TABLE II. Angular-momentum analysis of the total wave function of the three-nucleon bound state.

l	λ	L	S	α	Probability (%)
0	0	0	1/2	A	88.1
2	2	0	1/2	A	1.6
0	0	0	1/2	−	0.9
2	2	0	1/2	−	0.1
1	1	0	1/2	+	0.9
2	0	2	3/2	−	2.7
0	2	2	3/2	−	0.8
1	1	2	3/2	+	1.6
3	1	2	3/2	+	0.4
1	3	2	3/2	+	1.1

cluding some of them directly in the Faddeev amplitude (then the two-nucleon interaction in odd states occurs); in fact, one can wonder if neglecting components of the Faddeev amplitude which appear in the total wave function is not somewhat inconsistent.

The use of configuration space makes the solution of Faddeev equations for bound states straightforward. In this case the potential occurs directly instead of the physically equivalent two-body t matrix. Moreover, the method we described takes advantage of the appearance of the energy as an eigenvalue, and does not set a limit on the manageable number of components of the Faddeev amplitude. Our results for the properties of the three-nucleon bound state are in general agreement with those obtained from another exact (but fundamentally different) method; they confirm the relevance of some higher orbital momentum states in predicting the change form factor.

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Decoupled Yrast States in Odd-Mass Nuclei*

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We have studied a system consisting of a particle coupled to a rotating core. Although a complete decoupling of particle and core occurs only at very small deformations (coupling strengths), a "favored" high-spin band of levels tends to be decoupled over a much larger region of deformation. This can be understood as a simple Coriolis effect, and seems to be borne out remarkably well in some odd-A La isotopes.

In each of a series of five odd-A La isotopes, a band of levels based on a state with $I=\frac{11}{2}$ has been found[1,2] which has spin values increasing monotonically from the base level by $2\hbar$ per state, and energy spacings very similar to the even-even Ba isotope with one less proton. The correspondence in energy is rather remarkable and is shown in Fig. 1. The experimental data consist of in-beam γ-ray studies[1] following reactions Sn(^{14}N, $3n$)La, and proton-transfer-reaction studies[2] using the reactions Ba(α, t)La. The first work revealed the cascades of stretched-$E2$ γ rays, and the second indicated that these cascades were systematically based on $\frac{11}{2}$ states. This band might be explained by a particle-core weak-coupling model,[3] but this model seems improbable for nuclei with core spacings as small as those in Fig. 1, and has not been notably successful even in more favorable cases. It is the purpose of this note to show that this kind of spectrum can arise as a general Coriolis effect without the weak-coupling assumption.

To demonstrate the physical effect involved, we consider qualitatively a particle coupled to a rotating core.[4] The principal influence of the rotation on this coupling can be expressed by introducing the Coriolis effects, and these depend essentially on the Coriolis force $\vec{R}\times\vec{j}$, where $\vec{R}$ and $\vec{j}$ are the core and particle angular momenta. But for those states where $\vec{R}$ is coupled almost parallel to $\vec{j}$, achieving the maximum I, $\vec{R}\times\vec{j}$ is nearly zero. Thus, the rotation has little effect on the coupling of the particle to the core for this orientation of the two, and the energy changes of the system just reflect the core energy changes. Such a band of levels, which we will refer to as rotation-particle decoupled, has just the properties displayed in Fig. 1. To find under what conditions this band is likely to occur as a low-lying feature of the spectrum, we will examine a simple Hamiltonian that contains the essential features.

For a particle coupled to an axially symmetric deformed core which can rotate, we can write the Hamiltonian as

$$H = H_{\mathrm{intr}} + \frac{\hbar^2}{2\mathfrak{J}}\vec{R}^2 = H_{\mathrm{intr}} + \frac{\hbar^2}{2\mathfrak{J}}[I(I+1) - \Omega^2] + H_C + \frac{\hbar^2}{2\mathfrak{J}}(\langle\vec{j}^2\rangle - \Omega^2), \tag{1}$$

where the symbols have their usual meaning,[5] and H_{intr} represents the Hamiltonian of the particle in the absence of any rotation of the core. Equation (1) is derived by noting that $\vec{R} = \vec{I} - \vec{j}$, and the Coriolis operator H_C can be written as

$$H_C = -2(\hbar^2/2\mathfrak{J})[\vec{I}\cdot\vec{j} - \Omega^2] = -(\hbar^2/2\mathfrak{J})[I_+ j_- + I_- j_+]. \tag{2}$$

In diagonalizing the right-hand side of Eq. (1), we use a basis system where I and Ω (the projection of j on the nuclear symmetry axis) are constants of the motion, and this corresponds to the usual one for deformed nuclei. We must evaluate H_{intr} in this system and, in doing so, will take j to be a good quantum number. This assumption is not essential, but is rather good for the high-j orbitals we want

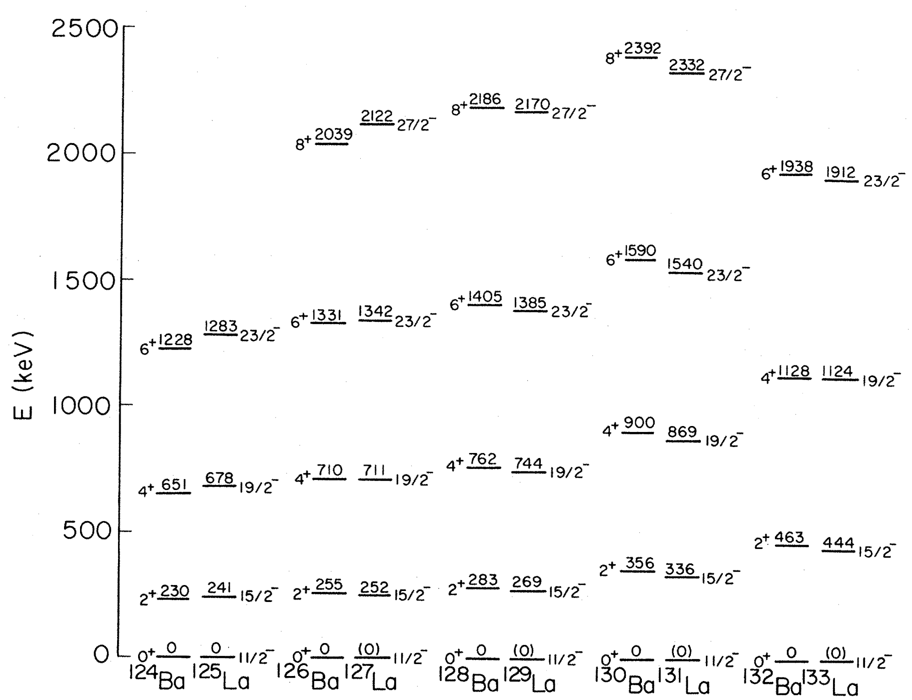

FIG. 1. Comparison of ground band levels in some Ba isotopes with the negative-parity bands in the neighboring La nuclei. In most cases (energy zero in parentheses) the La $\frac{11}{2}$ level is not the ground state, and its energy has been subtracted from all levels shown for that isotope.

to consider and permits simplifications in both $H_{\rm intr}$ and the Coriolis matrix elements. We can express $H_{\rm intr}$ by giving the energy of the system as a function of Ω; that is, as a function of the orientation of j to the symmetry axis of the core. For a core with quadrupole deformation, the Nilsson calculations[6] correspond to an evaluation of these energies, but if the deformation β is not too large, then we can use the limiting approximation[5]

$$E(\Omega)=E_0(nlj)+\frac{206\beta}{A^{1/3}}\left(\frac{3\Omega^2-j(j+1)}{4j(j+1)}\right)\text{ MeV}. \quad (3)$$

The numerical coefficient $206/A^{1/3}$ in Eq. (3) gives reasonable agreement with the Nilsson solutions for the $h_{11/2}$ and $i_{13/2}$ orbitals up to around $\beta=0.3$, and since we consider only a single j shell, the $E_0(nlj)$ can be taken to be zero. We also include the effects of the pairing correlations, which are given by

$$H_{\rm intr}(\Omega)=\{[E(\Omega)-\lambda]^2+\Delta^2\}^{1/2}-\Delta, \quad (4)$$

and the usual[5] UV factor for H_C. Using Eq. (4), we can diagonalize the right-hand side of Eq. (1) for any values of I, provided we have fixed $\hbar^2/2\mathfrak{J}$, β, λ, and Δ.

We can reduce these four parameters to two essential ones in the following way. There is a very general empirical relationship between $\hbar^2/2\mathfrak{J}$ and β (as defined from the $E2$-transition lifetime) that essentially all even-even nuclei follow.[7] For the purpose of our survey we will use this relationship to eliminate one of these variables. This gives

$$6(\hbar^2/2\mathfrak{J})\equiv E_{2+}\approx 1225/A^{7/3}\beta^2\text{ MeV}. \quad (5)$$

We use β explicitly here only to interconnect $\hbar^2/2\mathfrak{J}$ and $E(\Omega)$, and one could take the point of view that Eqs. (3) and (5) just define an empirical relationship between these quantities. However, some deformation is implicitly required for Eq. (1) to have any physical meaning. Of the remaining parameters, Δ is not very important and we take it always to be 0.8 MeV; so that the remainder of this note will consist of an attempt to understand Eq. (1) as a function of β and λ.

Let us first consider the limiting cases of large and small values for β. For large β ($\gtrsim 0.3$), the Coriolis effects are relatively unimportant since $\hbar^2/2\mathfrak{J}$ is small [Eq. (2)], and the different Ω components are widely separated. This results in the usual strong-coupling limit of pure rotational bands with good Ω values. As we move toward smaller β, the Coriolis effects become more important and perturb this structure, mixing Ω values. Our model should be reliable throughout this region. At the other limit, $\beta=0$, $H_{\rm intr}$ is a constant, and although the diagonalization of the right-hand side of Eq. (1) appears complex, the left-hand side is now transparent, giving the result that the energies are just those due to the rotation of the core. It is, of course, true that the core of an odd-A nucleus is somewhat different from that of the adjacent even-even nucleus because of the blocked level. In some cases this might be a large effect, but the data in Fig. 1 suggest that it can also be a rather small effect. In this limit there are states at each core energy having spin values that range from $|R-j|$ to $R+j$, and R is a good quantum number. This is just the case of complete decoupling discussed by Vogel.[8] Near this limit, Eq. (1) gives results identical to those of a weak-coupling model[3] with (apart from the pairing effects) a pure quadrupole-quadrupole particle-core interaction. The relevant feature of such a model is that the core state is split into a multiplet centered on the core energy, but not otherwise dependent on that energy nor on any other property of the core. Thus, even though the pure-rotational core energies given by Eq. (1) are not realistic for this region, the splitting of the core levels will be reasonable, if the real interaction is quadrupole-quadrupole (plus the pairing effects) and if our estimate of the effective quadrupole strength [Eqs. (3) and (5)] is about right. Thus, we will also examine Eq. (1) in this region of small β values, but we should at best only take seriously the positions of the various levels relative to the position of the related core state.

In Fig. 2 we have plotted the solutions to Eq. (1) for $j=\frac{11}{2}$, $\lambda=E(\frac{1}{2})$, and $\beta=0\to+0.35$. The outstanding feature is the coincidence of the energies of the "favored" high-spin states ($I=j, j+2, j+4, \cdots$) with those of the core states for all β values. The wave functions of these states show that they do tend to be decoupled from the core for all β values in Fig. 2; whereas the other states only decouple when β is small, $\lesssim 0.1$. The effect of varying λ has been studied and indicates that the tendency to decouple is strongest when λ is low. It is apparent from simple geometrical considerations that the states with low Ω values are much more important in constructing a state with $\vec{R}$ parallel to $\vec{j}$ than are those with high Ω values. In prolate nuclei these essential states lie low in the spectrum when λ is low. We found that at $\beta=+0.25$ (the largest value in the La nu-

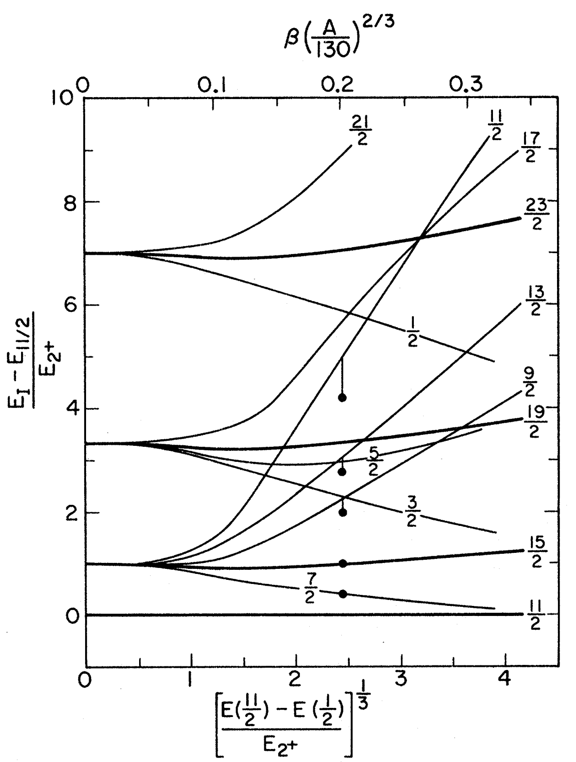

FIG. 2. Results of diagonalizing Eq. (1) for various β values, given for the lowest state of each spin up to $I=\frac{21}{2}$ (the second $I=\frac{11}{2}$ state is included). The ordinate is the eigenvalue less the lowest $I=\frac{11}{2}$ eigenvalue, in units of E_{2+}. The abscissa is β (top) or the cube root of the total splitting of the $h_{11/2}$ orbital in units of E_{2+} (bottom). The Fermi surface λ is always located on the $\Omega=\frac{1}{2}$ state, so that in the limit of very large β, the levels will become a pure $\Omega=\frac{1}{2}$ band with a decoupling parameter of -6. The dots show the effect of diagonalizing the Hamiltonian $H=H_{\text{intr}}+(\hbar^2/2\mathfrak{J})[\vec{R}^2-B\vec{R}^4+C\vec{R}^6]$, instead of Eq. (1), where B and C were adjusted to fit the lowest few levels in ^{126}Ba.

clei) the decoupling of the favored states should occur provided λ is anywhere below $\sim E(\frac{5}{2})$. In fact, λ is clearly far below this in the La region. Within the framework of our calculations, failure of the lowest favored states to decouple occurs only where β and λ are large enough, so that these states tend to have pure Ω values rather than pure R values. In addition to dramatic changes in the energies of the other yrast states ($I=j+1, j+3, \cdots$), this causes the energy, $E_{I=j+2}-E_{I=j}$, to increase from $6\hbar^2/2\mathfrak{J}$ (pure R) to $(4j+6)\hbar^2/2\mathfrak{J}$ (pure Ω). An exception occurs if $\Omega=\frac{1}{2}$, where the partial decoupling gives $(2j+5)\hbar^2/2\mathfrak{J}$ for this energy difference; generally about midway between the above limits.

In conclusion, we believe the remarkable energy coincidences shown in Fig. 1 for some La isotopes can be understood as due to rotation-particle decoupling of the observed states. The present calculations indicate that this should be a widespread effect for the high-j unique-parity orbitals ($h_{11/2}$, $i_{13/2}$, etc.) occurring over a wide range of prolate deformations at the beginning of major shells (nearly empty orbitals) and oblate deformations at the end of major shells (nearly full orbitals). It may also occur for lower-j orbitals, but only over more limited regions. It should be kept in mind that we are considering only one aspect of nuclear motion, and others not considered here will almost certainly obscure these effects for β values smaller than some (presently unknown) limit. Nevertheless, if this is the correct explanation for the La spectra in Fig. 1, it indicates that the particle sees the core states essentially as rotational states. If this kind of odd-A spectrum occurs rather widely (for which there is at present some indication), it could provide evidence that rotation is a basic feature of the core states over wider regions of the periodic table than generally thought.

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