tailed balance, we find

$$\sigma_{\gamma_0 n}(\text{back}) \approx 0.1 \text{ mb}.$$

Because of the assumptions made, this result is a very approximate estimate of σ(back). However, the cross section actually measured is sufficiently close to this value to suggest that the data for ^{29}Si establish qualitatively the relationship between correlations and background cross sections developed by Lane.

In summary, the simultaneous observation in ^{29}Si of localized neutron and radiative strength, strong partial-width correlations, and a background cross section observable through its interference effects may be attributed to the presence of an isolated common doorway state consisting of a $2p_{3/2}$ neutron coupled to the ^{28}Si ground state. If this explanation is correct, it poses further questions. The observed $E1$ strength is distributed over an ~500-keV interval, a substantial fraction of a single particle width. Yet the observed photon and neutron strengths are only $\lesssim 10\%$ of the total expected for the $2s_{1/2} \rightarrow 2p_{3/2}$ transition. This suggests that the strength of this transition either is fractionated and spread over a large region of excitation or that the transition is inhibited for some reason. Further experimental study of $p_{3/2}$ strength in ^{29}Si would clarify this point.

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Condensed π^- Phase in Neutron-Star Matter*

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It is argued that at some density, estimated at about 1 baryon/F^3, superdense nuclear matter will make a transition to a phase of approximately equal numbers of protons, neutrons, and π^- particles, the latter condensed in one or two plane-wave states of momentum ≈ 170 MeV/c. These conclusions are based on the conventional theory of the pion-nucleon interaction.

The properties of superdense matter in neutron-star interiors have been estimated by several authors, using various approaches.[1-5] However, these approaches have all begun from a simple basic picture for the matter, one in which the matter consists (for densities up to about 1 baryon/F^3) of a Fermi gas primarily composed of neutrons, with some protons and electrons, which interact with each other through the standard nucleon-nucleon potentials.

As densities grow larger than nuclear densities (0.15 baryons/F^3), the assumptions of these models become shakier. Such phenomena as large contributions from many-body forces, or an exotic ground state, or real mesons as constituents of the matter, all become more likely at higher densities.

As an example, let us consider the possibility of a condensed phase of π^- particles neutralized by an equal number of protons, the system also containing an equal number of neutrons. The interactions of a π^- of moderate momentum $(0.5m_\pi c$

$<k<1.5m_\pi c$) in a medium with $\rho_p \approx \rho_n$ will be dominated by attractive P-wave effects.[6] The optical potential of Auerbach, Fleming, and Sternheim[7] for a π^- in nuclear matter yields the following dispersion relation for the π^- within the nucleus (ρ is in baryons/F^3):

$$k^2(1-6\rho)+0.7\rho m_\pi\omega+m_\pi^2=\omega^2. \tag{1}$$

At a density only slightly greater than nuclear densities, there will be a solution with $\omega=0$ to (1) even for moderately low k, apparently indicating the possibility that the proton charge can be neutralized with pions, at no cost in pion energy. However, the phenomenological optical potential is not a correct starting point for discussing our new ground state. The nucleon pole graphs, which are responsible for the greater part of the P-wave scattering lengths underlying the $k^2\rho$ term in (1), have an energy denominator the variation of which is not taken into account in (1). This denominator becomes very small for small ω, which is exactly the region of interest in the present problem.

In the present note we report some considerations which we believe point in a more definite way to the likelihood of a π^- condensation at some density. The interaction used will be unrealistically simplified, but the argument will be free from the perturbation-theoretic problems of the optical-potential approach, and it will correctly take into account the meson-theoretic sources of the P-wave attraction discussed above. Consider first a Hamiltonian for free neutrons, protons, and pions, H_0, to which is added an interaction term H_1 giving the coupling of two modes only of the π^- field to the neutron-proton system. One mode has momentum k in the $\hat{z}$ direction; the other has momentum $-k$ in the $\hat{z}$ direction,[8]

$$H=H_0+\frac{\sqrt{2}\,ifk}{m_\pi(2\omega_k V)^{1/2}}\sum_{\vec{q}}[p^\dagger(\vec{q}-k\hat{z})\sigma_3 n(\vec{q})a^\dagger(k\hat{z})-n^\dagger(\vec{q})\sigma_3 p(\vec{q}-k\hat{z})a(k\hat{z})$$
$$-p^\dagger(\vec{q}+k\hat{z})\sigma_3 n(\vec{q})a^\dagger(-k\hat{z})+n^\dagger(\vec{q})\sigma_3 p(\vec{q}+k\hat{z})a^\dagger(-k\hat{z})]. \tag{2}$$

Here $n^\dagger(q), n(q), p^\dagger(q), p(q)$ are the creation and annihilation operators for neutrons and protons of momentum q, which we shall take as nonrelativistic except in the computation of kinetic energies. $a_k^\dagger$ and a_k are the respective creation and annihilation operators for π^- particles. The parameter $f=1.1=[4\pi(0.088)]^{1/2}$. At this stage we are leaving out all interactions except with the mesons of the anticipated condensed phase.

Now we ask for the ground-state energy of a system with baryon number N in our volume V, subject to the constraints of exact charge neutrality and no macroscopic currents. We distinguish two types of states, of which state I consists of all free-neutron levels filled up to the Fermi momentum k_F appropriate to the density $\rho=N/V$. The expectation value of the Hamiltonian in this state will be denoted $E(k_F)$. In the limit of infinite volume $E(k_F)$ is an actual eigenvalue of H.

State II will have to be described at length; it consists of a superposition of states of different numbers of protons, neutrons, and π^-'s which we individually denote by

$$|m_1,\alpha;m_2,\beta\rangle=|m_1 p\text{'s}(\uparrow), m_1\pi^-\text{'s}(-k\hat{z}),\ (N/2-m_1)n\text{'s}(\uparrow),\alpha;$$
$$m_2 p\text{'s}(\downarrow), m_2\pi^-\text{'s}(k\hat{z}), (N/2-m_2)n\text{'s}(\downarrow),\beta\rangle. \tag{3}$$

Here the vertical arrows denote the z component of baryon spin; α and β denote various ways of assigning protons and neutrons to plane-wave states. We make these assignments in the following way: The $N/2-m_1$ spin-up neutrons occupy states up to the k_F level (of which there are $N/2$) in any way, the different ways being distinguished in the label α. For the spin-up protons we consider a set of states of momenta $\{\vec{q}+k\hat{z}\}$, where $\vec{q}$ runs over the same basic Fermi sphere that we used for the neutron states. We now fill only those spin-up proton levels corresponding to $\vec{q}$'s that were not used for the spin-up neutrons (this exactly uses up the m_1 spin-up protons).

For the spin-down neutron states we use the same states $\{\vec{q}\}$ as for the spin-up neutron states, the exact assignments being described by the index β. For the spin-down proton states we use the set $\{\vec{q}-\hat{z}\vec{k}\}$ again avoiding all the q's which were utilized in the spin-down neutron sea.

The phases of these states are defined with respect to some standard ordering of the modes in the basic Fermi sea, $q_1\uparrow, q_1\downarrow, q_2\uparrow, q_2\downarrow, q_3\uparrow, \cdots$, as

$$|m_1, \alpha; m_2, \beta\rangle = \cdots [p^\dagger(\vec{q}_j + k\hat{z}, \uparrow) \text{ or } n^\dagger(\vec{q}_j, \uparrow)] \cdots [p^\dagger(\vec{q}_2 + k\hat{z}, \uparrow) \text{ or } n^\dagger(\vec{q}_2, \uparrow)][p^\dagger(\vec{q}_1 - k\hat{z}, \downarrow) \text{ or } n^\dagger(\vec{q}_1, \downarrow)][p^\dagger(\vec{q}_1 + k\hat{z}, \uparrow) \text{ or } n^\dagger(\vec{q}_1, \uparrow)] \times a^\dagger(k\hat{z})^{m_2} a^\dagger(-k\hat{z})^{m_1}|0\rangle, \qquad (4)$$

where the $p^\dagger$ and $n^\dagger$ are the creation operators for protons or neutrons in the indicated modes. In (4) it is made explicit that for every mode, q-spin, either a neutron state is occupied or a proton state is occupied in the displaced mode $\vec{q} \pm k\hat{z}$, but not both.

Now we form the normalized state:

$$|\psi_{\rm II}\rangle = \mathfrak{N} \sum_{m_1, m_2, \alpha, \beta} (i)^{m_1+m_2} f(m_1) f(m_2) |m_1, \alpha; m_2, \beta\rangle, \qquad (5)$$

where $0 < m_1, m_2 < \frac{1}{2}N$ and the function $f(m)$ (a) is peaked around $m = \frac{1}{4}N$, (b) varies negligibly for $\Delta m = 1$, and (c) in the limit of large N becomes negligible for $|m - \frac{1}{4}N| > \sqrt{N}$. An example would be $f(m) = \exp[-(m - \frac{1}{4}N)^2/\sqrt{N}]$.

A rough description of $|\psi_{\rm II}\rangle$ in words is as follows: A state of approximately $N/2$ protons, $N/2$ neutrons, and $N/2$ π^-'s, which is a superposition of states in which only one is occupied of any two corresponding states in the (displaced) proton Fermi sea and in the neutron sea. All possible such states with $N_p \approx N_n$ are superposed with equal weight, with, however, a cutoff imposed when the number of protons becomes very different from the number of neutrons. The phases have been arranged so that every term in the expectation value of $H_{\rm I}$ has the same sign.

We can show that in a certain limiting sense as $N \to \infty$, $V \to \infty$, $N/V \to$ const, the state $|\psi_{\rm II}\rangle$ is an actual eigenstate of the Hamiltonian. But, rather than get preoccupied with the mathematics of such limits, we shall note that the expectation value $\langle\psi_{\rm II}|H|\psi_{\rm II}\rangle$ gives, in any event, an upper bound on the ground-state energy. We retain only terms which contribute a finite amount of energy per particle in the $N \to \infty$ limit:

$$\langle\psi_{\rm II}|H|\psi_{\rm II}\rangle = E'(k_{\rm F}) - \frac{N}{2m_\pi}\frac{fk\sqrt{N}}{(\omega_k V)^{1/2}} + \frac{N}{2}\omega_k. \qquad (6)$$

$E'(k_{\rm F})$ here is the nucleon rest plus kinetic energy which differs from the energy of case I, $E(k_{\rm F})$, by the effects of the displacements of the proton Fermi sea. We have

$$E'(k_{\rm F}) \leq E(k_{\rm F}) + \tfrac{1}{2}Nk^2/2M, \qquad (7)$$

the equality holding in the nonrelativistic limit.

The second term on the right-hand side of (6) comes from the interaction term in (2). Its sign can be changed freely by changing phases in the wave function [as can be seen by considering the transformation $p(\vec{q}) \to -p(\vec{q})$ in (2)]. The third term on the right-hand side of (6) is the meson energy.

Writing N/V as the total baryon density ρ, we see from (6) that for low densities or small coupling the type-I state (no mesons) will have the lower energy, but for

$$\frac{f\sqrt{\rho}}{m_\pi} > \frac{\omega_k^{3/2}}{k} + \frac{k\sqrt{\omega_k}}{2M}, \qquad (8)$$

we find $\langle\psi_{\rm II}|H|\psi_{\rm II}\rangle$ is less than the energy of the type-I state. The minimum value of the right-hand side of (8) is $1.73\sqrt{m_\pi}$ at $k = 1.2m_\pi$. Thus for $\rho > 3m_\pi^3 \approx 1$ F^{-3} we predict a ground state of the type II with the meson momentum given by $k = 1.2m_\pi$.

At a density of 1 neutron/F^3, and in the absence of interaction (except for the weak interaction) a free-neutron gas would have lowered its energy slightly by making a transition to a conventional state with about 10% protons, 10% Σ^- particles, and 80% neutrons.[2,4] However, at a very slightly higher total baryon density our p, n, π^- phase would again win out. Furthermore, if we want to include hyperons we can play the same game with $\Sigma^-\Lambda$ states that we already played with n and p states and end by achieving still lower energies in a condensed pion state.

The state (3) (along with an infinite number of degenerate variants) has the lowest energy of any condensed pion state we have found which does not have a huge macroscopic electromagnetic current density. If we take only one pion mode, $-k\hat{z}$, say, and we displace our spin-down proton states by $+k\hat{z}$ [instead of by $-k\hat{z}$ as in (4)], we can gain an additional factor of $\sqrt{2}$ in the negative term on the right-hand side of (6), pointing to an onset of the condensed state at $\rho = \frac{1}{2}$ F^{-3}.[9] However, this state has an immense current flowing in the magnetic energy will preclude its formation.
magnetic energy will preclude its formation.

Next we briefly consider the question of interactions. It can be argued that the energies arising from nucleon-nucleon interactions depend mostly on the total density of nucleons and should therefore not differ greatly in the pure neutron state I and in the exotic state II at a given total baryon density. In the state II we should also take into account interactions of the π^- particles with the nucleons, beyond those interactions included in (2), and with each other. The question of how much of the observed attraction felt by π^- in nuclear matter is already taken into account in (2) and is a difficult one in our opinion. However, there would seem to be at least some residual attraction even after those Born terms implicit in Eq. (2) have been removed. As for the pion-pion interactions, the $I=2$ $\pi\pi$ S-wave interaction is thought to be repulsive, but quite small.[10] Thus, we come to no definite conclusion as to whether the formation of a condensed π^- phase will be encouraged or discouraged by interactions which have been omitted in (2).

If a condensed π^- phase should in fact occur in neutron star matter, the consequences would obviously be far reaching. If we calculate the pressure from our formula (6) using

$$p=\frac{N^2}{V}\frac{d}{dN}\left(\frac{E}{N}\right),$$

and evaluate at $\rho=1\ \mathrm{F}^{-3}$ (our guess for the onset of the condensed phase), we obtain only 20% of the pressure of the free Fermi gas at that density. The interactions could do much to restore the pressure; however, some reduction would seem certain.

In addition to the large effects on the equation of state, the existence of the condensed phase could give rise to many strange phenomena depending on the locally defined directionality of the matter, provided by the wave vector of the pion field.

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