Using guage invariance, a formulation analogous to the Ginzburg-Landau theory of superconductivity can be constructed and is presently under study.

Here we have emphasized the existence of a critical density ρ_c for the formation of an infinitesimal but macroscopic pion field condensate. In a more complete treatment, we anticipate modifications due to S-wave interactions[3] and also shifts of the nucleon energy associated with the noncondensate virtual-pion interactions. This latter effect is probably the most important. The modification of the nucleon propagators produced by the π^- condensate will reduce the binding energy associated with virtual pion exchange. At small values of the condensate density this change in the nucleon correlation energy will appear on the right-hand side of Eq. (15) as a positive term proportional to xN. This will shift the critical density to higher values. A similar effect is well known in strong coupling superconductors[7] where the superconducting condensation energy is less than that predicted by the original BCS theory due to the interference of the pair condensate with the *normal* electron correlation energy.

It is a pleasure to acknowledge many useful discussions with R. F. Sawyer.

*Research supported by the U. S. Air Force Office of Scientific Research under Grant No. 71-2007.

[1]R. F. Sawyer, preceding Letter [Phys. Rev. Lett. 29, 382 (1972)].

[2]E. H. Auerbach, D. M. Fleming, and M. M. Sternheim, Phys. Rev. 162, 1683 (1967).

[3]Here we have neglected the repulsive S-wave interaction because we expect the P-wave part to dominate for the relevant pion moment of order m_π. In addition, residual P-wave attractions, beyond the baryon pole terms already taken into account by our Hamiltonian Eq. (1), should cancel part of the S-wave repulsion. It may be that the remaining S-wave effects increase the critical density for pion condensation above that calculated here, but we believe that the nature of the transition can be understood from our simple model. Furthermore, as the proton concentration increases, the S-wave contributions decrease. Here, for clarity, we treat the nucleons in the nonrelativistic limit, which is a reasonable approximation over the density region of major interest to us.

[4]Actually, at a finite pion concentration the minimum energy occurs for a value of K different from that associated with ρ_c. Details of this will be reported elsewhere.

[5]To complete our solution, the constraint Eq. (13) implies that $2(\mu_2-\mu_1')=\overline{\omega}_K(\rho/\rho_c)^{1/2}(1-2x^*)/(1-x^*)^{1/2}$ and the splitting condition between the η and γ states becomes $(\rho/\rho_c)/1-x^*)\geq(2\mu/\omega_K)^2$, which is satisfied for $\rho\geq\rho_c$.

[6]It may be that a standing π^- condensate of the type used by Sawyer is favored. In this case ρ_c is increased by $\sqrt{2}$ and one must of course take into account the Coulomb interactions associated with local charge density deviations.

[7]D. J. Scalapino, in *Superconductivity I*, edited by R. Parks (Marcel Dekker, New York, 1969), Chap. 10.

Electromagnetic and Weak Masses*

Steven Weinberg
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 19 June 1972)

In theories with spontaneously broken gauge symmetries, various masses, or mass differences may vanish in zeroth order as a consequence of the representation content of the fields appearing in the Lagrangian. These masses or mass differences can then be calculated as finite higher-order effects. The mechanism for cancelation of divergences in second-order fermion masses is described explicitly. The weak interactions play an essential role in canceling infinities in electromagnetic masses.

The idea that electromagnetism is responsible for mass differences within isotopic multiplets, and possibly also for the whole mass of the electron, has historically proved very attractive but not very fruitful. In the context of perturbation theory, the trouble is that when some mass or mass differences vanish in zeroth order, there generally is either some symmetry in the theory which keeps the mass or mass difference zero in all higher orders (as in electrodynamics with zero electron mass), or else the higher-order corrections turn out to be infinite (as for the neutron-proton mass difference with equal zeroth-order masses). In renormalizable theories such infinities may be eliminated by introducing counter terms in the Lagrangian, but the masses are then free parameters.

The recent development[1] of renormalizable unified theories of the weak and electromagnetic interactions based on spontaneously broken gauge symmetries offers us a way out of this impasse. Consider some invariance group G [such as local SU(2), SU(2) ⊗ U(1), etc., but possibly including discrete symmetries] which forbids the appearance of a particular mass or mass difference term in the Lagrangian. Imagine G to be spontaneously broken down to some subgroup S of unbroken symmetries (such as electromagnetic gauge transformations) which allow the appearance of this mass or mass difference for the physical particles. Suppose, however, that even after the spontaneous breaking of G down to S, the zeroth-order contribution to this mass or mass difference vanishes *for all possible G-invariant and renormalizable Lagrangians* constructed from a given set of fields belonging to given representations of G. Then the higher-order corrections to this mass or mass difference must be finite, because there are no counter terms available to absorb ultraviolet divergences in these corrections, so that any infinity here would be inconsistent with the renormalizability of the theory.[2]

Let us see how this works for fermion masses. If a mass or mass difference term in the Lagrangian is forbidden by the underlying invariance group G, then the only way it can appear in zeroth order is through the vacuum expectation values of a set of spin-0 fields φ_i, which are generally inversely proportional to coupling constants. The only renormalizable interaction of a set of spin-$\frac{1}{2}$ fields ψ_n with the φ_i is of the Yukawa form

$$L_Y = -\psi^\dagger\gamma_4\Gamma_i\psi\varphi_i, \qquad (1)$$

where $(\Gamma_i)_{nm}$ are a set of matrices, proportional to one or more coupling constants. (Without loss of generality, we shall take the φ_i to be Hermitian, so that $\gamma_4\Gamma_i$ is Hermitian. Note that Γ_i may contain terms proportional to γ_5.) The zeroth-order fermion mass matrix is then

$$m = m_0 + \Gamma_i\langle\varphi_i\rangle_0, \qquad (2)$$

where m_0 is the bare mass matrix which must appear in the original Lagrangian unless forbidden by G. We may suppose that the ψ_n and φ_i furnish representations D_F and D_B (perhaps reducible) of G, and the $(\Gamma_i)_{nm}$ are then proportional to the coefficients which couple the various components of D_B to $D_F{}^* \otimes D_F$. The bare-mass matrix m_0 must be invariant under G, so it is absent unless $D_F{}^* \otimes D_F$ contains the identity representation.

How can zeroth-order mass relations arise in such theories? In general, any possible mass matrix m which is invariant under the unbroken subgroup S may be expanded as in Eq. (2) in a series of S-invariant vectors belonging to various irreducible components of $D_F{}^* \otimes D_F$. Further, the vacuum expectation values $\langle\varphi_i\rangle_0$ are usually unconstrained (except by S) if we allow free variations of the constants in the Lagrangian. Hence, no zeroth-order mass relations (except S invariance) are expected if there is a scalar-field multiplet for *each* irreducible component of $D_F{}^* \otimes D_F$. However, if there are no scalar fields in the theory which transform according to one or more of the irreducible representations in $D_F{}^* \otimes D_F$ (and if these are representations which contain S-invariant vectors), then the absence of these representations in Eq. (2) will generate one or more zeroth-order mass relations. Such mass relations are not mere consequences of the unbroken symmetries of the theory, and finite corrections will generally appear in higher order. Thus, apart from possible "accidental" constraints on $\langle\varphi_i\rangle_0$, *it is the representation content of the scalar multiplet appearing in the Lagrangian which determines the pattern of broken fermion mass relations.*

To take an extreme case, we might suppose that *none* of the scalar fields in the theory belong to any of the irreducible components of $D_F{}^* \otimes D_F$, so that there are no Yukawa couplings at all. The zeroth-order fermion masses then arise only from the bare mass matrix m_0, and are therefore equal within irreducible representations of G. However, the vacuum expectation values of the scalar fields that do appear in the Lagrangian will give the vector gauge bosons a G-noninvariant mass matrix, so that emission and absorption of virtual gauge bosons can produce *finite* G-noninvariant fermion mass splittings. Just such a model has been considered by 't Hooft,[1] and indeed, he finds a finite fermion mass splitting of second order in the gauge coupling constant. In 't Hooft's model the Yukawa couplings are forbidden by a reflection symmetry R, under which all fermion and vector fields are even, and all scalar fields are odd. (This symmetry was not explicitly stated by 't Hooft, but it is implicit in his Lagrangian.) The equality of the zeroth-order fermion masses can thus be attributed here to the absence of scalar fields even under R.

Georgi and Glashow[3] have considered quite a different model, in which one of the scalars which could have a Yukawa coupling is absent, and in consequence one of the fermions has a vanishing zeroth-order mass. This "electron" picks up a mass of second order in the Yakawa couplings, which is found to be finite.

The above models are useful only as illustrations, but it is not difficult to construct a semirealistic model of the neutron-proton mass difference along these lines. Consider an SU(2) ⊗ SU(2) ⊗ U(1) gauge group with generators $\vec{T}_L$, $\vec{T}_R$, and Y, the charge being given by $T_{L3}+T_{R3}+Y/2$. The left- and right-handed nucleon fields have $T_L=\frac{1}{2}$, $T_R=0$, $Y=+1$ and $T_L=0$, $T_R=\frac{1}{2}$, $Y=+1$, respectively. In general, Yukawa couplings would be possible here for two independent Hermitian scalar fields with $T_L=T_R=\frac{1}{2}$, $Y=0$, so if we suppose that the theory only has *one* such field, we have a zeroth-order mass relation, which requires equal nucleon masses. In order to break the gauge group down to electromagnetic gauge invariance, we may introduce another scalar multiplet, with $T_L=\frac{1}{2}$, $T_R=0$, $Y=+1$, and allow all neutral scalar-field components to have nonvanishing vacuum expectation values. The theory then contains *six* heavy intermediate bosons (two each with charges +1, 0, −1) and a massless photon, but if we assume that the leptons all have $T_R=0$, then no gross conflict with the known properties of the semileptonic strangeness-conserving weak interactions need arise. A finite neutron-proton mass difference is expected to arise in second order.

Although the finiteness of the corrections to mass relations in such theories rests on very general considerations, it is of interest to see in detail how the divergences will drop out in actual calculations. We consider a general renormalizable and gauge-invariant Yang-Mills Lagrangian

$$\mathcal{L}=-\psi^\dagger\gamma_4\gamma^\mu(\partial_\mu+it_\alpha A_{\alpha\mu})\psi-\tfrac{1}{2}(\partial_\mu\varphi+i\theta_\alpha\varphi A_{\alpha\mu})_i(\partial^\mu\varphi+i\theta_\alpha\varphi A_\alpha{}^\mu)_i$$
$$-\tfrac{1}{4}F_{\alpha\mu\nu}F_\alpha{}^{\mu\nu}-F(\varphi)-\psi^\dagger\gamma_4\Gamma_i\psi\varphi_i. \tag{3}$$

Here $(t_\alpha)_{nm}$ and $(\theta_\alpha)_{ij}$ are the matrices representing the Lie algebra of G on the fields ψ and φ, respectively; $A_\alpha{}^\mu$ is the gauge vector field; $F_\alpha{}^{\mu\nu}$ is its gauge-covariant curl; and $F(\varphi)$ is an arbitrary G-invariant quartic polynomial in φ. (Our choice of kinematic terms here requires that t_α is Hermitian; θ_α is imaginary and antisymmetric, and the structure constants are totally antisymmetric. The gauge coupling constants are included in t_α, θ_α, and in the structure constants. The matrix t_α may contain terms proportional to γ_5.) The G invariance of the Yukawa coupling requires

$$[t_\alpha,\gamma_4\Gamma_i]+(\theta_\alpha)_{ij}\gamma_4\Gamma_j=0. \tag{4}$$

We choose to work in the "unitarity gauge," in which the field components φ_i corresponding to Goldstone bosons are absent:

$$\varphi_i(\theta_\alpha)_{ij}\langle\varphi_j\rangle_0=0. \tag{5}$$

The zeroth-order vector-boson mass matrix is then

$$\mu_{\alpha\beta}{}^2=-(\theta_\alpha\langle\varphi\rangle_0)_i(\theta_\beta\langle\varphi\rangle_0)_i. \tag{6}$$

Now, we are only interested here in corrections to the mass matrix which change its representation content. In particular, higher-order effects will force us to renormalize the coupling constants in Γ_i, and will change the values of the $\langle\varphi_i\rangle_0$, but these corrections will not change the representation content of $\Gamma_i\langle\varphi_i\rangle_0$, and therefore cannot affect the mass relations which arise in zeroth order from a specification of the representation content of the fields. The only one-loop diagrams which need be taken into account here are those in which a gauge vector boson or a scalar boson is emitted and absorbed from a fermion line. A straightforward calculation gives the self-energy matrix here as

$$\Sigma(p)=\frac{-i}{(2\pi)^4}\int d^4k\Big[-\gamma^\mu t_\beta[i\gamma_\lambda(p-k)^\lambda+m]^{-1}\gamma^\nu t_\alpha\Big(\frac{g_{\mu\nu}+\mu^{-2}k_\mu k_\nu}{k^2+\mu^2}\Big)_{\beta\alpha}$$
$$+\Gamma_i[i\gamma_\lambda(p-k)^\lambda+m]^{-1}\Gamma_j\Big(\frac{\Pi}{k^2+M^2}\Big)_{ij}\Big]. \tag{7}$$

M is the zeroth-order mass matrix of the scalar bosons, and Π is a projection matrix, which eliminates Goldstone bosons from the sum over scalar-meson fields. Inspection of Eqs. (5) and (6) shows that

$$\Pi_{ij}=\delta_{ij}+(\theta_\alpha\langle\varphi\rangle_0)_i\,\mu_{\alpha\beta}^{-2}(\theta_\beta\langle\varphi\rangle_0)_j. \tag{8}$$

In calculating the mass matrix from (7), we may recall that a term in $\Sigma(p)$ with a factor $ip_\lambda\gamma^\lambda+m$ on the extreme left or right will induce a fermion-field renormalization, but cannot shift the poles of $[ip_\lambda\gamma^\lambda+m-\Sigma(p)]^{-1}$, and hence may be dropped. With this understanding, we find that the divergent parts of $\Sigma(p)$ consist of a quadratically divergent term Q arising from the $k_\mu k_\nu$ term in (7),

$$\gamma_4 Q=\frac{-i}{2(2\pi)^4}\int d^4k\left(\frac{\mu^{-2}}{k^2+\mu^2}\right)_{\beta\alpha}[t_\beta,[t_\alpha,\gamma_4 m]], \tag{9}$$

plus logarithmically divergent terms L_1, L_2, L_3, L_4 arising respectively from the $g_{\mu\nu}$ and $k_\mu k_\nu$ terms in the vector propagator and from the δ_{ij} and $\Pi_{ij}-\delta_{ij}$ terms in the scalar propagator. Equations (2), (4), (6), and (8) imply that L_2 and L_4 cancel. The remaining terms turn out to have the matrix structure

$$\gamma_4 L_1\propto t_\alpha\gamma_4 m t_\alpha,\ \gamma_4 m t_\alpha t_\alpha,\ t_\alpha t_\alpha\gamma_4 m; \tag{10}$$

$$\gamma_4 L_3\propto\gamma_4\Gamma_i\gamma_4 m\gamma_4\Gamma_i,\ \gamma_4 m\gamma_4\Gamma_i\gamma_4\Gamma_i,\ \gamma_4\Gamma_i\gamma_4\Gamma_i\gamma_4 m. \tag{11}$$

(Details will be published elsewhere.) Since $\gamma_4 m$ is a linear combination of matrices $\gamma_4\Gamma_i$ satisfying Eq. (4), it is easily seen that (9), (10), and (11) are linear combinations of the *same* $\gamma_4\Gamma_i$ matrices. Thus, the divergent part of $\Sigma(p)$ has the same representation content as the zeroth-order mass matrix, and so cannot enter into the corrections to the zeroth-order mass relations.

As a practical matter, we expect that scalar-boson exchange will be much smaller than vector-boson exchange, because Eqs. (2) and (6) show that ratios of typical elements of Γ_i and t_α are of the same order as the ratio of the fermion masses m to the vector-boson masses μ. In addition, once we extract the quadratic divergence (9), the $k_\mu k_\nu$ term in (7) is smaller than the $g_{\mu\nu}$ term by a factor of order $(m/\mu)^2$. (This explains how it is possible for the logarithmic divergence in this term to cancel part of the logarithmic divergence in scalar exchange.) Thus, if $m\ll\mu$, *the corrections to zeroth-order fermion-mass relations may be calculated taking into account only vector-boson exchange, and dropping the $k_\mu k_\nu$ term in the vector propagator.*

However, matters may not be so simple. It is attractive to suppose that all dimensionless couplings of elementary scalar and vector bosons are of order e, in which case the zeroth-order fermion masses would be of the same order as the intermediate vector-boson masses μ, say roughly 50 GeV. It is entirely possible that there exists a class of such "super-fermions" (and indeed a number of interesting models[4] require the presence of heavy leptons or quarks). In this picture there must be zeroth-order relations which require the masses of the observed leptons and hadrons to vanish. Second-order weak and electromagnetic effects would produce masses of order $\alpha\mu$, a few hundred MeV, but the electron mass must for some reason remain zero until fourth order.

In any case, one lesson definitely emerges from this work: It is pointless to try to evaluate electromagnetic mass differences without taking weak interactions into account. In particular, it is only when we sum over all gauge fields $A_{\alpha\mu}$ in (10) that the logarithmic divergence drops out of the corrections to mass relations. (However, this does not introduce parity violations of order α, because γ_5 terms in the mass matrix can always be eliminated by redefining the fermion fields.) Perhaps the weak interactions also invalidate various one-photon-exchange theorems, such as the Sutherland theorem for $\eta\to 3\pi$ and the $\Delta I\leq 2$ rule.

I am grateful for valuable discussions with H. Georgi, S. Glashow, R. Jackiw, F. E. Low, and J. Primack.

*Work supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-3069.

[1]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Physics,* edited by N. Svartholm (Almquist and Wiksells, Stockholm, 1968), p. 367; G. 't Hooft, Nucl. Phys. B35, 167 (1971); B. W. Lee, Phys. Rev. D 5, 823 (1972); S. Weinberg, Phys. Rev. Lett. 27, 1688 (1971); etc.

[2]This has already been remarked by S. Weinberg, Phys. Rev. D 5, 1962 (1972), note added in proof. However, the models discussed in this reference do not seem to lead to realistic theories with an electron mass which vanishes in zeroth order.

[3]H. Georgi and S. Glashow, private communication.

[4]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970); H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972); B. W. Lee, to be published; J. Prentki and B. Zumino, to be published; C. Bouchiat, J. Iliopoulos, and P. Meyer, to be published; etc.