experiments roughly agrees with their results.

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Torque on a Rayleigh Disk Due to He II Flow*

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A model for He II flow past a Rayleigh disk is presented which assumes that superfluid potential flow and normal fluid Helmholtz flow occur. A torque which increases as the temperature decreases is predicted, agreeing with Pellam's measurements in rotating helium below T_λ. New measurements of the torque on a Rayleigh disk in nonrotating helium are presented, indicating Helmholtz flow above T_λ, a continuous torque at the λ point, and increasing torque below T_λ.

The Rayleigh disk has been used in several experiments[1-7] to probe the local velocity field of He II. However, the flow pattern around the disk is still not well understood.[8] Particularly puzzling are Pellam's experimental results[1-4] in rotating He II, which indicate a decreasing torque as the temperature is increased toward the λ point.

We present here a simple model for the flow of He II past a Rayleigh disk, which predicts a temperature-dependent torque below T_λ as observed by Pellam. New experimental results are also presented in nonrotating He II, which are in at least qualitative agreement with this model.

The classical flow pattern for a perfect (nonviscous) liquid around a Rayleigh disk is potential flow. The superfluid component of He II provides the only real liquid capable of perfect potential flow, and experiments[7,9] indicate that at sufficiently low velocity, potential flow does take place. A flat rectangular disk (width w and height d) exposed to pure potential flow will experience a torque of the form[10]

$$\tau = \tfrac{1}{8}\pi w^2 d\rho v^2 \sin 2\theta, \qquad (1)$$

where ρ is the fluid density, v the undisturbed fluid velocity, and θ the angle between the disk and fluid velocity.

Normal viscous liquids *do not* move past a Rayleigh disk with potential flow. Instead, flow separation with a velocity discontinuity occurs at the disk edges and a stagnant region exists behind the disk.[11] By direct observation, Kitchens *et al.*[12] have seen velocity fields of this general character when the normal component of He II flows past a flat plate. This type of flow is known as Helmholtz flow, and its importance and relevance to superfluid hydrodynamics has been suggested by Craig.[8] An idealized version of this type of velocity field can be analyzed quantitatively. If it is assumed that the wake is composed of dead liquid (velocity zero) and extends to infinity, then the torque[13] on a flat disk is

$$\tau = \frac{3\pi w^2 d\rho v^2 \sin 2\theta}{8(4+\pi \sin\theta)^2}. \qquad (2)$$

It is important to note that for identical experimental parameters, the torque due to potential flow is a factor of $\frac{1}{3}(4+\pi\sin\theta)^2$ larger than for Helmholtz flow.

In light of the above results we propose the following model for He II flow past a Rayleigh disk. At sufficiently low velocity the superfluid component exhibits pure potential flow around the Rayleigh disk. The normal fluid always displays Helmholtz-type flow. The net torque on the disk is the sum of the superfluid and normal fluid contributions:

$$\tau = \frac{\pi\rho w^2 d \sin 2\theta}{8}\left[\rho_s v_s^{\,2} + \frac{3\rho_n v_n^{\,2}}{(4+\pi\sin\theta)^2}\right], \qquad (3)$$

where the subscripts s and n refer to the superfluid and normal fluid components, respectively. For helium above the λ point the torque is given by the Helmholtz expression.

Figure 1 shows torque versus temperature as predicted by this model for an attack angle $\theta = 45°$. These results have been normalized to unity at the low-temperature limit. Between 4.2°K and the λ point the torque increases by about 15% as a result of the change in the helium density. At T_λ the torque is continuous since $\rho = \rho_n$ at T_λ and ρ is continuous. Below T_λ is increases rapidly with decreasing temperature. The significant increase in torque below T_λ is due to the temperature dependence of the superfluid and normal fluid densities and the order-of-magnitude difference between the torques due to potential and Helmholtz flow.

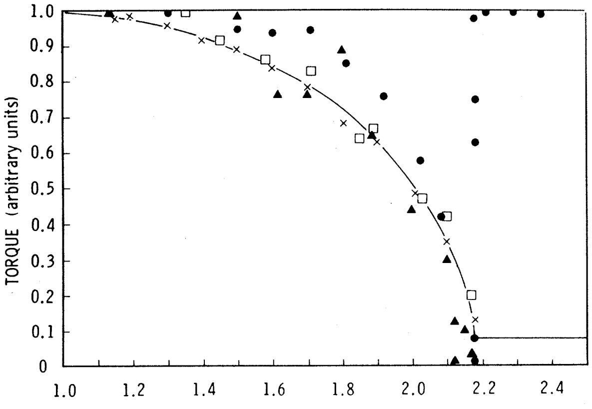

FIG. 1. Relative (normalized) torque on a Rayleigh disk versus temperature. The solid curve represents Eq. (3) for $\theta = 45°$. The circles, squares, triangles, and crosses represent data from Refs. 1, 2, 3, and 4, respectively.

In Fig. 1 we also present the results from four experiments by Pellam[1-4] in which he measured the torque on a Rayleigh disk suspended in a container of rotating He II. Pellam's results, normalized to unity at the low-temperature limit, are in excellent agreement with our model below the λ point. Thus, the previously unexplained temperature-dependent torque, observed by Pellam below the λ point, can be understood by assuming potential and Helmholtz flow by the superfluid and normal fluid components, respectively. This model does not explain the sudden jump in torque at T_λ shown in Ref. 1.[14]

In order to provide another test of this model and to study the relevance of Helmholtz flow to He II hydrodynamics, we have measured the torque on a Rayleigh disk in nonrotating He II flow. In this manner we intend to eliminate any effects which the rotational properties of He II might have contributed to Pellam's experiments.[15] Our apparatus is shown in Fig. 2. We use a rectangular disk 0.1 mm thick, measuring 2.5 mm wide and 2.0 mm high, suspended vertically in a 1.0-cm-diam channel. The suspension consists of a platinum-wire torsion fiber (torsion constant 1.75×10^{-4} dyne cm/deg) attached to a rigid quartz rod on which are mounted the Rayleigh disk, a mirror and an eddy-current damping plate. The channel is connected to a vertical tube in which a displacement plunger can be moved across the helium level to force liquid flow through the channel. Torque due to liquid flow past the disk is measured by observing the deflection of a laser beam reflected by the mirror.

The results of our measurements of torque versus temperature at a velocity of 0.5 cm/sec and attack angle of 40° are presented in Fig. 3. These results show a continuous torque at the λ point and an increasing torque as the temperature is lowered below T_λ. Between T_λ and 1.65°K, the measured torque increases by a factor of 3.5. The theoretical curve obtained from our model shows an increase of 10.7 over the same temperature range. We have also measured the torque

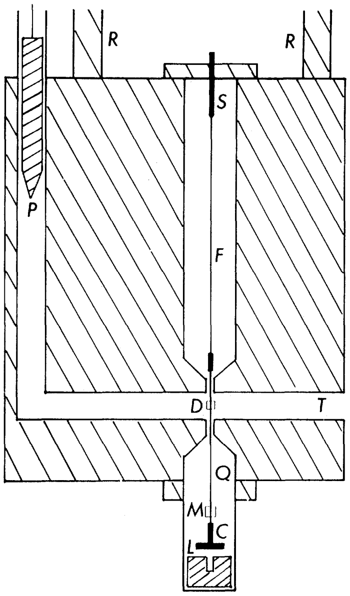

FIG. 2. Schematic diagram of the Rayleigh-disk suspension system and flow channel including the following: Rayleigh disk (D), torsion fiber (F), damping plate (C), damping magnet (L), mirror (M), quartz rod (Q), plunger (P), flow channel (T), suspension point (S), and support rods (R)

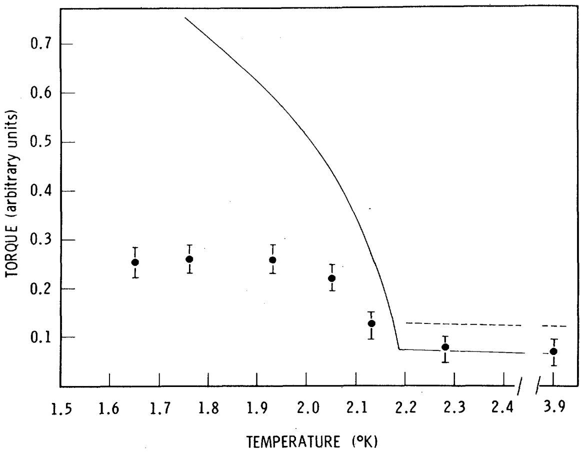

FIG. 3. Torque on a Rayleigh disk versus temperature. The solid curve represents Eq. (3) for $\theta = 40°$ and is normalized to the experimental values above T_λ. The circles indicate our experimental data at a velocity of 0.5 cm/sec and attack angle $\theta = 40°$. The dashed line represents the absolute magnitude of the torque expected for Helmholtz flow [Eq. (2)].

versus temperature at velocities of 0.75 and 1.01 cm/sec. Again the torque is continuous through the λ point. However, the measured increase in torque between T_λ and 1.65°K decreases with velocity. At a velocity of 0.75 cm/sec the torque at 1.65°K was 2.0 times the T_λ value, while at 1.01 cm/sec the factor was only 1.5. We believe that this quantitative difference between our model and the experimental results below T_λ is due to superfluid velocities in excess of the critical velocity for the breakdown of pure superflow. Many measurements of He II flow indicate a critical velocity less than 0.5 cm/sec for a 1.0-cm-diam channel.[16] It is well known that at critical velocity pure potential flow breaks down and turbulence and vorticity begin to form. As the velocity is further raised, the vorticity and turbulence rapidly increases.[16] Our measurements are in general agreement with these observations, since below T_λ the torque as a function of the velocity show a magnitude intermediate between potential and Helmholtz flow and approaches the Helmholtz value as the velocity increases.

Measurements of the absolute magnitude of the torque are also useful in determining the character of the flow taking place. Pellam[4] indicates that the torque at 1.1°K was about 20% less than the value predicted for potential flow. In our experiments the magnitude of the torque above T_λ is about 40% less than the predicted Helmholtz value (Fig. 3). We believe that the 40% disagreement in magnitude between our experimental results and the theory is not significant. The theory assumes a flat, symmetrical disk, but the actual disk used in our experiment has a 0.010-in. fiber and a blob of epoxy on one surface. The effects of this asymmetry are observable. When liquid flow approaches the flat side of the disk, the torque is 30% larger than when the fiber and epoxy side face upstream. Therefore, our results clearly show that Helmholtz flow and not potential flow occurs above T_λ.

In conclusion, we have shown that the torque on a Rayleigh disk depends strongly upon the character of the velocity field. If He II flow occurs with potential superflow and Helmholtz normal flow, a temperature-dependent torque will exist below T_λ which is in agreement with Pellam's observations. Our initial experimental results indicate that Helmholtz flow takes place above T_λ, and that below the λ point a torque intermediate between potential and Helmholtz flow occurs. We believe that this torque is due to superfluid flow above the critical velocity. We will now attempt to make measurements with a more sensitive torsion fiber so that superfluid flow below critical velocity can be studied. We also hope to be able to clearly differentiate between potential and Helmholtz flow by measuring the torque on the

Rayleigh disk as a function of attack angle, since Helmholtz flow leads to an angular variation of torque substantially different from potential flow.

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Collisionless Wave-Particle Interactions Perpendicular to the Magnetic Field*

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The collisionless interaction between particles undergoing $\vec{E}\times\vec{B}$ drift and Kelvin-Helmholtz instabilities propagating across a strong magnetic field is investigated using a test-wave technique. Orbit perturbations in phase space are observed with increasing instability amplitude. We also present the first direct measurements of finite–Larmor-radius effects.

Wave-particle interactions such as Landau damping have been extensively studied for many years,[1] principally for waves propagating in a plasma in a direction parallel to the magnetic field. The possibility of particles interacting with electrostatic waves or instabilities propagating strictly perpendicular to B has not been extensively studied theoretically,[2] since particles in general cannot move across a strong magnetic field in synchronization with the wave.

In this paper we wish to report experiments that demonstrate strong collisionless interactions between cross-field (Kelvin-Helmholtz[3]) instabilities and plasma ions whose guiding centers move across the magnetic field. By means of a test wave at a frequency much larger than the instability frequency, changes in the velocity shear profile (the free-energy source) have been measured in the presence of strong instabilities. This change in the profile can be correlated with the saturation of the Kelvin-Helmholtz instability.

The experiment was performed in a Q-machine plasma column operated in the single ended mode with a specially designed segmented hot end plate (the ionizer). This type of ionizer has been described in detail elsewhere,[4] although it should be noted that the dimensions are somewhat different in the present experiment: The center of the gap is now at $r = 1.45$ cm, and the gap width is 0.22 cm. The temperature difference across the gap is $\Delta T/T \approx 1\%$, while the density perturbation in the gap region is generally about 10%. The