}, $2\omega_{ce}$, $3\omega_{ce}, \cdots$. However, when Eq. (6) is not valid, this effect will tend to widen the frequency distribution. Because of this nonlinear interaction, rf radiation with wavelength λ can be produced from a bunch of electrons with a scale length greater than λ.

The theoretical considerations and experimental results described here suggest a new mechanism for coherent rf emission. This mechanism can be of some importance in explaining nonthermal rf radiation from celestial objects.[15] Some of the observed rf emissions from astrophysical phenomena cannot be explained by any simpler mechanism.[16] The rippled magnetic field can be produced in astrophysical phenomena by any one of several instabilities which have been detected in laboratory plasmas.

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Ising-Model "Metamagnet" and Tricritical Susceptibility Exponent*

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A spin-$\frac{1}{2}$ Ising-model "metamagnet" is studied by the method of high-temperature expansions. The phase boundary in the H-T plane is obtained for a simple cubic lattic with in-plane ferromagnetic interactions but between-plane antiferromagnetic coupling, and a tricritical point is located. Along the critical line, the staggered susceptibility appears to have an exponent $\frac{5}{4}$ (consistent with the universality hypothesis), while at the tricritical point, the direct susceptibility shows a tricritical exponent of $\frac{1}{2}$.

Griffiths[1-2] has recently called attention to the existence of "tricritical points" in the phase diagram of metamagnetic systems. At the tricritical temperature T_t, the phase transition changes abruptly from second to first order. Such behavior has been observed in real materials such as $FeCl_2$,[3] $Ni(NO_3)_2 \cdot 2H_2O$,[4] and dysprosium aluminum garnet.[5] The tricritical point is characterized by its own set of exponents; in particular, we may expect that critical exponents will change *discontinuously* at T_t from their values on the second-order phase boundary, along which they remain constant.[6,7]

Previous theoretical work has been restricted to Landau's "classical theory"[8] or molecular theory,[9] and some very recent Monte Carlo studies of models with tricritical points.[10] Riedel[11] has presented a scaling theory near the tricritical point consistent with experimental work on He^3-He^4 mixtures,[12] a system whose phase diagram is thermodynamically closely analogous to that of metamagnetic materials.[1] An Ising Hamiltonian has been proposed for the He^3-He^4 system[13] and analyzed by series expansions,[14,15] although

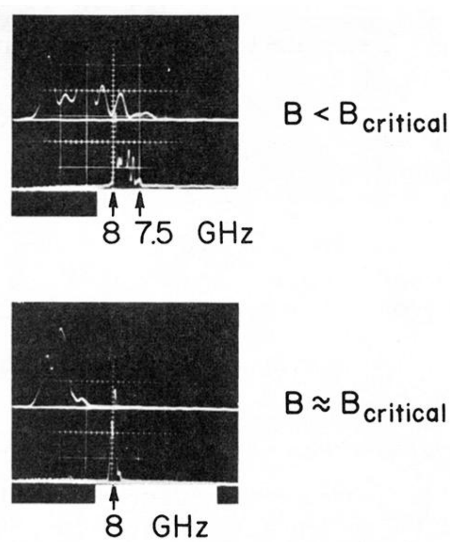


FIG. 2. Dispersed microwave traces in the X band obtained under the following conditions: (top) $B < B_c$ and (bottom) $B \approx B_c$. In each condition the time scale for the top traces is 50 nsec/div and for the bottom traces, 200 nsec/div. In addition, time zero for the bottom traces in each condition was the time the current had started.

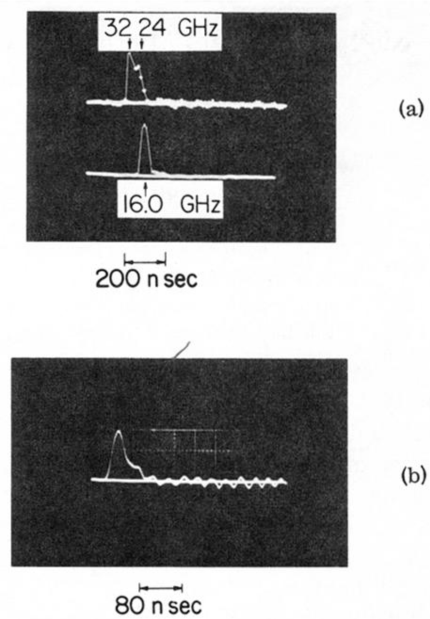


FIG. 3. (a) Top curve, dispersed microwave trace in the K_a band, and bottom curve, dispersed microwave trace in the K_u band. (b) Undispersed microwave trace at the E band. The magnetic field $B \approx B_c$.

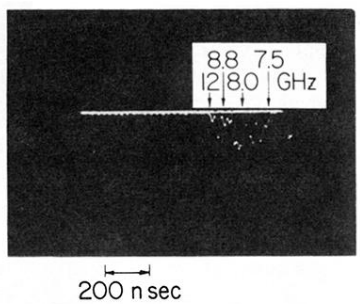
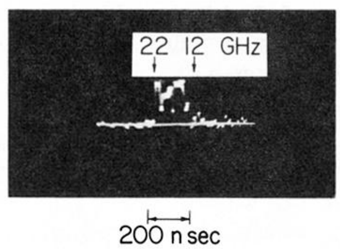


FIG. 4. Dispersed microwave trace in the *X* band, obtained when a rippled magnetic field with random pitches was used.



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