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## Differential and Direct Differential Elastic Scattering Cross Sections for Electrons and Potassium Atoms\*

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In a modulated crossed-beam experiment we have elastically scattered unpolarized electrons of 3.3 eV energy from spin-polarized K atoms. A measurement of the polarization of the scattered electron yields  $|f(\theta)|^2/\sigma(\theta)$ . Our measured values of  $|f|^2/\sigma(\theta)$  show a significant angular shift relative to the theoretical curve in forward-angle scattering ( $\theta = 20^{\circ}-40^{\circ}$ ). This shift is not apparent in measurements of the differential scattering cross section  $\sigma(\theta)$  over the same angular range, which, however, do show a significant angular shift in the range  $\theta = 50^{\circ}-120^{\circ}$ .

Elastic scattering of electrons from one-electron atoms, such as the alkalis and hydrogen, is customarily described using two scattering amplitudes, the direct amplitude  $f(\theta)$  and the exchange amplitude  $g(\theta)$ .<sup>1</sup> This involves neglecting spinorbit coupling, electron-nuclear spin and electron-electron coupling during the collision, as well as the influence of the core electrons on the scattered electron. The differential scattering cross section is written

$$\sigma(\theta) = \frac{1}{2} |f(\theta)|^2 + \frac{1}{2} |g(\theta)|^2 + \frac{1}{2} |f(\theta) - g(\theta)|^2.$$
(1)

A complete solution to the collision problem requires the knowledge of the magnitude of these two amplitudes as well as the phase difference between them; therefore three independent measurements are necessary. In this paper we report the first successful measurement of the square of the direct scattering amplitude for lowenergy electron collisions with such one-electron systems; we also report here an accurate relative differential cross section measurement at the same energy.

To determine the scattering cross section one measures the intensity of the scattered electrons; one can also specify the spin state of one or both of the incident particles and measure the polarization of either of the scattered particles. In particular, in the experiment described here, unpolarized electrons are scattered from potassium atoms whose polarization is  $P_A$ , and the spin polarization of the scattered electrons  $P_{e'}$  is measured. The relation between the scattering amplitudes and the measured electron polarization is given by

$$|f(\theta)|^2 / \sigma(\theta) = 1 - P_{e'} / P_A.$$
<sup>(2)</sup>

The scattering cross section  $\sigma(\theta)$  was measured in this experiment by summing over the scattered electron spin states.

A separate experimental program, which has been underway for some time at New York University,<sup>2</sup> yields measurements of the polarization of the scattered atoms,  $P_A'$ , in a similar collision; this leads to the exchange amplitude

$$|g(\theta)|^2 / \sigma(\theta) = 1 - P_A' / P_A.$$
(3)

The experiments reported here utilize a highvacuum crossed-beam apparatus; a schematic diagram is shown in Fig. 1. A mechanically modulated thermal potassium beam is polarized by passage through a hexapole magnet.<sup>3</sup> To make differential analysis of the low-energy scattered electrons possible, the magnetic field in the center of the scattering chamber is reduced with Helmholtz coils to less than 0.2  $\mu$ T. The necessary vertical axis of spin quantization is defined by a pair of small Helmholtz coils which adds a



FIG. 1. Schematic diagram of the apparatus.

1.0- $\mu$ T field to the center of the interaction region.

The K beam leaving the hexapole magnet has a calculated polarization of 78.5%. In the low-field region near the center of the scattering chamber, hyperfine coupling with the nuclear spin reduces the polarization by a factor of  $(2I + 1)^{-1}$ ; for natural K with  $I = \frac{3}{2}$  the beam polarization would be 19.6%. The molecular component in this polarized beam is estimated to be less than 0.2%. The alkali beam intensity is monitored with a hot-wire detector above the interaction region. The multistage electron gun<sup>4</sup> can be rotated in a plane perpendicular to the alkali beam over a range of scattering angles from  $-130^{\circ}$  to  $+130^{\circ}$ . The energy resolution of the gun is about 0.3 eV as determined by retarding-potential measurements.

After scattering from the alkali atoms, the electrons pass into a filter lens<sup>5</sup> which removes unwanted inelastically scattered electrons. The collecting system accepts electrons from a volume which is larger than the overlap region of electron and atomic beams; hence no geometrical corrections of the angular distribution data are necessary.

After the electrons leave the filter lens they are accelerated through a potential of 100 kV and spin analyzed by Mott scattering.<sup>6-8</sup> The physical observable in this experiment is the ratio of the number of electrons counted by the two detectors in the Mott scattering chamber. This ratio is labeled  $\chi$  (=  $N_1/N_2$ , where subscripts 1 and 2 refer to the left and right detectors); for a single measurement the electron polarization is

$$P_{e'} = \frac{1}{S} \left( \frac{\chi - 1}{\chi + 1} \right), \tag{4}$$

where S is the asymmetry function,<sup>7,8</sup> which is a function of energy, target thickness, and scattering angle.

To allow for compensation of the large background counting rate, the scaler gating is synchronized to the rotation of the alkali beam chopping wheel so that "signal-plus-background" and "background" counting rates are separately measured.

To compensate for any instrumental asymmetries in each electron polarization measurement, two measurements of  $\chi$  are made, one with the spin axis up, one with the spin axis down. This is accomplished by inverting the small magnetic field in the interaction region. The expression for  $\chi$  used in formula (4) is then replaced by  $\chi' = [\chi(t)/\chi(t)]^{1/2}$ . This procedure has been checked by replacing the gold foil in the Mott chamber with an aluminum foil. This reduces the asymmetry due to polarization but leaves those due to instrumental effects.<sup>7</sup> In all cases scattering electrons from the aluminum target yields a measured value of  $\chi = 1.00$  within statistical error.

The internal error in each measurement is dominated by statistical fluctuations. For a choppedbeam experiment of this type the standard (rms) deviation  $\Delta P$  of the measured electron polarization  $P_{e'}$  is<sup>9</sup>

$$\Delta P = \frac{1 - (SP_e')^2}{4|S|} \sum_{j=1,2} \sum_{i=1,2} \frac{(E_{ij} + B_{ij})^{1/2}}{E_{ij} - B_{ij}} .$$
 (5)

The subscripts *i* and *j* refer to the number of counts recorded by scaler *i* with magnetic quantization field in a direction *j* (1, up; 2, down).  $E_{ij}$  refers to measurements with the atomic beam on (signal plus background) and  $B_{ij}$  refers to measurements with the atomic beam off (background). In this experiment, for example, with 3.3-eV electrons at  $\theta = 40^\circ$ ,  $E_{ij} \simeq 12 \sec^{-1}$ ,  $B_{ij} \simeq 10 \sec^{-1}$ .

In Fig. 2, values of  $|f|^2/\sigma$  computed from our measurements of  $P_e'$  are plotted,<sup>10</sup> as well as the results of a theoretical computation<sup>11</sup> for 3 and 4 eV energy. The error bars represent 1 standard deviation error for the measurement of  $P_{e'}$ . The atomic beam polarization  $P_A$  used in this calculation was the theoretical value (~0.20). A precise check on this value has not been accomplished, and it should be regarded as an upper limit. Low-



FIG. 2. Measured values of  $|f|^2/\sigma(\theta)$  versus  $\theta$  for an electron energy of 3.3 eV. Continuous and dashed curves, theoretical values from the work of Karule and Peterkop (Ref. 11) for 3 and 4 eV energy, respectively. Error bars represent 1 standard deviation in the counting statistics of the measurement of  $P_e'$  [see Eq. (5)].

er values would tend to reduce  $|f(\theta)|^2/\sigma$ , with greatest effect at the largest angle studied. We are continuing our measurements at larger angles but these are difficult since the scattering cross section decreases very rapidly with increasing angle.

We have also measured the angular distribution of elastically scattered electrons in a manner similar to that just described. However, for this measurement the signals from the two detectors were added so that no spin analysis was done. To increase the counting rate a thick tungsten target was substituted for the thin gold foil in the Mott scattering chamber. The entrance aperture of the filter lens system was reduced so that the solid angle subtended was represented by an angular range of  $\Delta \theta = 4^{\circ}$  (rather than 10°). Measurements were made over an angular range from 30° to 126°.

Our data at 3.3 eV shown in Fig. 3 can be compared with theoretical results<sup>11</sup> as well as recently published experimental values.<sup>12</sup> Since these are relative differential scattering cross sections, the ordinates of the figures are plotted in arbitrary units. Furthermore, the experimental points and the theoretical curve are normalized so that the secondary maximum between 90° and 120° scattering angle is equal to 1.0. One can see rather close agreement of our experimental values with those of Gehenn and Wilmers.<sup>12</sup> The



FIG. 3. Measured values of the elastic scattering cross section  $\sigma(\theta)$  at 3.3 eV. Continuous curves, from the theory of Karule and Peterkop (Ref. 11). Dashed curves, experimental values from the work of Gehenn and Wilmers.

differences which do exist could possibly be explained by the slight differences in the electron energy. Error bars in the work of Gehenn and Wilmers were not published.

There is general agreement of our experimental values for both  $\sigma(\theta)$  and  $|f|^2/\sigma(\theta)$  with the twostate close-coupling theory of Karule and Peterkop. An angular shift of about 10° relative to the theoretical curve in the 3-eV data is clearly indicated in the plot of  $\sigma(\theta)$ , particularly in the positions of the minimum and maximum at  $60^{\circ}$  and  $100^{\circ}$ . This shift is not apparent at the forward angles. A significant angular shift at the forward angles is clearly apparent in the data for  $|f(\theta)|^2/$  $\sigma(\theta)$ . There is some evidence from the results of a recent four-state close-coupling theory for sodium<sup>13</sup> that a relative shift may arise because of an insufficient number of states used in the closecoupling expansion. Further close-coupling computations of this type for potassium would be valuable.

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## Evidence of Mode-Mode Coupling and Nonlocal Shear Viscosity in a Binary Mixture near the Consolute Point\*

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Linewidths of Rayleigh scattering from a binary liquid system of *n*-hexane plus nitrobenzene have been measured at two fixed scattering angles over a temperature range such that  $\epsilon \equiv (T - T_c)/T_c$  extends from  $10^{-2}$  to  $10^{-6}$ . The ratio of the linewidths at two angles as a function of the correlation length  $\xi$  have been compared with recent self-consistent mode-mode coupling calculations of Kawasaki and Lo. The theory takes into account the nonlocality of shear viscosity near the critical point, and the data clearly establish the predicted effect in the critical region.

It has been well established both theoretically<sup>1</sup> and experimentally<sup>2</sup> that the decay rate  $\Gamma_k$  of the concentration fluctuation  $C_k$  deviates appreciably from the hydrodynamic expression  $K^2D$  as one approaches the critical region, as defined by  $K\xi \ge 1$ . In fact, an explicit  $K\xi$  dependence of the diffusion coefficient  $D(K\xi)$  has been given by Kawasaki<sup>1</sup> as

 $\Gamma_{k} = K^{2} D(K\xi) = (k_{B} T / 6\pi \eta^{*}) \xi^{-3} K_{0}(K\xi), \qquad (1)$ 

where

$$K_0(x) = \frac{3}{4} \left[ 1 + x^2 + (x^3 - x^{-1}) \tan^{-1} x \right]$$
(2)

and  $\eta^*$  is loosely called the "high-frequency" shear viscosity. Experimental work of Berge *et al.*<sup>3</sup> indicated that with reasonable choices of  $\xi_0$  and  $\nu$  in the defining equation  $\xi = \xi_0 \epsilon^{-\nu}$ ,  $\eta^*$  could be taken as a constant in fitting the Rayleigh linewidth data over the entire temperature range covered. Since then there have been new measurements<sup>4</sup> indicating that far away from the critical point  $\eta^*$  approaches  $\eta(T)$ , the hydrodynamic shear viscosity, and near the critical point it deviates from  $\eta(T)$  but not entirely temperature independently. Based on these observations, Kawasaki and Lo<sup>5</sup> recently made an attempt to clarify the meaning of  $\eta^*$  by solving self-consistently the coupled equations of  $C_k$  and  $V_q$ , the transverse local velocity fluctuation.

Far away from the critical point, the time dependence of these two fluctuating modes assumes the hydrodynamic expression  $C_k(t) \sim \exp(-K^2Dt)$