

table in the $1f_{7/2}$ shell. The additional masses required may be measured using the $(p, {}^6\text{He})$ reaction, and current plans are to use the techniques we have developed for the $({}^3\text{He}, {}^6\text{He})$ measurements in investigating the $(p, {}^6\text{He})$ reaction.

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Three-Nucleon Bound State from Faddeev Equations with a Realistic Potential

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A method for solving Faddeev equations in configuration space is used to study the state of three nucleons bound in a Reid potential. Including D states for the spectator particle gives a binding energy of 6.6 MeV, probabilities $P_{S'}=1.9\%$, $P_D=7.9\%$, and a charge radius $r_c({}^3\text{He})=1.97$ fm; the dip in the charge form factor is found at $q^2=14.5$ fm⁻².

Exact treatment of the three-nucleon bound state can provide a significant test for realistic two-nucleon interactions.

We here give an exact solution of the Faddeev equations for three nucleons bound via the Reid soft-core potential¹ acting on the states 3S_1 , 3D_1 , 3D_2 , 1S_0 , and 1D_2 . We consider the six components $[\psi_{l\lambda L}^\alpha(x, y)e_{S^\alpha}]_{1/2}$ of the Faddeev amplitude listed in Table I. Distances and orbital momenta are denoted by x and l for the interacting pair, and y and λ for the spectator particle; l and λ are coupled to total momentum L . The spin isospin state e_{S^α} is characterized by total spin S and by α which stands for A , $-$, or $+$ according to whether e_{S^α} has complete antisymmetry, mixed symmetry and antisymmetry, or symmetry, respectively, under the exchange of two interacting par-

TABLE I. Independent components of the Faddeev amplitude included in the complete computation.

Component	l	λ	L	S	α
1	0	0	0	1/2	A
2	2	2	0	1/2	A
3	0	0	0	1/2	$-$
4	2	2	0	1/2	$-$
5	2	0	2	3/2	$-$
6	0	2	2	3/2	$-$

ticles. Then, writing the Faddeev equations in configuration space² turns them into a set of six partial differential equations³ for the six unknown functions of Table I. Typically, the first equation, which involves the spatially symmetric component, has the following form²:

$$\left[\frac{\hbar^2}{m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + E - V(x) \right] \psi_{000}^A(x, y) = V(x) \int_{-1}^1 \frac{xy}{x'y'} \psi_{000}^A(x', y') du + \Phi(x, y), \quad (1)$$

where

$$x' = \frac{1}{2}(x^2 - 2\sqrt{3}xyu + 3y^2)^{1/2}, \quad y' = \frac{1}{2}(y^2 + 2\sqrt{3}xyu + 3x^2)^{1/2}.$$

Here the potential $V(x)$ is the average interaction in 3S_1 and 1S_0 states; the function $\Phi(x, y)$ is a short-

hand notation for contributions from the five other components. The integrodifferential set may also be written

$$H\psi(x, y) = E\psi(x, y), \quad (2)$$

where ψ is the vector of the six $\psi_{i\lambda L}^\alpha$ considered and H is a linear operator involving second-order partial derivatives and one-dimensional integrals.

We solve the eigenvalue problem (2) by the so-called inverse iteration. Starting from an initial guess $\psi^0(x, y)$ and expanding it in the eigenfunctions of H , one can see that the iteration

$$(H - p)\psi^{k+1}(x, y) = \psi^k(x, y) \quad (3)$$

makes $\psi^k(x, y)$ converge towards the eigenfunction $\psi(x, y)$ corresponding to the eigenvalue E closest to the number p . Equation (3) can be turned into finite-difference equations—for a set of N values of $\psi^{k+1}(x, y)$ on a grid—which we then solve by a relaxation method. Such a method does not involve explicitly the complete matrix of the linear set.

We have used a set of 1927 discrete points in the (x, y) plane for each of the six components $\psi_{i\lambda L}^\alpha(x, y)$ and imposed those to be zero on the boundaries $x=0$, $y=0$, and $(x^2 + y^2)^{1/2} = 25$ fm, respectively. Our results were stable with respect to variations in energy guess p (we took -30 MeV), discretization mode, and cutoff radius. More details about the method will be given in Ref. 3, but it is important to point out the usefulness of the relaxation method which does not impose any practical limit on the number of points or components considered.

We have done a first computation with only components 1, 2, and 3 of Table I. It gave binding energy $E_B = -6.39$ MeV, ${}^3\text{He}$ charge radius $r_c({}^3\text{He}) = 2.00$ fm, and the minimum of ${}^3\text{He}$ charge form factor at $q^2 = 15.2$ fm $^{-2}$ (Fig. 1). The energy we obtained is in agreement with the calculation of Fischbach *et al.*⁴ However, our values for the probabilities were $P_{S'} = 2.0\%$ and $P_D = 7.8\%$, against 1.8 and 5.8%, respectively.

Adding all the $\lambda = 2$ components listed in Table I to our computation had little influence on the energy and probabilities which became $E_B = -6.64$ MeV, $P_{S'} = 1.9\%$ and $P_D = 7.9\%$. This behavior, different from what Harper, Kim, and Tubis⁵ have found, may be due to the contributions of our $\lambda = 2$ components in all symmetry states, while the $\lambda = 2$ components considered by these authors contribute only to P_D . On the other hand, some computations we have done for the spatially symmetric components $\psi_{000}^A(x, y)$ and $\psi_{220}^A(x, y)$

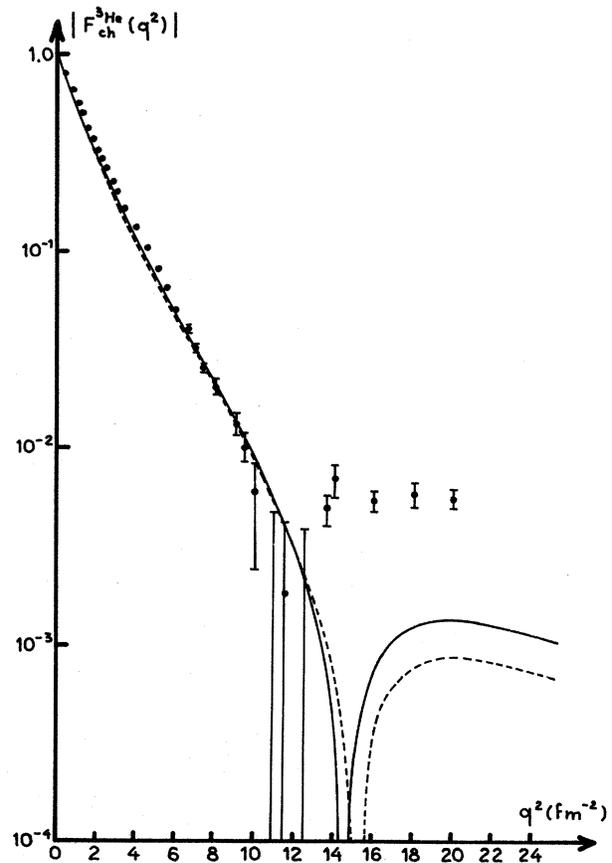


FIG. 1. Charge form factor of ${}^3\text{He}$ without (dashed curve) and with (solid curve) $\lambda = 2$ components. Experimental data are taken from McCarthy *et al.* (Ref. 7). We used the analytic form of Janssens *et al.* (Ref. 8) for the nucleon charge form factors.

with repulsive-core potentials may indicate that the range shortness of the repulsive part in the Reid soft-core potential is responsible for the weakness of the energy variation when $\lambda = 2$ components are added. However, we obtained charge radii $r_c({}^3\text{He}) = 1.97$ fm and $r_c({}^3\text{H}) = 1.72$ fm and the minimum of the ${}^3\text{He}$ charge form factor at $q^2 = 14.5$ fm $^{-2}$ (Fig. 1) which manifests the importance of including $\lambda > 0$ components for some crucial observables.⁶ We found a second maximum in the form factor at $q^2 = 20$ fm $^{-2}$, too small by a factor of 3 (instead of a factor of 8 as in Ref. 5).

We give in Table II the probabilities of the total-wave-function components which were used to compute the observables. One concludes from this table that the component $\psi_{220}^- e_{1/2}^-$ could have been discarded from the computation, but that $\psi_{220}^A e_{1/2}^A$ and $\psi_{022}^- e_{3/2}^-$ contribute noticeably to the total wave function. The odd l and λ components turn out to be important, and we are now in-

TABLE II. Angular-momentum analysis of the total wave function of the three-nucleon bound state.

l	λ	L	S	α	Probability (%)
0	0	0	1/2	A	88.1
2	2	0	1/2	A	1.6
0	0	0	1/2	-	0.9
2	2	0	1/2	-	0.1
1	1	0	1/2	+	0.9
2	0	2	3/2	-	2.7
0	2	2	3/2	-	0.8
1	1	2	3/2	+	1.6
3	1	2	3/2	+	0.4
1	3	2	3/2	+	1.1

cluding some of them directly in the Faddeev amplitude (then the two-nucleon interaction in odd states occurs); in fact, one can wonder if neglecting components of the Faddeev amplitude which appear in the total wave function is not somewhat inconsistent.

The use of configuration space makes the solution of Faddeev equations for bound states straightforward. In this case the potential occurs directly instead of the physically equivalent two-body t

matrix. Moreover, the method we described takes advantage of the appearance of the energy as an eigenvalue, and does not set a limit on the manageable number of components of the Faddeev amplitude. Our results for the properties of the three-nucleon bound state are in general agreement with those obtained from another exact (but fundamentally different) method; they confirm the relevance of some higher orbital momentum states in predicting the change form factor.

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Decoupled Yrast States in Odd-Mass Nuclei*

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We have studied a system consisting of a particle coupled to a rotating core. Although a complete decoupling of particle and core occurs only at very small deformations (coupling strengths), a "favored" high-spin band of levels tends to be decoupled over a much larger region of deformation. This can be understood as a simple Coriolis effect, and seems to be borne out remarkably well in some odd- A La isotopes.

In each of a series of five odd- A La isotopes, a band of levels based on a state with $I = \frac{1}{2}$ has been found^{1,2} which has spin values increasing monotonically from the base level by $2\hbar$ per state, and energy spacings very similar to the even-even Ba isotope with one less proton. The correspondence in energy is rather remarkable and is shown in Fig. 1. The experimental data consist

of in-beam γ -ray studies¹ following reactions $\text{Sn}(^{14}\text{N}, 3n)\text{La}$, and proton-transfer-reaction studies² using the reactions $\text{Ba}(\alpha, t)\text{La}$. The first work revealed the cascades of stretched- $E2$ γ rays, and the second indicated that these cascades were systematically based on $\frac{1}{2}$ states. This band might be explained by a particle-core weak-coupling model,³ but this model seems im-