Turbulent-like Spectrum of Ion Waves in a Beam-Plasma System*

Y. Amagishi
Department of Physics, Shizuoka University, Shizuoka, Japan

(Received 24 July 1972)

A quasi-one-dimensional spectrum of ion waves excited by a slow-electron beam is investigated experimentally. The power spectrum in a region of spatial wave decay is found to fall off with increasing frequency as $f^{-2.3\pm0.2}$. The result is compared with some nonlinear theories.

The spectrum of electron waves generated by a beam-plasma interaction was studied experimentally by Apel,[1] and consequently the power of peaks at harmonics of the fundamental frequency was found to be proportional to $f^{-5.5\pm0.5}$ in a spatially decaying region. This result is in good agreement with the power law with respect to the plasma turbulence,[2,3] if all waves have the same phase velocity. In this Letter, it is reported that the power spectrum of ion waves in a spatially decaying region is not proportional to f^{-5}, but to $f^{-2.3\pm0.2}$.

The experiment is carried out in a cylindrical Pyrex tube 7.6 cm in diameter and 45 cm long. The Ar plasma is produced by a dc discharge in the plasma source and is diffused into the interaction region (16 cm long) along a magnetic field of 120 G. The ion gyroradius is greater than the plasma diameter. The plasma density is in the range 10^8–10^9 cm^{-3}, and the electron temperature is about 3 eV. The background pressure of Ar gas is 0.4–1.6 mTorr. Instabilities of ion waves are generated by a slow-electron beam of diameter 5 mm at the gun. The feature of the gun, which has been described in detail elsewhere,[4] is that it can produce an electron beam with a slow velocity of the same order as that of thermal electrons in a plasma, and the velocity can be controlled independently of the beam density in the range 0–20 eV. The signals of ion waves are received from a movable plane probe of a molybdenum plate (1 mm^2), which is faced

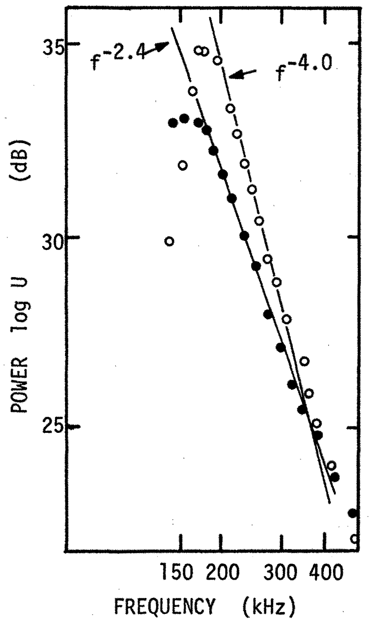

FIG. 1. Typical ion-wave spectra after careful calibration. Open circles, at $x=4$ cm from the gun; closed circles at $x=9$ cm. A maximum-peak shift toward low frequency with increasing x was observed, and in a decay process, a smaller slope occasionally appeared in a narrow part near the peak as exhibited in S. Watanabe and H. Tanaca [J. Phys. Soc. Jap. 26, 1331 (1969)] (not appearing in this figure).

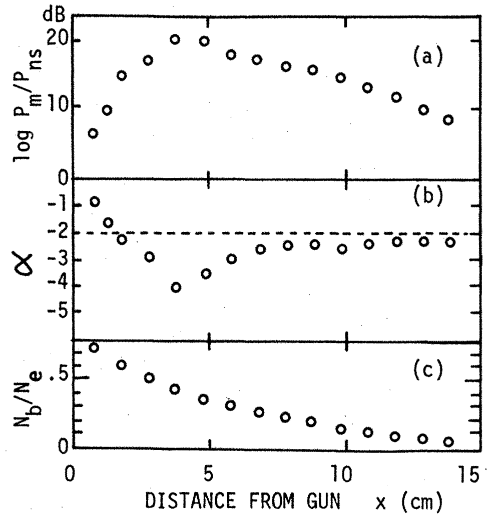

FIG. 2. (a) Maximum intensity P_m normalized by noise intensity P_{ns} plotted versus x. (b) Exponent α of the power spectrum U_f ($\propto f^\alpha$) in the high-frequency portion ($\alpha<0$). This graph indicates that α is close to -2.3 in the decay region. (c) Electron beam density N_b, normalized by the background electron density N_e versus x. The beam velocity is about 4 eV, which is nearly equal to the electron temperature of the plasma. Ar gas pressure, 8×10^{-4} Torr.

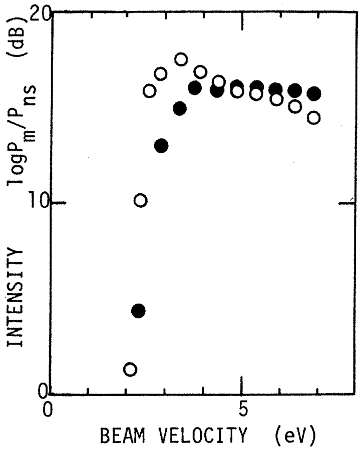

FIG. 3. Dependence of the maximum intensity on beam velocity. Open circles, $x=3$ cm (growing position); closed circles, $x=9$ cm (decaying position).

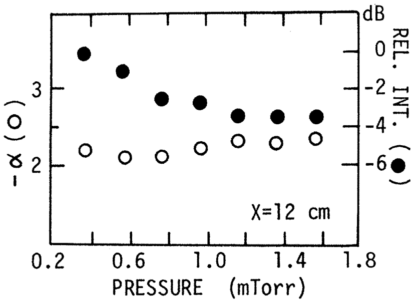

FIG. 4. Influence of neutral gas pressure on the turbulent state.

to the gun and coated on the rear side, and analyzed on a Stoddart NM25T radio interference and field intensity meter (covering frequency range, 150 kHz to 32 MHz). In this experiment, the probe bias is floating, because the potential applied to the probe results in the acceleration of the slow-electron beam. The influence of the floating probe immersed in the beam on the beam velocity or spectrum is negligible.

A previous experiment[4] with this gun has shown that at a small distance from the gun, i.e., in the spatial growth region, the unstable-stable boundary as a function of beam velocity and frequency is adequately described by a dispersion relation derived by applying the theory established by Jackson,[5] and the instability generated within the boundary is identified with the so-called ion-acoustic wave mode.

Typical spectra of ion waves, which are for the same modes as described above, are shown in Fig. 1 in the frequency range 150–500 kHz. Above 500 kHz up to the ion plasma frequency (1.1 MHz), ion waves are so strongly damped that their power is not observed distinctly. From the electrostatic dispersion relation of ion waves, the wave number of ion waves below 300 kHz is small compared with the Debye wave number ($k < k_D$). Maximum peaks appear in the vicinity of 150 kHz, below which the spectrum decreases with decreasing frequency. Here, the discussion of the power spectrum is focused on the region of small wave number but above the frequency of the maximum peak. In Fig. 2, the spatial variation of the spectrum along the axial distance is shown. As the distance x from the gun increases, spatial growth is first observed, followed by decay at $x \gtrsim 6$ cm. Note that in the decaying region the exponent α of the power U_f at large f ($U_f \propto f^{\alpha}$) is roughly -2.3 ± 0.2. Figure 3 shows the relation between the saturation level and beam velocities. In the growth region, the maximum intensity of the fluctuation first increases, followed by decay with increasing beam velocity. This tendency is evident from the dispersion relation for three-component plasma.[4] However, in the turbulentlike region, i.e., in the decaying region, the maximum intensity first increases rapidly with the beam velocity, but soon approaches a constant value independent of the variation of the beam velocity. Moreover, the value of the exponent α in the turbulentlike state remains in the range from -2 to -2.5 when the pressure is changed, keeping the electron density constant, while the maximum intensity is somewhat varied (Fig. 4). This suggests that the slope of the turbulent spectrum is not mainly dominated by electron- or ion-neutral collisions, but its intensity is affected.

Vedenov[2] and Kadmotsev[6] have given theoretical and reasonable arguments for the existence of the power spectrum of ion-acoustic waves in the strong turbulent state as f^{-1} and $f^{-1}\ln(f_{pi}/f)$, respectively. However, these evaluations, based on three-dimensional grounds, give too small a slope to be applied to our results. The spectrum obtained experimentally is rather quasi one-dimensional; or, in other words, the wave number is almost along the beam direction, as shown in Fig. 5. This suggests that the above evaluations are not appropriate for the explanation of the experiments. Another theory associated with the spectrum of ion waves is that of Nishikawa and Wu.[7] They predicted that once the fluctuation has grown to a certain level, the trapping of an elec-

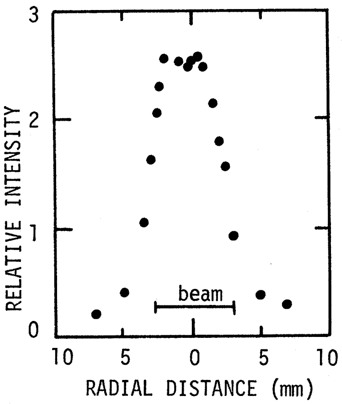

FIG. 5. Dependence of the maximum intensity on radial direction in the turbulentlike region ($x = 7$ cm).

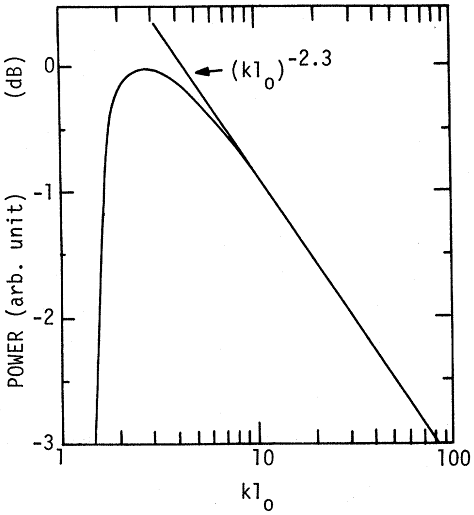

FIG. 6. Power spectrum of ion waves calculated from the theory of Nishikawa and Wu (Ref. 7).

tron in a potential of ion waves may set in and serve as a saturation mechanism, while Coulomb collisions of electrons are very efficient in suppressing the trapping. In other words, a statistical balance between the trapping and Coulomb collisional effects determines a saturation state. Therefore, the saturation level should be independent of the beam velocity and collisions with neutrals. The power spectrum, based on such considerations as the assumptions of the quasi-one-dimensional case and $k \ll k_{\rm D}$, obeys the following relation[7]:

$$U_f \propto \left| \int_0^\infty dl\, l^{4/3} \exp(-2l/l_0) \cos(kl) \right| ,$$

where the notation is the same as in Ref. 7. The right-hand side of the above equation is plotted in Fig. 6, which shows the power decaying with increasing frequency as $f^{-2.3}$. The experimental value of the exponent α and its independence of both the beam velocity and collisions with neutrals are consistent with this prediction. Moreover, considering that the trapping condition

$$\omega_b/\omega_s \simeq (M/m)^{1/2} (n\lambda_e^3)^{-1/4} > 1 ,$$

given in Eq. (10) of Ref. 7 (which is a necessary but not satisfactory condition for trapping) is easily satisfied for our experimental parameters, the results presented here seem to support the new prediction, including the trapping effect of electrons in an ion-wave potential trough. However, it is somewhat questionable whether the theory of Nishikawa and Wu is applicable to our experiments for the following reasons: (1) According to the theory, collisions with neutrals are not effective for the determination of saturation level; but in the experiment, as shown in Fig. 4, this collisional effect is not negligible even though the slope has not been affected. (2) The condition $k \ll k_{\rm D}$ is not necessarily satisfied in the vicinity of 300 kHz.

In conclusion, we have observed an ion-wave spectrum whose slope closely resembles the result of Nishikawa and Wu. However, further considerations, especially those including the effect of ion-neutral collisions, are needed to interpret the experimental results, because such collisions in a partially ionized plasma are expected to contribute to the determination of saturation level and the stabilization mechanism.

The author would like to thank Professor S. Kojima for his encouragement, Professor K. Nishikawa for his useful comments, and Mr. K. Murai for his technical assistance.

---

*Work performed at Tokyo University of Education, Tokyo, Japan.

[1] J. R. Apel, Phys. Rev. Lett. 19, 744 (1967).

[2] A. A. Vedenov, *Theory of Turbulent Plasma* (Iliffe Books, London, 1968), p. 93.

[3] F. F. Chen, Phys. Rev. Lett. 15, 381 (1965).

[4] Y. Amagishi, J. Phys. Soc. Jap. 29, 1592 (1970).

[5] E. A. Jackson, Phys. Fluids 3, 786 (1960).

[6] B. B. Kadomtŝev, *Plasma Turbulence* (Academic, New York, 1965), p. 71.

[7] K. Nishikawa and C. S. Wu, Phys. Rev. Lett. 23, 1020 (1969).