son, in *Proceedings of the Twelfth International Conference on Cosmic Rays, Hobart, 1971* (Ref. 1), Vol. 1, p. 215.

[4]W. R. Webber, S. V. Damle, and J. Kish, Astrophys. Space Sci. 15, 245 (1972).

[5]M. Casse, L. Koch, N. Lund, J. P. Meyer, B. Peters, A. Soutoul, and S. N. Tandon, in *Proceedings of the Twelfth International Conference on Cosmic Rays, Hobart, 1971* (Ref. 1), Vol. 1, p. 241.

[6]T. Saito, J. Phys. Soc. Jap. 30, 1243, 1535 (1971).

[7]M. M. Shapiro and R. Silberberg, Annu. Rev. Astron. Astrophys. 8, 323 (1970).

[8]L. P. Moskaleva, G. A. Fedoseev, and A. N. Khalemskii, Yad. Fiz. 12, 872 (1970) [Sov. J. Nucl. Phys. 12, 472 (1971)].

[9]B. G. Cartwright, M. Garciz-Munoz, and J. A. Simpson, to be published.

[10]M. M. Shapiro, R. Silberberg, and C. H. Tsao, in *Proceedings of the Twelfth International Conference on Cosmic Rays, Hobart, 1971* (Ref. 1), Vol. 1, p. 221.

Approach to Factorization and Scaling in Inclusive Reactions*†

T. Ferbel‡

Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

(Received 14 June 1972)

We present a compilation of existing data on inclusive reactions which provide evidence for the factorization hypothesis in the "central" region of pion production but only at infinite beam momentum. The approach to limiting behavior in the central region is consistent with that expected on the basis of the Mueller-Regge formalism.

Several groups have recently examined the question of the validity of the hypothesis of limiting fragmentation, scaling behavior, and the factorization hypothesis in inclusive pion-production reactions.[1] In general, the conclusion which has been reached is that these principles appear to hold to good accuracy in the fragmentation region of momentum space; these principles, however, do not appear to be valid for small values of emitted-pion momentum in the center-of-mass system (i.e., small $x = p_l^*/p_{l,\text{incident}}^*$). In fact, several groups have pointed out that, particularly for reactions such as

$$p + p \to \pi^- + \text{anything}, \tag{1}$$

$$\pi^+ + p \to \pi^- + \text{anything} \tag{2}$$

scaling near $x = 0$ is badly violated.

We wish to make here several comments and observations concerning inclusive pion-production reactions at accelerator energies, and point out that a great simplification results when the invariant cross section for these reactions is extrapolated to infinite momentum. The results of this extrapolation strongly suggest that, to better than 10% accuracy, factorization of inclusive reactions obtains at $x = 0$, and consequently at all values of x.

Figure 1 presents a plot of the normalized invariant single-particle production cross sections (integrated over p_t) at $x = 0$ as a function of $p^{-1/4}$ (p is the bombarding momentum in the laboratory system). We define the ordinate in the graph, the parameter c, as follows:

$$c = \frac{1}{\sigma_T} \int_{\text{all } p_t} E^* \frac{d^2\sigma}{dp_l^* dp_t} dp_t \bigg|_{p_l^* = 0}.$$

The starred variables are c.m. quantities, p_t^* is

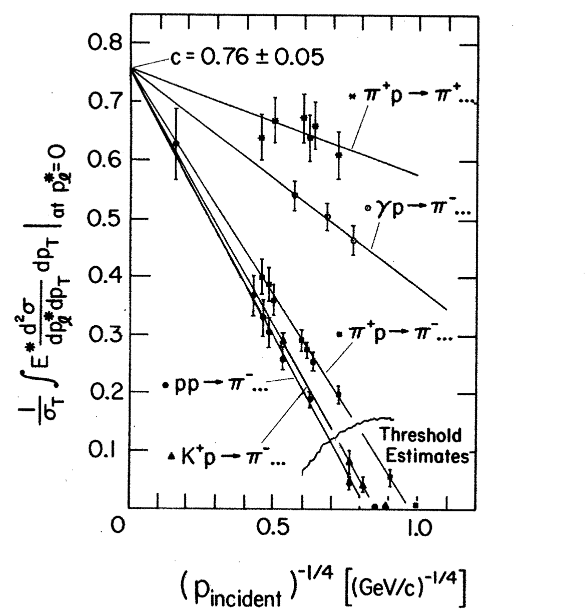


FIG. 1. Normalized invariant single-particle inclusive cross section at $x = 0$ as a function of beam momentum. In the normalization we use the following asymptotic values of σ_T: for pp, 39.8 mb; π^+p, 23.4 mb; K^+p, 17.4 mb; and γp, 99 μb.

the transverse momentum, p_l^* is the longitudinal momentum, and σ_T is the total cross section for the incident channel at asymptotic energies. We have converted to this form[2] all the available data for Reactions (1), (2), and the following:

$$\gamma + p \to \pi^- + \text{anything}, \tag{3}$$

$$K^+ + p \to \pi^- + \text{anything}, \tag{4}$$

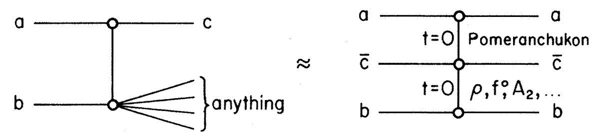
$$\pi^+ + p \to \pi^+ + \text{anything}. \tag{5}$$

We note that all the experimental points from each reaction appear to fall on a straight line; furthermore, all straight lines appear to extrapolate back to a single point in the limit of $p \to \infty$. The straight lines drawn in Fig. 1 are not best fits to the data, but are just to guide the eye. The error bars provide estimates of uncertainty in each data point (some data were obtained directly from published graphs, others were obtained by extrapolation—error bars reflect all uncertainties).

We also note that for Reactions (1) and (2), where at least five measurements are available, the extrapolation through the data points to *small* p intercepts the estimated values of c near threshold. [No inclusive measurements are available for $p < 2$ GeV/c; we have therefore estimated the values of c at low beam momenta by taking the known total π^- production cross section $\sigma(\pi^-)$ (not the "invariant" type) and multiplying this number by the ratio of $c\sigma_T/\sigma(\pi^-)$ for the lowest-energy experiment available. The latter ratio is somewhat energy dependent and the error bars on the "threshold" estimated data points reflect this source of uncertainty.] Because there is only one momentum point available for Reaction (4) we have consequently used the threshold value of c as a crutch point in drawing the straight line.

Our reason for using a $p^{-1/4}$ form in the extrapolation is based on Mueller's ideas concerning the "central" region of particle production. It is expected that at high energy the remnant dependence near $x \simeq 0$ would be of the form $p^{-1/4}$ if a three-particle elastic-scattering Mueller graph is relevant for the description of the central region (see Fig. 2). It is, of course, somewhat astounding that the $p^{-1/4}$ dependence appears to be valid down to almost threshold momenta. (Mueller-Regge graphs which contain two three-particle vertices, rather than at least one Pomeranchukon-particle-particle vertex, must therefore not contribute significantly in these reactions.)

The inescapable conclusion one can draw from these results is that the Mueller-Regge formalism for inclusive reactions is valid for the entire domain of x momentum space. Furthermore, it appears that when the scaling limit is reached (far above the intersecting-storage-rings range of energies!), inclusive cross sections factorize for all values of x (the limit having already previously been reached at lower energies for the fragmentation regime of large x values).[3]



$$S_{ab} = (E_a^* + E_b^*)^2 \propto p \qquad S_{\bar{c}b} = M_c^2 + 2M_c E_b^* \propto p^{1/2}$$

$$F_c(p_T^2, p_l^* = 0, S_{ab}) \approx S_{\bar{c}b}^{\alpha(0)-1} \approx p^{-1/4}$$

FIG. 2. Slowest energy-dependent Mueller-Regge graph appropriate for the central region.

It is hoped that this note will serve to stimulate experimenters to re-examine their data in more detail so as to quantify more fully the present partially qualitative findings. It is also hoped that an explanation can be found for the premature onset of the $p^{-1/4}$ behavior, the significance of the numerical value of the parameter c, and the slopes observed in Fig. 1. Our results also imply certain simple statements concerning, for example, multiplicity growth and mean multiplicities at large momenta (if $\bar{n} \sim c \ln p$, then multiplicity is independent of incident channel). Also, tests of the Pomeranchuk theorem will be possible when substantial data on similar reactions in the K^-p, $\bar{p}p$, and π^-p channels become available.

As a final word of warning we note that what we have indicated is a consistency of the presently available data with a rather simple $p^{-1/4}$ behavior. We have not excluded other forms of approach to limiting behavior. This can only be done if experiments publish their values of c (using our definitions for x as a convenience), with appropriate errors, so that fits can be performed to the data in order to extract the exact form of the energy dependence.

I wish to thank the National Accelerator Laboratory and Argonne National Laboratory for the hospitality extended to me during my stay at these laboratories. I also thank Ed Berger for many pleasant discussions. Finally, I thank my colleague Paul Slattery for reading this note and

for making helpful suggestions.

*This note is based on a more extensive review paper presented at the International Colloquium on Multiparticle Reactions at Zakopane (June 1972).

†Work supported in part by the U. S. Atomic Energy Commission.

‡John S. Guggenheim Fellow.

[1]See Proceedings of the Conference on Inclusive Reactions at Davis, California, February 1972 (unpublished); and papers in Proceedings of the Fourth International Conference on High-Energy Collisions, Oxford, England, April 1972 (unpublished).

[2]The data used in Fig. 1 are derived from the following sources: pp data from A. Bertin *et al.*, Phys. Lett. 38B, 260 (1972); British-Scandinavian Intersecting Storage Rings Collaboration, in Proceedings of the Fourth International Conference on High-Energy Collisions, Oxford, England, April 1972 (unpublished); J. Hanlon *et al.*, Nucl. Phys. B41, 317 (1972); H. J. Mück *et al.*, DESY Report No. F1-72/1, 1972 (unpublished); Scandinavian Bubble Chamber Collaboration, to be published; E. Gellert, Lawrence Berkeley Laboratory Report No. LBL-784, 1972 (unpublished). γp data from K. C. Moffeit *et al.*, SLAC Report No. SLAC-PUB-1004, 1971 (unpublished). π^+p data from D. J. Crennell *et al.*, Phys. Rev. Lett. 28, 643 (1972); W. D. Shephard, to be published; R. Stroynowski (ABBCCHLVW Collaboration), CERN Report No. CERN/D.Ph.II/PHYS 72-18, 1972 (unpublished); S. Stone *et al.*, Phys. Rev. D 5, 1621 (1972); M. Alston-Garnjost *et al.*, Phys. Lett. 39B, 402 (1972). Also CERN-HERA reports and UCRL-20 000 series for "threshold" estimates.

[3]This statement is strictly true only for "produced" as opposed to "leading" particles. In particular, Reaction (5) contains the elastic and pseudoelastic channels near $x\approx 1$ which are not present in the other four reactions. It is therefore somewhat surprising that Reaction (5) behaves similarly to the other reactions. This may be accidental, and it may, in fact, turn out that the cross section for Reaction (5) will start falling towards Reaction (2) and eventually reach to the same limiting value of c (as expected on the basis of the results from the intersecting storage rings).

Determination of Resonance Parameters from Partial-Wave Amplitudes*

Philip W. Coulter
Department of Physics, The University of Alabama, University, Alabama 35486
(Received 15 May 1972)

We present a simple means of separating a partial-wave amplitude into a resonant term plus an inelastic background. The separation is exact if only one inelastic channel is present, but it should be a good approximation in the presence of more than one inelastic channel. Inequalities are obtained relating the elastic and inelastic widths to the background inelasticity. Comments are made regarding the application of the formalism.

Masses and widths of resonances can be obtained only from phase-shift analyses of partial-wave amplitudes. Although there are now several independent phase-shift analyses of πN scattering,[1] there are still no reliable values for the masses and widths of resonances in the πN system.[2] One problem is that the errors in the phase shifts are large. Another problem arises in separating the resonance from the background. The latter problem has been discussed by Dalitz[3] and Michael.[4] They have shown that in principle the background can be separated from the resonance by using unitarity. Their formalism is difficult to apply in practice and has generally been ignored in determining resonance parameters.

The purpose of this paper is to suggest an alternative formalism for determining resonance parameters which is easy to use. The shortcoming of the formalism presented here is that it is strictly applicable only if there are two open (two-body) channels. In the present phase-shift analyses of πN scattering, the phase shift and inelasticity for the elastic channel only are obtained. Until more information is available about the inelastic channels, a more sophisticated approach cannot be profitably applied. We believe that an application of the formalism presented here can give the most reliable determination of the resonance parameters available from the present data.

We follow the usual procedure of writing the scattering amplitude as a resonant term plus a background term which is assumed to be unitary.[5] If the background term also includes a resonance, the separation may be repeated. We write

$$T_{ij}=T_{ij}{}^{B}+\exp(2i\alpha_{ij})\frac{\frac{1}{2}(\Gamma_i\Gamma_j)^{1/2}}{m-W-i\frac{1}{2}\Gamma},\qquad(1)$$