Criteria for the Elimination of Discrete Ambiguities in Nuclear Optical Potentials*

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(Received 8 June 1972)

The elastic scattering of composite projectiles by nuclei is considered in the semiclassical limit. Criteria based on this description are obtained for the incident energy and angular range of data necessary to eliminate discrete ambiguities in optical potentials for such projectiles. We demonstrate the effectiveness of these criteria for elastic α-scattering data.

It has been known for a number of years that the interaction responsible for the elastic scattering of nucleons and nucleon clusters by nuclei can be represented by an optical potential. Although the optical model has on the whole been quite successful, persistent difficulties have existed in the determination of the strength of the interaction for composite projectiles (nucleon clusters) because of the well-known discrete ambiguities in the values of the real well depth of the optical potentials.[1] Recent investigations[2-4] have indicated that these ambiguities can be eliminated by measuring differential cross sections at "sufficiently" high energies and at "sufficiently" large scattering angles. We have observed that the elastic differential cross sections satisfying these sufficiency requirements, when plotted as a ratio to the appropriate Rutherford cross section, exhibit a characteristic monotonic, almost exponential falloff pattern at angles beyond those characterized by diffraction oscillations. A semiclassical description of the scattering is able to reproduce this behavior; using this description, we have been able to formulate quantitative criteria for the bombarding energy and angular range of data necessary for the elimination of these discrete ambiguities.

The physical basis of our criteria can be simply illustrated by considering the classical description of the scattering of a particle by a central force. In this limit the differential cross section is given by the familiar relation[5]

$$\sigma(\theta) = (b/\sin\theta)\,|(d\theta/db)|^{-1}, \qquad (1)$$

where b is the impact parameter and θ is the deflection angle, which is generally a function of the strength of the interaction, the impact parameter, and particle energy. If the radial form of the interaction is similar to an attractive Woods-Saxon form, and its central depth is large compared to the incident energy, "spiral scattering" will occur; i.e., for some impact parameters the deflection angle of the scattered particles will exceed 180°. If the interaction potential is approximately energy independent, spiral scattering will cease to occur if the energy is increased sufficiently. The scattering will then be characterized by a maximum deflection angle Θ, which will decrease with increasing bombarding energy. No particles will be observed beyond this angle, and a measurement of this angle can be utilized to determine the strength of the interaction.[6]

In the semiclassical description of scattering one can similarly define a deflection angle.[7] If the maximum deflection angle Θ is less than 180°, the differential cross section exhibits an almost exponential falloff[8] which commences at angles somewhat less than Θ. We will show that elastic differential-cross-section data satisfying the following criteria enable one to eliminate discrete ambiguities: (1) The measurements must be performed at a bombarding energy high enough that the falloff appears, i.e., that Θ is less than 180°; (2) the measurements must be extended to angles beyond Θ. Conversely, if a set of potentials constituting a discrete ambiguity is known, one can use the above criteria to specify the bombarding energy and angular range of data required to determine whether a given potential in that set can be either unambiguously defined or else eliminated.

In the semiclassical limit nuclear scattering can be formulated in the Jeffreys-Wenzel-Kramers-Brillouin (JWKB) approximation.[7] The lth partial wave is identified with a particle trajectory having an impact parameter $b = (l+\frac{1}{2})/k$. The particle deflection for this impact parameter is related to wave-mechanical quantities via the relation

$$\theta_l = 2\,d(\delta_l^R + \sigma_l)/dl, \qquad (2)$$

where θ_l is the deflection angle of the particle, δ_l^R is the real part of the nuclear phase shift, and σ_l is the Coulomb phase shift of the lth par-

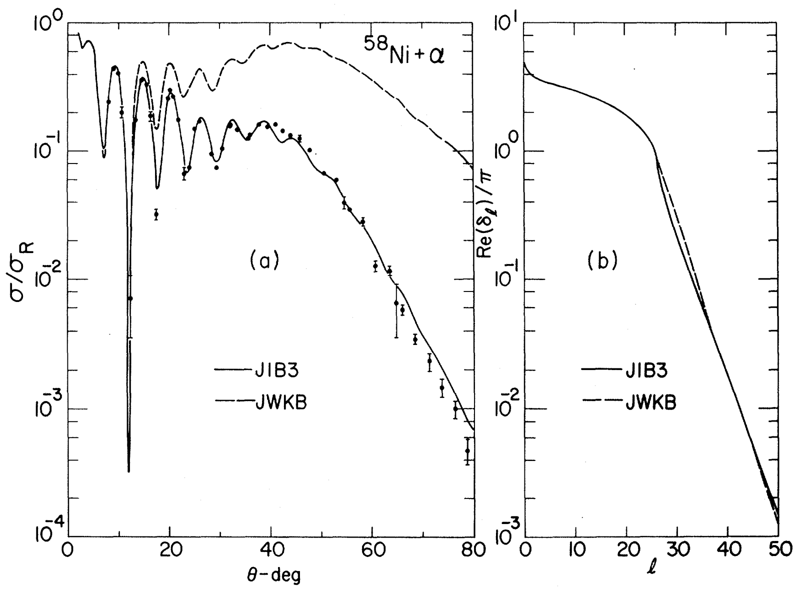

FIG. 1. Comparison of semiclassical (JWKB) and conventional (JIB3) optical-model calculations with 139-MeV α-scattering data. (a) Ratio of differential cross section to Rutherford cross section. (b) Real nuclear phase shifts. The optical-model parameters used in both calculations are $V = 116.7$ MeV, $r_0 = 1.246$ fm, $a = 0.793$ fm, $W = 21.15$ MeV, $r_0' = 1.590$ fm, and $a' = 0.569$ fm.

tial wave. The maximum θ_l for all l is then Θ.[9]

The ability to define the angle Θ for a given set of elastic scattering data is based on the adequacy of the semiclassical description. We now demonstrate this adequacy for data exhibiting the falloff pattern described above. (In the present paper we consider only a single set of such data; similar results for a number of other cases will be presented in a more detailed report.[10]) We have performed JWKB and exact optical-model calculations for the scattering of 139-MeV α particles by ^{58}Ni using an optical potential having the conventional six-parameter Woods-Saxon form.[11] The parameters used in the calculations were obtained by fitting the data[12] using a modified version of the code[13] JIB3. The semiclassical cross section was obtained by summing the individual partial-wave amplitudes calculated using JWKB phase shifts.[14]

The data and calculated cross sections are shown in Fig. 1(a). The zeroth-order JWKB calculation reproduces the *shape* of the cross section, including the large-angle falloff, fairly well. As can be seen from Fig. 1(b), the real phase shifts obtained from the two calculations are in good agreement. The discrepancy in *magnitude* at large angles is primarily due to discrepancies in the imaginary phase shifts. This has been substantiated by model calculations with $W = 0$. In this case, the imaginary phase shifts vanish identically and the two calculations produce essentially identical cross sections. We note that these cross sections still retain the characteristic falloff pattern at large angles, indicating that the *falloff is not caused by strong absorption, but is indeed associated with the real part of the potential.*

In obtaining an optical potential to fit the entire range of the above data, there was no evidence of a discrete ambiguity (i.e., only a single parameter "family" was found). This illustrates our earlier statement that if the cross sections exhibit the falloff pattern, one can eliminate the ambiguities. For the above Woods-Saxon potential the maximum deflection angle Θ, as calculated using Eq. (2), was 63°, somewhat beyond the point at which the falloff in the cross section begins.

To determine the angular range of data required to eliminate ambiguities, we performed a series of χ^2 searches using code JIB3 for various truncated sets of the data shown in Fig. 1(a). For each set we performed a series of grid searches in which V was held fixed at various values and the remaining parameters were varied to minimize χ^2/N, where N is the number of degrees of freedom. The first grid search was performed using 29 data points extending to 41°; succeeding searches included data to 61° (just below Θ), 65° (just beyond Θ), and 80°. This represented in-

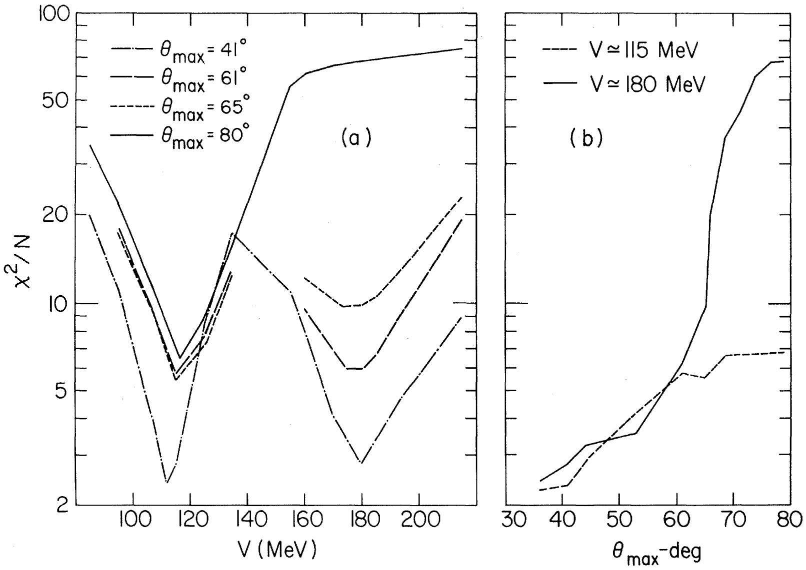

FIG. 2. (a) Minimum χ^2/N values obtained from grid searches on V based on four subsets of the data truncated at differing θ_{max}. (b) Minimum χ^2/N for the potential "families" in the neighborhood of $V=115$ MeV and $V=180$ MeV as a function of θ_{max} (includes results from additional data subsets).

creases of 12, 2, and 6 data points, respectively. For each grid, χ^2/N was plotted as a function of V. The results are shown in Fig. 2(a). For the 41° searches, two minima of comparable χ^2 were obtained at $V \approx 115$ MeV and $V \approx 180$ MeV. When data to 61° are included, χ^2/N at each of these minima roughly doubles, but they remain essentially equal to each other. However, as one includes data to 65° (only two additional data points), the χ^2/N minimum at $V \approx 180$ MeV increases by approximately 50%, whereas the one at $V \approx 115$ MeV remains essentially unchanged. Including data to 80° eliminates any trace of a minimum in χ^2/N in the neighborhood of $V = 180$ MeV.

An alternate illustration of the necessity for extending measurements beyond Θ is presented in Fig. 2(b). For the parameter "families" characterized by $V=115$ MeV and $V=180$ MeV, the minimum χ^2/N is plotted as a function of θ_{max}, the data truncation limit. As can be seen, the values of χ^2/N for the two families increase slightly with θ_{max}, but still remain within about 20% of each other until θ_{max} reaches Θ. Beyond Θ, however, χ^2/N for the higher family increases to over 10 times its value at $\theta_{max} = \Theta$, whereas χ^2/N for the lower family increases by less than 30%.

The incident energy required to observe the falloff pattern for a specific nucleus depends on its size. For fixed bombarding energies, Θ increases with increasing A, possibly even exceeding 180° (i.e., ceasing to exist for large A).[15] Thus, energies sufficiently high to eliminate ambiguities in lower-A nuclei may not suffice to eliminate them in nuclei of higher A. This effect has been noted previously[2] when it was observed that a large number of potential "families" were found which satisfactorily described the scattering of α's by ^{208}Pb at 139 MeV, an energy sufficiently high to eliminate the ambiguities for ^{58}Ni. On the other hand, if one is able to eliminate the ambiguities in the prescribed way for a target nucleus of a given A, then one should be able to eliminate the ambiguities for all lower-A nuclei at the same incident energy. Recent results[16] for the scattering of 139-MeV α particles by ^{12}C bear this out. The A dependence of the falloff, and thus of Θ, is also evident in a number of intermediate-energy elastic scattering investigations.[4]

Although we have primarily been discussing intermediate-energy elastic α scattering, the prescription we have outlined here for eliminating discrete ambiguities is suitable for other composite projectiles, such as d and ^{3}He. Data satisfying the outlined criteria will thus permit comparison of the relative interaction strengths of different projectiles, both macroscopically through optical potentials, and microscopically, through density-dependent calculations. We believe that such comparisons will be extremely useful in furthering our understanding of the nuclear elastic scattering interaction.

We would like to thank H. G. Pugh for fruitful discussions regarding the semiclassical treatment of scattering, P. G. Roos and N. S. Wall for assistance and encouragement, G. Jacob and G. Tibell for helpful suggestions, and N. R. Yoder for greatly facilitating the computation procedures. Comments by Koheleth have proven helpful in placing the present results in perspective.

*Work supported in part by the U. S. Atomic Energy Commision.

[1]R. M. Drisko, G. R. Satchler, and R. H. Bassel, Phys. Lett. 5, 347 (1963); also see examples in D. F. Jackson and C. G. Morgan, Phys. Rev. 175, 1402 (1968).

[2]D. A. Goldberg, H. G. Pugh, P. G. Roos, and S. M. Smith, Bull. Amer. Phys. Soc. 16, 1184 (1971).

[3]P. P. Singh, R. E. Malmin, M. High, and D. W. Devins, Phys. Rev. Lett. 23, 1124 (1969); C. B. Fulmer and J. C. Hafele, Bull. Amer. Phys. Soc. 16, 99 (1971).

[4]G. Hauser *et al.*, Nucl. Phys. A128, 81 (1969); B. Tatischeff and I. Brissaud, Nucl. Phys. A155, 89 (1970).

[5]H. Goldstein, *Classical Mechanics* (Addison-Wesley, Reading, Mass., 1950), p. 82.

[6]Alternatively, the particle trajectory limit of the scattering process can be viewed as the geometrical-optics limit of wave optics. In this limit, the nucleus acts like a refracting sphere which is capable of deflecting light rays up to a maximum angle Θ. The real refractive index of the nuclear lens is then characterized by this maximum angle of refraction.

[7]R. M. Eisberg and C. E. Porter, Rev. Mod. Phys. 33, 190 (1961); K. W. Ford and J. A. Wheeler, Ann. Phys. (New York) 7, 259, 287 (1959).

[8]In our optical analogy, this may be likened to the illumination of the region forbidden by geometrical optics; such illumination appears, for example, in the shadow region of a Fresnel diffraction pattern.

[9]Reference 7 discusses the case of a maximum *positive* deflection angle caused by the repulsive Coulomb force; they refer to this angle as the "rainbow" angle. In the present work, Θ is the maximum *negative* deflection produced by the nuclear potential. The nuclear "rainbow" angle is generally much larger than the Coulomb "rainbow" angle for nucleon cluster projectiles.

[10]S. M. Smith and D. A. Goldberg, to be published.

[11]$U(r) = -V(1+e^x)^{-1} - iW(1+e^{x'})^{-1}$, where $x = (r - r_0 A^{1/3})/a$ and $x' = (r - r_0' A^{1/3})/a'$; to this is added the Coulomb potential $V_C(r)$ of a uniformly charged sphere.

[12]D. A. Goldberg, S. M. Smith, H. G. Pugh, P. G. Roos, and N. S. Wall, to be published.

[13]F. Perey, private communication.

[14]The JWKB complex nuclear phase shifts are given in conventional notation by

$$\delta_l = k\int_{r_2}^{R} [1 - \hbar^2(l+\tfrac{1}{2})^2/2\mu r^2 - U(r) - V_C(r)]^{1/2}\,dr - k\int_{r_1}^{R} [1 - \hbar^2(l+\tfrac{1}{2})^2/2\mu r^2 - 2\eta/kr]^{1/2}\,dr,$$

where r_2 and r_1 are defined by the equations

$$1 - \hbar^2(l+\tfrac{1}{2})^2/2\mu r_2^2 + V(r_2) - V_C(r_2) = 0,$$

$$1 - \hbar^2(l+\tfrac{1}{2})^2/2\mu r_1^2 - 2\eta/kr_1 = 0.$$

[15]W. H. Miller, J. Chem. Phys. 51, 3631 (1969) has shown that spiral scattering ceases at an energy given by $E = [V(r) + (r/2)dV(r)/dr]_{\max}$. Thus, if the real optical potentials of two nuclei differ essentially only in their radial extent (i.e., $r_0 A^{1/3}$ for a Woods-Saxon form), the larger will require a greater energy to make $\Theta < 180°$.

[16]G. Tibell *et al.*, Bull. Amer. Phys. Soc. 17, 465 (1972).