which, although not the best cooling, is the best recorded. The time taken to reduce the field to zero was about 2 h. As is clear from Fig. 1 at the lower fields, the hexagonal nuclei act as a considerable thermal load on the cooling and thus H/T is not constant. The solid line in Fig. 3 represents the ideal isoentropic cooling curve calculated from the heat-capacity data. The experimental points follow this line reasonably well except at the lowest temperatures where flux jumps in the magnet, and too fast a demagnetization rate caused considerable heating. The inset shows the entropy of the system for 0 and 18.6 kOe. With starting conditions of 65 kOe and 20 mK it should be possible to reach 1 mK. After such a demagnetization the warmup times will be extremely long as the heat capacity at 1 mK is ~2 J/mole K, and by virtue of the ordered hexagonal sites does not fall below 1 J/mole K for more than two decades in temperature.

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Side-Jump Mechanism for Ferromagnetic Hall Effect*

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We show the equivalence of the side-jump model of the ferromagnetic Hall effect recently proposed by Berger and the anomalous velocity contribution obtained by Luttinger and others. Specifically, starting from the cited anomalous velocity expression, we give a derivation (to the lowest Born order) of the (vector) side jump which an electron undergoes during a collision with an impurity for a system of ferromagnetically polarized free electrons, each interacting with the impurity centers, together with the concomitant spin-orbit interaction. The side jump obtained is independent of the strength and form of the impurity potential and agrees with the result of Berger.

There have been two main approaches to the anomalous Hall effect of ferromagnetic metals and alloys, namely, an itinerant electron model of Smit[1] and Luttinger,[2] and a localized electron model of Kondo[3] and Maranzana.[3] In this paper we confine ourselves to the former model; we assume that the conduction electrons are ferromagnetically polarized. In particular, we will consider the scattering by impurity centers only.

There are two important contributions to the anomalous Hall effect; one is the skew scattering[1,2] contribution which can be explained according to the Boltzmann equation by extending the collision kernel to higher Born orders. The other is the nonclassical anomalous velocity[2,4] contribution which has no classical interpretation in terms of the Boltzmann equation. In either case, the transverse Hall current is caused by the spin-orbit interaction. The anomalous velocity contribution was first obtained by Luttinger[2] (his Eq. 3.26) using rigorous quantum transport theory.[5] However, there has been no simple intuitive picture of the underlying physical mechanism for this contribution. Recently, Berger[6] proposed a

side-jump mechanism for this, whereby an electron undergoes a small side jump due (according to him) to the anisotropic time delay caused by the spin-orbit interaction during each collision. As a demonstration, he calculated the side jump of an electron for a system of ferromagnetically polarized free electrons, each interacting with impurity centers, and spin-orbit interactions associated with the gradients of the potential of said centers. He assumed a Gaussian wave packet for the incoming electron and a three-dimensional square-well potential for the impurity potential. The purpose of this Letter is to give a rigorous theoretical justification to this significant step taken by Berger, by showing the equivalence between the anomalous velocity and the side-jump contribution for the case explicitly treated by Berger.

The appropriate form of the anomalous velocity (β component) for this case is given by[7]

$$\omega_k^{\beta} = 2\pi N\hbar^{-1}\textstyle\sum_{k'}\delta(\epsilon_k-\epsilon_{k'})\,\mathrm{Im}[H_{kk'}D_{\beta}H_{k'k}], \qquad D_{\beta}\equiv\partial/\partial k_{\beta}+\partial/\partial k_{\beta}', \tag{1}$$

where N and ϵ_k are, respectively, the total number of impurities and the energy of a free electron with a momentum $\hbar\vec{k}$, and

$$H' = V(\vec{r}) + (\hbar/4m^2c^2)\vec{\sigma}\cdot\nabla V(\vec{r})\times\vec{p}. \tag{2}$$

The first term represents the potential of a single impurity and the second term the spin-orbit coupling; $\vec{\sigma}$ is the Pauli matrix.

Physically, the anomalous velocity $\vec{\omega}_k$ is the expectation value of the spin-dependent part of the velocity operator which results from the *noncommutativity* of the coordinate operator with the spin-orbit interaction. In order to obtain a nonvanishing expectation value of the commutator, it is necessary to take into account the perturbation of the basic plane-wave states by the scattering potential. This perturbation is conveniently expressed by the introduction of "steady-flow" states $|k\rangle^+$ defined by the Lippmann-Schwinger equation,

$$|k\rangle^+ = |k\rangle + (\epsilon_k - H_0 + i\hbar\alpha)^{-1}H'|k\rangle^+, \quad \alpha = 0^+, \tag{3}$$

where H_0 is a free-electron Hamiltonian and H' for simplicity, is a single-impurity scattering potential given by (2). The above-mentioned expectation value is then given (to the lowest order in H') by

$$\left\langle k\left|\frac{[\vec{r},H']}{i\hbar}\right|k\right\rangle^+ = -\frac{1}{\hbar}\sum_{k'}\left\{\mathrm{P}\frac{1}{\epsilon_k-\epsilon_{k'}}[\langle k'|H'|k\rangle\vec{D}\langle k|H'|k'\rangle + \mathrm{c.c.}] + i\pi\delta(\epsilon_k-\epsilon_{k'})[\langle k|H'|k'\rangle\vec{D}\langle k'|H'|k\rangle - \mathrm{c.c.}]\right\}, \tag{4}$$

where P stands for the principal part. The first terms in the square brackets turn out to be purely imaginary (to the first order in spin-orbit interaction; higher-order terms will be neglected hereafter); therefore the first square brackets can be dropped, and one is left with

$$\left\langle k\left|\frac{[\vec{r},H']}{i\hbar}\right|k\right\rangle^+ = \frac{2\pi}{\hbar}\sum_{k'}\delta(\epsilon_k-\epsilon_{k'})\,\mathrm{Im}[H_{kk'}\vec{D}H_{k'k}] \tag{5}$$

which is identical to (1) for the case of N impurity centers. It is to be emphasized that only the spin-orbit part of H' which does not commute with the coordinate operator contributes to (5).

One can immediately evaluate (1) using $D_{\beta}V_{k'k}=0$:

$$\omega_k^{\beta} = \frac{2\pi N}{\hbar}\sum_{k'}\delta(\epsilon_k-\epsilon_{k'})|V_{k'k}|^2\frac{\hbar^2}{4m^2c^2}\vec{\sigma}\cdot D_{\beta}\vec{k}'\times\vec{k}. \tag{6}$$

Assuming, for convenience, $\vec{\sigma}$ to be in the z direction, one obtains

$$\omega_k^{x} = \frac{\hbar^2\sigma_z}{4m^2c^2}\frac{2\pi N}{\hbar}\sum_{k'}\delta(\epsilon_k-\epsilon_{k'})|V_{k'k}|^2(k_y-k_y'), \quad \omega_k^{y} = -\frac{\hbar^2\sigma_z}{4m^2c^2}\frac{2\pi N}{\hbar}\sum_{k'}\delta(\epsilon_k-\epsilon_{k'})|V_{k'k}|^2(k_x-k_x'). \tag{7}$$

For the special case under consideration, where $|V_{kk'}|^2\delta(\epsilon_k-\epsilon_{k'})$ depends only on $\theta_{kk'}$, the angle between $\vec{k}$ and $\vec{k}'$ (as well as the magnitude k and k') well-known spherical trigonometric analysis permits us to rewrite (7) as

$$\omega_k^{x} = \frac{\hbar^2\sigma_z}{4m^2c^2}\frac{k_y}{\tau_{\mathrm{tr}}}, \quad \omega_k^{y} = -\frac{\hbar^2\sigma_z}{4m^2c^2}\frac{k_x}{\tau_{\mathrm{tr}}}, \tag{8}$$

where the "transport" relaxation time τ_{tr} is given by

$$\frac{1}{\tau_{tr}}=\frac{2\pi N}{\hbar}\sum_{k'}\delta(\epsilon_k-\epsilon_{k'})|V_{k'k}|^2(1-\cos\theta_{k'k}). \quad (9)$$

One can put (8) in vector form:

$$\vec{\omega}_k=\Delta\vec{r}_k/\tau_{tr}, \quad \Delta\vec{r}_k\equiv\tfrac{1}{4}(\hbar/mc)^2\vec{k}\times\vec{\sigma}. \quad (10)$$

Noting that $1/\tau_{tr}$ is a transport collision rate, one can interpret $\Delta\vec{r}_k$ as a side jump of the electron (occurring during each collision) in the direction perpendicular to both the spin and momentum vectors. The side jump $\Delta\vec{r}_k$ of (10) agrees with that given by Berger[6a] assuming Born and short-range approximations, except that our result is larger by a factor of $\frac{3}{2}$.[8] It is to be noted that the final form of the side jump $\Delta\vec{r}_k$ does not depend on either the magnitude or the form of the impurity potential. This is in agreement with Berger's result which is independent of the depth and radius of the scattering potential well.

The magnitude of the side jump can be estimated by assuming $k\simeq 10^8$ cm^{-1} at the Fermi level:

$$|\Delta\vec{r}_k|\simeq 4\times10^{-14}\text{ cm} \quad (11)$$

which is too small to explain the data.[6a] However, as pointed out by Smit[1] and discussed by Berger,[6a] the small effect of the spin-orbit interaction ($H_v^{s\cdot o}$) associated with the impurity potential should be replaced by the spin-orbit interaction ($H_U^{s\cdot o}$) associated with the periodic lattice potential. The effect of $H_U^{s\cdot o}$ is to (electrically) polarize the electrons.[9] The combined effect of the dipole moment and impurity potential is to produce an effective spin-orbit interaction,[10] which is enhanced by a factor of 10^4 compared to $H_v^{s\cdot o}$, thereby enhancing the magnitude of the side jump by the same factor. A more detailed treatment of this enhancement effect will be postponed for forthcoming work.[11]

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Coherent Scattering in a Random-Network Model for Amorphous Solids

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Coherent scattering regions of approximately 10–25 Å in size have been observed in a variety of amorphous solids. It has been suggested that these observations provide evidence for the microcrystalline model. In this note we show that the observations of coherently scattering regions are not incompatible with a random-network model.

High-resolution dark-field electron microscopy of amorphous Si, Ge, Ge-Te alloys, and SiO_2 has revealed the presence of coherently scattering regions of approximately 10–25 Å in diameter.[1-5] In the case of amorphous silicon and germanium the coherently scattering regions (CSR's) cannot be related to the diamond cubic crystallites as the calculated interference functions do not agree with the experimental values.[6-8] It has been suggested, however, that the CSR correspond to the wurtzite-structure crystallites, and a better match with the calculated and experimental values of the interference function can then be obtained.[5] An alternative approach which generates radial distribution functions that match the experimentally determined ones is to consider some form of a random-network model.[9-15] In this, no structural order of the type present in the micro-