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[8]From now on we will replace κ_0 by the renormalized reciprocal correlation length κ in all G_0 Green's functions.

[9]The factor 1.07 in Eq. (21) is $(1-q^{-1}\tan^{-1}q)/2q^2$ evaluated at $q^2=-\frac{1}{9}$. It comes from evaluating the renormalization parameter at $k^2=-\kappa^2$ and would be 1 if Z_ψ^{-1} were evaluated at $k^2=0$.

[10]The power law has, of course, only been generated to leading logarithmic order by the screening approximation. However, the higher-order terms can be obtained by continuing the n^{-1} expansion. These also generate further corrections to η in powers of n^{-1}.

[11]There is a small difference, less than 10%, due to the fact that Z_ψ^{-1} is evaluated for $k^2=-\kappa^2$ rather than $k^2=0$.

Transition from Bulklike Behavior to Josephson-Junction–like Behavior in Superconducting Microbridges

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The behavior of superconducting Sn and Sn-In microbridges has been studied as a function of physical parameters of the evaporated films. For a proper choice of mean free path and bridge size, a regime of ideal Josephson-junction-like behavior appears just below T_c. The data are in good agreement with recent calculations by Baratoff, Blackburn, and Schwartz.

The behavior of superconducting "weak links" has been the subject of several recent experimental[1,2] and theoretical[3-5] studies. Although both dc[1] and ac[2] experiments indicate oscillatory behavior of the phase near the critical temperature T_c, the problem is complicated by the fact that weak links exhibit bulk superconducting properties under some experimental conditions and Josephson-junction–like properties under others. By measuring the temperature dependence of the critical supercurrent of Sn and Sn-In alloy microbridges, we have determined that there is a region of temperature below T_c within which the bridges show near-ideal Josephson-junction–like behavior; this is further supported by simultaneous monitoring of the quality of the ac Josephson effect. The width of this junctionlike regime can be changed by varying the coherence length, either by altering the mean free path of the film via thickness and impurity content, or by changing the temperature. In the latter case, the bridges are shown to make a smooth transition to bulk superconductorlike behavior as the temperature is decreased.

Figures 1 and 2 show the critical current versus temperature data for a number of ~0.5-μm ×0.5-μm superconducting microbridges with varying mean free paths l. For the pure Sn films of Fig. 1, l was decreased by decreasing the film thickness, while for Fig. 2, l was shortened by alloying In into the Sn. The transition temperature T_c was determined by linearly interpolating the critical current to zero from about 20 mK below T_c, which was the highest temperature where the critical current I_c could be determined to ±20%. This was typically within 5 mK of the temperature at which the dc supercurrent vanished to within the 1 μA resolution of our apparatus. Three theoretical curves are shown in Fig. 1 for comparison with our data. The upper curve is the temperature dependence of the critical current density for an ideal Josephson junction between two identical superconductors,[6,7]

$$J_c(T)=\tfrac{1}{2}\pi R_n^{-1}\Delta(T)\tanh[\Delta(T)/2k_BT], \qquad (1)$$

where R_n is the normal-state resistance and $\Delta(T)$ is the superconducting energy gap. At a reduced temperature $(t\equiv T/T_c)>0.95$, J_c is linear in T_c-T for this case. The middle (solid) curve is the critical current of a bulk superconductor, neglecting the current dependence of the gap parameter. Using the temperature dependence of the two-fluid model, the critical current density may be written as

$$J_c(T)=\frac{cH_c(0)}{4\pi\lambda_L(0)}(1-t^2)^{3/2}(1+t^2)^{1/2}, \qquad (2)$$

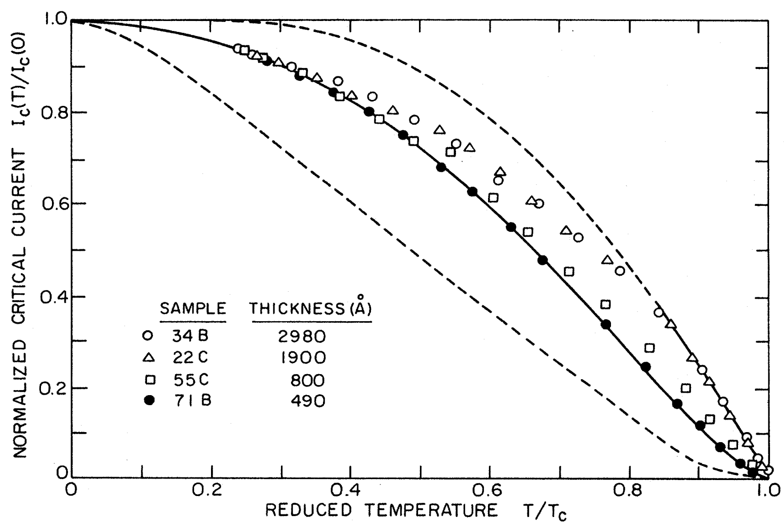

FIG. 1. Normalized critical current versus reduced temperature for pure Sn microbridges as a function of film thickness. The mean free path is varied by size effects. The upper broken curve is the theoretical behavior of an ideal Josephson junction, the middle (solid) curve that of a bulk superconductor, and the lower broken curve that of a bulk superconductor with the gap parameter depressed by the current.

where $H_c(0)$ and $\lambda_L(0)$ are the critical field and the London penetration depth at 0 K, and $J_c(0) = cH_c(0)/4\pi\lambda_L(0)$ is the maximum critical supercurrent density. The lower (broken) theoretical curve is calculated using a gap-dependent free-energy model based on microscopic theory[8,9] with the depression of the gap parameter by the current taken into account.[10] The transition from bulklike to junctionlike behavior as the temperature is increased is clearly shown for the clean samples, as is the repression of this transition as the mean free path is decreased.

Our samples consisted of thin Sn or Sn-In-alloy films evaporated (in vacuo) onto a glass substrate cooled to liquid-nitrogen temperatures; film thicknesses varied from 500 to 3000 Å. A bridge was scribed on this film with a fine-point diamond tool mounted on a specially designed micromanipulator.[11] Bridge dimensions were measured by a scanning electron microscope at a magnification between 3000 and 10 000; film thicknesses were measured with a Fizeau interferometer. For the Sn-In–alloy samples, the residual resistivity ratio was measured and the

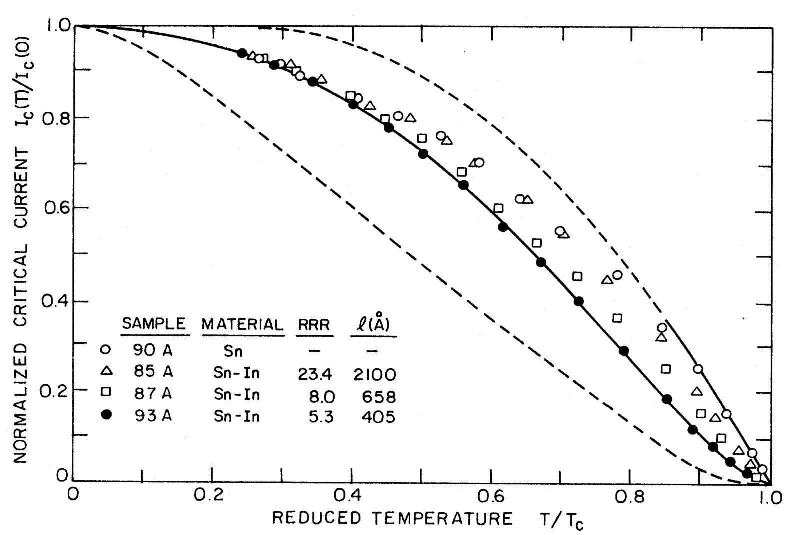

FIG. 2. Normalized critical current versus reduced temperature for Sn-In–alloy microbridges. The mean free path is varied by impurities. The theoretical curves are the same as Fig. 1.

mean free path estimated by using Matthiessen's rule. The maximum concentration of In in our Sn-In films was estimated to be less than 3 at.%.[12] The ambient magnetic field at the sample was reduced to less than 3 mG with a shield made of high-permeability material. Temperatures were measured with a calibrated germanium resistor, which was estimated to be absolutely accurate within 10 mK, and were regulated to within a few mK with an ac Wheatstone bridge circuit.[13] The sample was installed inside a cylindrical, TE_{011}-mode resonant cavity at the position where the microwave electric field is maximum. The microwave electric field E_{rf} was applied parallel to the bridge direction. Relative microwave power was read from the setting of a precision attenuator. The heights of the rf-induced current steps were measured from x-y recordings of the I-V curves; these curves were measured by a conventional four-terminal method using a constant-current biasing circuit.

Baratoff, Blackburn, and Schwartz[3] (BBS) have recently solved the Ginzburg-Landau equations for weak links using a simplified one-dimensional model. The link is considered to occupy region B which differs from the electrode (bulk) region A only in having a shorter mean free path $l_B < l_A$, and therefore a smaller coherence length $\xi_B < \xi_A$. Their solutions are completely determined by the normalized bridge length $d = L/2\xi_A$, where L is the physical length of the bridge, and a "weakness" parameter $\gamma \equiv \chi_B/\chi_A$, where χ is Gor'kov's[14] universal function of the impurity parameter l/ξ_0. The weak link B is treated as being identical to A except in having a lower critical current density, and thus γ represents the decrease in effective cross-sectional area for supercurrent flow in the bridge region. BBS predict that a sinusoidal current-phase relation will obtain whenever the bridge is "weak," but shorter than the bulk coherence length ξ_A; these conditions will be met when $\gamma < d < \gamma^{1/2}$.[15] By varying the mean free path with impurity or film thickness, we could alter the BBS parameters γ and d readily; note that the effective bridge length increases for dirty samples and decreases for clean ones. The Gor'kov impurity function can be approximated to within 20%[9] by

$$\chi \approx (1 + \xi_0/l)^{-1}, \tag{3}$$

where ξ_0 is the BCS coherence length (2300 Å for Sn). We may then approximate γ by

$$\gamma = \frac{1 + \xi_0/l_A}{1 + \xi_0/l_B}. \tag{4}$$

The size effect reduction of l_B must be taken into account for pure samples, as l_A is comparable to the width of the bridge. For the clean samples shown in Figs. 1 and 2 (34B, 22C, 90A), l_A was about 0.3 μm. Since these bridges are about 0.6 μm long by 0.5 μm wide, $\gamma \sim 0.6$ and $d \sim 1$ at low temperature. As $t \to 1$, ξ_A increases, and the BBS criterion $\gamma < d < \gamma^{1/2}$ is met; therefore there is a region below T_c where the bridges behave like junctions. For the very dirty samples (55C, 71B, 87A, 93A) the bridge width is $\gg l_A$, and $\gamma \approx 1$ at all temperatures. Thus, the geometrical constriction is virtually nonexistent, and the BBS criterion cannot be met even in the limit $t \to 1$ where the temperature-dependent coherence length is much longer than the bridge length. In other words, for small, clean bridges such that $l \gtrsim$ the bridge width, there will be a regime for $t < 1$ in which the bridges will behave like Josephson junctions, having a sinusoidal current-phase relationship and a critical current which decreases linearly for $0.95 < t \leq 1$.

In addition to the above mentioned features, e.g., a linear dependence of I_c upon $1 - t$ for clean samples, a bulklike temperature dependence for dirty samples, and a continuous transition from bulk dependence to linear dependence as t is increased, which can be varied by varying l, there is an independent set of measurements confirming the junctionlike behavior in the linear region. An 8.8-GHz rf signal was applied to our samples, and the position and height of the rf-induced current steps produced was measured. A near-ideal ac Josephson effect with rf-induced steps only at multiples of $\hbar\omega/2e$ was observed in the region where the linear temperature dependence occurs. The rf power dependence of the step heights is identical to that observed by Gregers-Hansen *et al.*[2] on very small bridges of similar geometry; this rf dependence is adequately described by a model based on a sinusoidal current-phase relationship, $J_c(\Delta\varphi) \propto \sin\Delta\varphi$. Outside the linear regime, rf steps appear at harmonics and subharmonics of the rf frequency; these steps are weak and have a very complex rf power dependence. Again this correlates well with the known behavior of very long bridges, i.e., bulklike behavior.

Our observations appear to be well explained by variation of the gap parameter in the bridge region as calculated by BBS. The linear variation of the critical current for a short weak bridge, $\gamma < d < \gamma^{1/2}$ arises from the boundary condition $J(\Delta\varphi) \propto \Delta^2 d\varphi/dx$ at the boundary. Since the bridge is shorter than ξ_B and $\Delta(x)$ must have zero slope at the center of the bridge, the depression of Δ with current is expected to be very small in this case. Thus, Δ at the boundary cannot differ greatly from the bulk equilibrium value which is proportional to $(1-t)^{1/2}$, even at the critical current I_c. For a bridge of fixed size, a constant phase change occurs at I_c, and $d\varphi/dx$ is nearly constant at the boundary. Therefore, $I_c \propto 1-t$ in this regime. Experimentally, our values of I_c were always about 40% smaller than the prediction of Eq. (1). However, there was some difficulty in determining the normal-state resistance R_n unambiguously, as the I-V characteristic of a bridge usually showed several regions at different current levels which might be interpreted as the asymptotic slope to give R_n. We used the slope of such a region at the lowest current bias to determine R_n, and this may introduce a systematic error into our calculations. We have also calculated the correction to be made in approximating our bridges by a one-dimensional model. This correction is due to nonuniform current distribution in the bridge. For $t > 0.9$, where meaningful comparisons with theory can be made, the correction is quite small,[16] $< 10\%$ for the thickest films, and can be neglected to within the accuracy of our comparisons with the theory.

In the bulklike behavior regime, our data closely fit the two-fluid model calculations. The lower broken curve of Fig. 1, derived from microscopic theory, assumes a uniform transition to the normal state over the entire bridge, and a consequent reduction of Δ over its whole volume. However, in most experimental situations the transition is nucleated by a local hot spot, negating the assumption of extended nonlocality. In addition, our bridges were only a few times ξ_A, and we would not expect much depression of Δ even at I_c. Earlier experiments on very long bridges[17] confirm these observations.

We have thus shown that for samples lying on the upper curve in Fig. 1 (which satisfy the condition $\gamma < d < \gamma^{1/2}$ required by BBS) a sinusoidal current-phase relationship $J(\Delta\varphi) \propto \sin(\Delta\varphi)$ is obtained. The theoretically predicted enhancement of I_c above the bulk value near T_c has also been observed; such enhancement for short, weak bridges is thought to be a measure of the Josephson-junction–like effect. This enhancement should be large to minimize bulklike effects which tend to obscure phase coherence across the bridge. For this reason ideal ac Josephson effects can be seen only near T_c in microbridges or other weak links of this type.

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