has been indicated above, and leads to the conjecture that the speed of sound is bounded asymptotically by $c/\sqrt{3}$, at least for a large class of relativistic interactions. This conjecture is made more plausible by the results of a recently completed calculation of the equation of state for a system of baryons which includes the pseudoscalar coupling[11]

$$L_{\rm int} = ig_0\bar{\psi}\gamma^5\psi\varphi$$

to second order in the coupling constant, where it was found that in the limit

$$\frac{(\text{baryon number density})^{1/3}}{\text{baryon rest mass}} \to \infty,$$

the speed of sound

$$v_s \to c/\sqrt{3}.$$

These results tend to strengthen the position that an upper bound to the equation of state of matter at high densities should be consistent with the requirement $v_s \leq c/\sqrt{3}$, at least for a broad class of interactions. It further suggests that the results based on models which yield less stringent limits be re-examined within the construct of relativistic many-body theory.

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Experimental Tests of Discrete Symmetries in the Decays $K^\pm \to \pi^+\pi^-e^\pm\nu_e$†

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Tests of the discrete symmetries P, C, CP, and T are obtained through comparison of the rare decays $K^\pm \to \pi^+\pi^-e^\pm\nu_e$ (K_{e4} decays). An important outgrowth of the experiment is a direct measurement of the low-energy pion-pion relative scattering phase, $\delta_0 - \delta_1$.

Since the discovery[1] of a violation of CP invariance in the decay $K_2^0 \to \pi^+ + \pi^-$ in 1964, there have been a number of experimental tests of the invariance of CP and T in systems other than the neutral kaons.[2] Within their sensitivity, none of these tests has provided any evidence that the Hamiltonian describing the interactions of elementary particles contains a CP- or T-invariance–violating part. In this Letter we present the preliminary results of a new test of CP invariance through comparison of the rare decays (branching ratio $\sim 3\times10^{-5}$) $K^\pm \to \pi^+\pi^-e^\pm\nu_e$ (K_{e4} decays). An important outgrowth of the experiment is a direct measurement of the low-energy pion-pion relative scattering phase, $\delta_0 - \delta_1$, where δ_0 and δ_1 are, respectively, the $I=0$, $l=0$ and $I=1$, $l=1$ phase shifts.

$K_{e4}^\pm$ decays, because they have four-body final states, provide tests of the discrete symmetries P, C, and T that do not involve any correlation of particle spin with one or more particle momenta. Furthermore, the $K_{e4}^{\pm}$ system is sensitive to a violation of CP invariance in the strangeness-changing hadronic axial-vector weak current, the only part of the weak Hamiltonian that has not been tested[3] in any way.[4,5]

The experimental arrangement for observing K_{e4} decays is shown in Fig. 1. A partially separated charged kaon beam of 1.8 GeV/c was produced at 10° from the G-10 target of the Brookhaven National Laboratory alternating gradient synchrotron. The incident kaons were electronically tagged by the beam Cherenkov and scintillation counters and their momenta measured to $\Delta p/p \simeq 1\%$ in a simple beam spectrometer as indicated. Acceptable kaon decays occurred in the first half of a wide-gap optical spark chamber located in a magnet with a central field of about 6 kG. The spark chamber had eight gaps and was 36 in. long by 32 in. wide by 18 in. high.

Immediately upstream from the spark chamber was a veto counter with an aperture for the incident beam. Directly downstream from the spark chamber was a four-counter hodoscope to indicate that a decay with more than one charged particle occurred in the spark chamber. Behind this front hodoscope were two large-volume, optically segmented, focusing threshold Cherenkov counters, $C_{\rm I}$ (eight segments) and $C_{\rm II}$ (sixteen segments); these distinguished electrons from pions and also allowed events with two electrons to be distinguished from events with one electron among the decay products. The average efficiency of each of these counters, weighted by the spatial distribution of electrons in K_{e4} decay, was approximately 98%. The sixteen-element

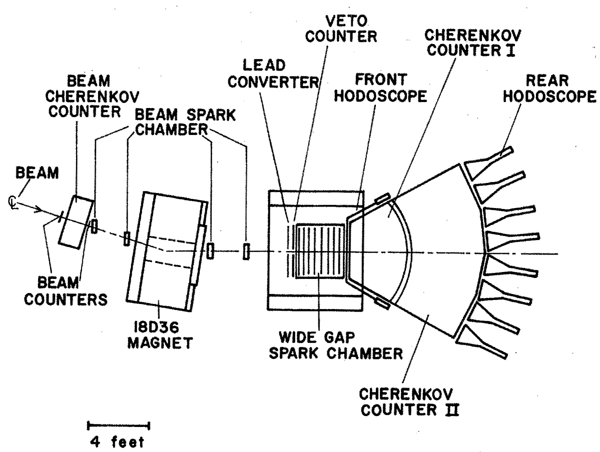

FIG. 1. Experimental arrangement for observing K_{e4} decays (plan view).

rear hodoscope was used in conjunction with the Cherenkov counters to identify events with three or more charged particles, of which one and only one was an electron.

Events were hand measured on image-plane digitizing machines with a least count of 5 μm. Spatial reconstruction was done using a modified version of the program TVGP and a separate program to determine the momentum of the incident kaon. Each event was tested for a fit to the K_{e4}, $K_{\pi 2}$, and τ hypotheses[6] by the kinematic fitting program SQUAW. All of the accepted K_{e4} events satisfy (i) a loose χ^2 criterion and (ii) the requirement that the particle types and trajectories in a given event are properly correlated with the counters and counter segments that in fact triggered the event. Two additional criteria, (iii) $P_\nu^{\rm lab} \geq 25$ MeV/c and (iv) if 120 MeV/$c^2 \leq M_{e\nu} \leq 160$ MeV/c^2, then $P_\nu^{\rm lab} \geq 80$ MeV/c, serve mainly to reduce the contamination of the K_{e4} sample to less than 1%, due mostly to τ decays. Of all possible K_{e4} decays, 21% are accepted by the apparatus and pass the selection criteria. The sample of events reported here is about $\frac{3}{4}$ of the total.

Five independent kinematic variables are required to specify a K_{e4} decay. The customary variables[7] are $M_{\pi\pi}$ and $M_{e\nu}$, the di-pion and di-lepton invariant masses; θ_π and θ_e, the polar angles of the π^+ (π^-) and e^+ (e^-) in K^+ (K^-) decay, specified in the di-pion and di-lepton rest frames, respectively; and φ, the azimuthal angle between the di-pion and di-lepton decay planes.

The measured one-dimensional distributions for K^+ and K^- decays in $\cos\theta_\pi$ and φ are presented in Figs. 2 and 3. The smooth curve through each of the distributions is a fit to the distribution; the form of the fit is dictated principally by locality of the lepton pair production.[8] The experimental one-dimensional distributions in $\cos\theta_e$, $M_{\pi\pi}$, and $M_{e\nu}$, which are not shown, are the same for K_{e4}^+ and K_{e4}^- within experimental error, which is expected on the basis of CPT invariance and an approximate semileptonic $|\Delta I| = \frac{1}{2}$ rule.

The data in Figs. 2 and 3 provide tests of the discrete symmetries P, C, CP, and T, which are independent of any dynamical assumptions about the K_{e4} decay interaction and also independent of CPT invariance.

Under the parity operation, $\varphi \to -\varphi$, while the polar angles θ_π and θ_e remain unchanged. Parity invariance therefore requires that the coeffi-

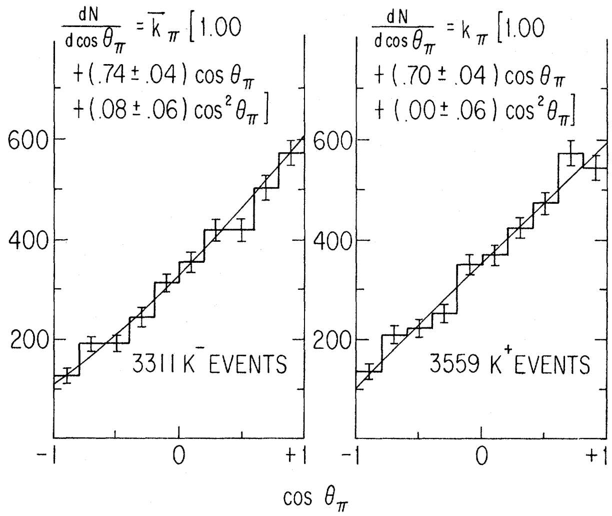

FIG. 2. Distributions in $\cos\theta_\pi$ for K^+ and K^- decays, both corrected for experimental detection efficiency. That correction changes the linear term observed in the raw data by about 16%. The correction is the same for K^+ and K^-.

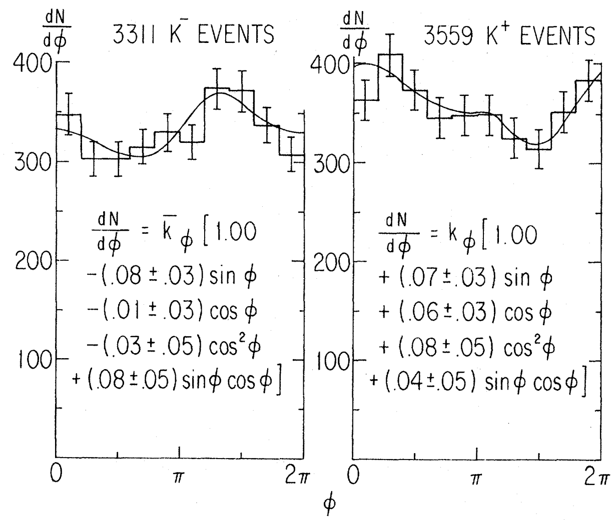

FIG. 3. Distributions in φ for K^+ and K^- decays, both corrected for experimental detection efficiency. The correction is the same for K^+ and K^-. It amounts to $+0.02$ in the coefficient of $\sin\varphi$, -0.06 in the coefficient of $\cos\varphi$, 0.02 in the coefficient of $\cos^2\varphi$, and 0.03 in the coefficient of $\sin\varphi\cos\varphi$.

cients of terms in the φ distribution which change sign when $\varphi \to -\varphi$ must be zero. The coefficient of the term in $\sin\varphi$ in K^+ and K^- independently is observed in Fig. 3 to be nonzero, which is direct evidence of a violation of parity invariance.

The operation C takes a particle into its antiparticle. Invariance under C therefore requires that the distribution in any variable should be the same for K^+ and K^-. The difference in algebraic sign of the $\sin\varphi$ terms in the φ distributions for K^+ and K^-, shown in Fig. 3, is a violation of C invariance.

Note that these violations of P and C invariance are observed without recourse to any spin correlation, unlike all previous observations of P- and C-invariance violations in semileptonic weak interactions and, more generally, in weak interactions involving fermions.[9]

Under the combined CP operation, $\varphi \to -\varphi$ and particle → antiparticle. Invariance under CP requires therefore that the distribution for K^+ at $+\varphi$ should be the same as the distribution for K^- at $-\varphi$. Within the experimental errors the data in Fig. 3 are consistent with CP invariance.

Under the operation T, spins and momenta are inverted. T invariance requires that the relative phase between any two interfering amplitudes either is zero, e.g., as in p-wave–p-wave interference leading to the term in $\sin\varphi\cos\varphi$, or is the phase shift due to the final-state interactions, e.g., as in $\sin(\delta_0 - \delta_1)$ in the s-wave–p-wave interference leading to the term in $\sin\varphi$; zero relative phase results in a zero coefficient of the $\sin\varphi\cos\varphi$ term. In Fig. 3 the coefficient of the term in $\sin\varphi\cos\varphi$ is seen to be consistent with zero and therefore consistent with T invariance.

We can obtain a quantitative limit on a possible CP-invariance violation from the data of this experiment. In the usual way, we write the matrix element of the hadronic current for $K_{e4}{}^+$ decay as

$$\langle \pi^+\pi^- | J_\mu | K^+\rangle = \frac{f}{m_K}(P_+ + P_-)_\mu + \frac{g}{m_K}(P_+ - P_-)_\mu + \frac{h}{m_K{}^3}\epsilon_{\mu\nu\rho\sigma}P_\nu{}^K(P_+ + P_-)_\rho(P_+ - P_-)_\sigma,$$

where P_+, P_-, and P^K are the π^+, π^-, and K four-momenta, respectively; f and g are dimensionless axial-vector form factors; and h is a dimensionless polar-vector form factor. Performing a partial-wave expansion of the form factors, retaining terms with $l \le 1$ and assuming the semileptonic $|\Delta I| = \frac{1}{2}$ rule, one finds that g and h induce transitions to di-pion states with $I=1, l=1$, while f contributes two terms, f_0 and f_1, which induce transitions to di-pion states with $I=0, l=0$ and $I=1, l=1$, respectively. For K^+ decay we write

$$f_0(K^+) = \tilde{f}_0 \exp[i(\delta_0 + \alpha_0)], \quad f_1(K^+) = \tilde{f}_1 \exp[i(\delta_1 + \alpha_f)], \quad g(K^+) = \tilde{g}\exp[i(\delta_1 + \alpha_g)], \quad h(K^+) = \tilde{h}\exp(i\delta_1),$$

where $\tilde{f}_0$, $\tilde{f}_1$, and $\tilde{g}$ are real, δ_0 and δ_1 are the phases of the π-π scattering amplitude, and α_0, α_f, and α_g are CP-invariance violating phases. Invariance under CPT requires that

$$f_0(K^-)=\tilde{f}_0\exp[i(\delta_0-\alpha_0)],\quad f_1(K^-)=\tilde{f}_1\exp[i(\delta_1-\alpha_f)],\quad g(K^-)=\tilde{g}\exp[i(\delta_1-\alpha_g)],\quad h(K^-)=\tilde{h}\exp(i\delta_1).$$

It has been observed[8] that the transition probability can be written in the form

$$I(M_{\pi\pi}, M_{e\nu}, \cos\theta_\pi, \cos\theta_e, \varphi)dM_{\pi\pi}\,dM_{e\nu}\,d\cos\theta_\pi\,d\cos\theta_e\,d\varphi,$$

where I contains all of the form factor and relative phase information and may be specified as an explicit function of θ_e and φ as follows:

$$I=I_1+I_2\cos2\theta_e+I_3\sin^2\theta_e\cos2\varphi+I_4\sin2\theta_e\cos\varphi+I_5\sin\theta_e\cos\varphi+I_6\cos\theta_e+I_7\sin\theta_e\sin\varphi+I_8\sin2\theta_e\sin\varphi+2I_9\sin^2\theta_e\sin\varphi\cos\varphi.$$

It is of particular interest[8] that

$$(\langle I_7\rangle/2\langle I_4\rangle)_{K^+}=\tan[(\delta_0-\delta_1)+(\alpha_0-\alpha_g)],\quad (\langle I_7\rangle/2\langle I_4\rangle)_{K^-}=-\tan[(\delta_0-\delta_1)-(\alpha_0-\alpha_g)],$$

where the angular brackets indicate that the quantity is averaged over the entire range of $M_{\pi\pi}$ (and $M_{e\nu}$) of the experiment; $\langle M_{\pi\pi}\rangle=327$ MeV/c^2. It should be emphasized that *all form factor information has exactly cancelled out of the ratios above.*

By fitting the experimental two-dimensional distribution in θ_e and φ for K^+ and K^- separately, we obtain[10]

$$\langle\delta_0-\delta_1\rangle=0.22\pm0.06\text{ rad},\quad \langle\alpha_0-\alpha_g\rangle=-0.02\pm0.06\text{ rad}.$$

This analysis also provides the explicit relationship $(2\langle I_8\rangle/\langle I_5\rangle)_{K^\pm}=\tan(\delta_0-\delta_1\pm\alpha_0)$, as well as a direct relationship between $\langle I_9\rangle$ and $\sin\alpha_g$ alone. We find, however, that $\langle I_5\rangle$, $\langle I_8\rangle$, and $\langle I_9\rangle$ to be consistent with zero within experimental error and are therefore unable to determine α_0 and α_g separately.

These results do not test that model[11] of CP-invariance violation in which $\alpha_0=\alpha_f=\alpha_g=\alpha$, i.e., in which α is a relative CP-invariance-violating phase between the strangeness-changing axial- and polar-vector currents. The measured value of the phase difference $\alpha_0-\alpha_g$ indicates that a large CP-invariance violation does not arise from the interference of two strangeness-changing axial-vector amplitudes in K_{e4} decay.

A detailed analysis of these data in which the form factors and the dependence of $\delta_0-\delta_1$ on $M_{\pi\pi}$ are extracted will be presented later.

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Charged-Particle Multiplicity Distribution from 200-GeV *pp* Interactions*

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From 2728 events of 205-GeV pp interactions found in 15 000 pictures taken with the 30-in. hydrogen bubble chamber at the National Accelerator Laboratory, a total cross section of 39.5 ± 1.1 mb was measured. The mean charged-particle multiplicity for inelastic pp collisions was measured to be 7.65 ± 0.17. The prong distribution from 2 to 22 prongs is broader than a Poisson distribution and has a width parameter $f_2^- = \langle n_-(n_- - 1)\rangle - \langle n_-\rangle^2 = 0.95 \pm 0.21$.

In this Letter we present data on pp interactions observed in the 30-in. hydrogen bubble chamber at the National Accelerator Laboratory. Although many aspects of this exposure are still being studied, we present the results on the charged-prong multiplicity distributions because of the current interest in such properties of high-energy interactions.

The proton beam from the separated-function synchrotron operating at 205 GeV was extracted and passed through a series of collimators and magnetic lenses to reduce the intensity from $\sim 10^9$ to a few protons per pulse. After momentum analysis and shaping, the beam entered the 30-in. hydrogen bubble chamber. The chamber was operated at a magnetic field of 27 kG and four views were photographed on 70-mm film.

The total of about 15 000 good pictures was divided into two equal samples which were independently analyzed by two groups of the authors. The film was scanned by the authors using all four views on a projection of 90% life size. The fiducial volume in space was about 40 cm along the beam direction. In the scan, the following were recorded: (1) the number of charged prongs, (2) associated V^0 decays and γ conversions, (3) charged decays of prongs from the main vertex, (4) obvious Dalitz pairs, recognized by the spiralization or bubble density and curvature of an electron track, (5) associated neutron stars, and (6) any secondary interactions of the particles from the primary vertex. The frames used in this experiment were quite free of background tracks, and the optical quality of the track images is outstanding in this chamber, so that no trouble was experienced in recognizing the events.

For each acceptable frame, a beam count was made. An average of 3.5 beam tracks entered