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Linear Stark Effect Due to Resonant Interactions of Static and Dynamic Fields

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We investigated the line shape of the Lyman-α transition of a hydrogen atom simultaneously subjected to a static and a perpendicular high-frequency electric field. It was found that resonance effects occur, producing a profile substantially different from the Stark spectrum of the fields acting independently. This result shows that the interpretation of turbulent hydrogen plasma spectra by the Blokhintsev (dynamic field only) theory is not generally valid.

In turbulent plasmas the radiating atoms are subjected to an oscillating high-frequency electric field originating from electronic plasma oscillations. It appears to be generally believed that in hydrogen plasmas the radiation spectrum in this situation is well described by the theory of the dynamical linear Stark effect developed by Blokhintsev.[1] However, the atoms are radiating also under the simultaneous influence of a quasistatic field due to slowly moving ions or to low-frequency ion-acoustic turbulence. The present paper points out that the Blokhintsev theory is not adequate, as the resonant interactions between the Stark separation induced by the quasistatic field and the oscillations of the dynamic field produce a spectrum quite different from what could be explained as the combination of the independent effects of these fields.

The problem thus posed is that of radiation of an atom with degenerate states under the combined effects of static and dynamic electric fields. For a pure high-frequency dynamical field, the Blokhintsev theory predicts the appearance of a series of satellites at the harmonics of the frequency of the applied field. An addition of a static field parallel to the high-frequency field is still readily described by a simple extension of this theory and merely produces a symmetric splitting of each satellite with shifts proportional to the static field. However, when the static field has a component perpendicular to the dynamical field, then new results, essentially different from the Blokhintsev theory, emerge. In another sense this situation can be viewed as the formation of static Stark-split states which are connected by the off-diagonal matrix elements of the perpendicular dynamic field. For typical magnitudes of the plasma quasistatic field the energy separation due to the component perpendicular to the dynamic field can be of the same order of magnitude as the plasma frequency and resonance effects will occur. In plasmas of atoms which are not subject to the linear Stark effect, the theories[2-5] that only consider the dynamic field work well, because the atomic levels are naturally well separated and are only negligibly shifted by the usual magnitude of the quasistatic field in the plasma.

As the important physical effects are related to the perpendicular component of the static field, for simplicity we will ignore the parallel component. The more general case will be discussed elsewhere. We will concentrate on the structure of the hydrogenic Lyman-α line, which possesses all the essential ingredients of the problem but is still simple enough to illustrate well the fundamental physical processes. This transition occurs from the fourfold-degenerate $n=2$ upper level to the nondegenerate ground state. We can ignore interactions between unequal n levels because of the large energy separations. Choosing a representation in terms of real spherical wave functions,[6] the states of interest are $|2s\rangle$, $|2p_z\rangle$, $|2p_x\rangle$, and $|2p_y\rangle$, where the x and z directions are along the dynamical field E_D and the static field E_S, respectively. The $|2p_y\rangle$ state radiates only at the unperturbed freqeuncy, since, being perpendicular to the plane of the fields, it is not affected by them and remains uncoupled to the other states. For the other states the polarization of the radiation is along the direction specified by the x or z label of the corresponding state, while the $|2s\rangle$ state does not radiate at all. In terms of the three interacting states, labeled as $|1\rangle \equiv |2s\rangle$, $|2\rangle \equiv |2p_z\rangle$, $|3\rangle \equiv |2p_x\rangle$, the nonvanishing components of the Hamiltonian matrix are

$$H_{12}=H_{21}=S, \quad H_{13}=H_{31}=D\sin\omega t, \tag{1}$$
$$S=E_S\mu, \quad D=E_D\mu, \quad \hbar\mu=e\langle 1|z|2\rangle=e\langle 1|x|3\rangle.$$

The Schrödinger equation is

$$|\Psi(t)\rangle=\sum_j c_j(t)|j\rangle, \quad i\dot{c}_j=\sum_k H_{jk}c_k. \tag{2}$$

Equations of somewhat analogous form but describing different physical situations have been considered elsewhere.[5,7]

We have numerically integrated Eqs. (2) for the three sets of initial conditions $(c_1(0), c_2(0), c_3(0)) = (1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$. This covers all of phase space, and in fact provides the solution for the nine elements $c_j{}^l(t)$ of the matrix $\langle j|U(t)|l\rangle \equiv U_{jl}$ (j being the state of interest and l the state occupied at $t=0$) of the time-evolution operator U which satisfies the matrix analog of Eqs. (2) with the initial condition $U_{jl}(0)=\delta_{jl}$. By an extension[8] of Floquet's theorem,[9] a system of equations with periodic coefficients such as (2) will have a solution of the form

$$c_j(t)=P_j{}^0(t)+P_j{}^+(t)e^{ikt}+P_j{}^-(t)e^{-ikt}, \tag{3}$$

where the $P_j{}^\alpha$ ($\alpha=+,0,-$) are periodic functions of period $\tau\equiv 2\pi/\omega$. The structure of Eq. (3) indicates that some of the radiation [associated with $P_j{}^0(t)$] is exactly at the unperturbed Ly-α frequency and at a series of frequencies shifted by the harmonics of ω, while the other two terms give rise to radiation at frequencies shifted by $\pm k$ from this series. The matrix $U_{jl}(\tau)$ has the three eigenvalues $e^{ik\tau}$, 1, and $e^{-ik\tau}$ and thus the evaluation of $\mathrm{Tr}U_{jl}(\tau)=1+2\cos k\tau$ provides a means of determining k. Thus it follows that k is multibranched. The replacement of k by $\pm(k\pm n\omega)$ merely results in a corresponding relabeling of the harmonics. A particular branch, designated k_0, that has the useful property that the strength in each harmonic varies continuously with S and D is obtained using the additional restrictions that $k_0(S,D)$ and $(\partial k_0/\partial S)_D$ be continuous.

In Fig. 1, k_0 is plotted versus S for various values of D. As D approaches 0, k_0/ω approaches a periodic ramp curve of unit height and width, which in fact is the image of the unperturbed Stark-shifted static components. For small S, but arbitrary D, k_0 can be shown to have the slope $(\partial k_0/\partial S)_{S\to 0}\to J_0(D/\omega)$. The extrema of k_0 are bounded by ± 1. The height of the first (lowest) maximum for moderate values of D can also be shown to be well approximated by $1-D/2$. As S is increased, successive extremum points of k_0 approach ± 1 closer and closer. The values of S and D corresponding to these extremum points, as will be seen, are the field strengths for which the resonant behavior becomes most pronounced.

Using the value of k_0 thus obtained, the numerical solution for the $c_j(t)$ can be separated in the form of Eq. (3), and the corresponding periodic functions of ω, $P_j{}^\alpha(t)$, extracted and Fourier

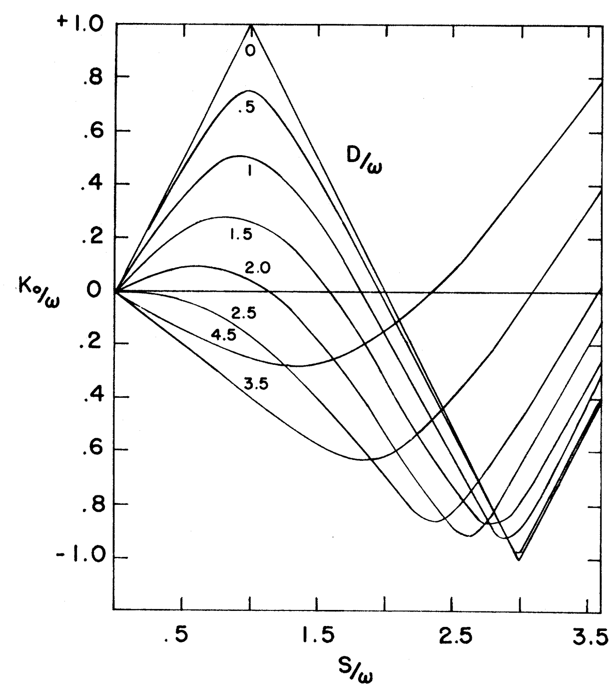

FIG. 1. k_0/ω versus S/ω for various values of D/ω.

analyzed.

Since the initial states are all equally likely, the initial density matrix ρ_{jl} is proportional to the unit matrix δ_{jl}. Therefore, at a later time $\rho_{jj}(t)$ is proportional to $\sum_l |\langle j|U(t)|l\rangle|^2$ and its Fourier transform $\rho_{jj}(\nu)$ can be expressed as

$$\rho_{jj}(\nu) \equiv \sum_{\alpha,n} I_{j,n}{}^{\alpha}\, \delta(\nu - \eta\omega - \alpha k_0), \qquad (4)$$

where now the coefficients $I_{j,n}{}^{\alpha}$ are proportional, for each frequency, to the intensity of radiation at that frequency, polarized in the direction determined by the state j.

The spectrum is such that the $I_{2,n}{}^{0}$ and $I_{3,n}{}^{\pm}$ harmonics vanish for even n, while the $I_{3,n}{}^{0}$ and $I_{2,n}{}^{\pm}$ harmonics vanish for odd n. A representative spectrum for $D/\omega = 1.5$ and variable S/ω is given in Figs. 2(a) and 2(b).

Some important features illustrated by these diagrams are the following:

(1) At $S=0$ the $|p_x\rangle$ state spectrum reduces to the Blokhintsev result, while all the intensity of the $|p_y\rangle$ state is at the unperturbed freqeuncy.

(2) As S increases from 0, the $I_{j,n}{}^{+}$ and $I_{j,n}{}^{-}$ harmonics which initially overlap and have equal intensity start to separate and all the harmonics change in relative strength. Approaching the vicinity of the $S=n\omega$ points, a resonance-type behavior occurs at points $S<n\omega$ whose positions coincide with the extrema of k_0. At these points, the behavior of the $|p_x\rangle$ state is characterized by the $I_{3,0}{}^{0}$ intensity going completely to zero with an abrupt transfer of the intensity to the $I_{3,-1}{}^{+}$ and to the $I_{3,+1}{}^{-}$ components. Also, the behavior of the $|p_z\rangle$ state is characterized by an abrupt transfer of intensity from the harmonics of $I_{2,m}{}^{+}$ and $I_{2,m}{}^{-}$ to the closest harmonic of $I_{2,m}{}^{0}$. As S increases, the frequency-jumps between the competing harmonics become very small, since the extrema of k_0 approach ± 1, and at the same time the intensity transition region becomes much narrower.

(3) For $S \gg \omega$ and D, the results reduce to the static Stark spectrum perturbed by a small dynamic field, with weak Baranger-Mozer–type satellites. Most of the intensity for both the $|p_x\rangle$ and $|p_z\rangle$ states is in the zeroth harmonic, while for the $|p_z\rangle$ state it is in the harmonic whose position, denoted by k_p, is approximated from perturbation theory by

$$k_p = S + SD^2/4(S^2 - \omega^2). \qquad (5)$$

(4) For the region where D, as well as S, is $\gg \omega$, the total field acts as the vector sum of the static and the root mean square of the dynamic field, and k_p is approximately given by

$$k_p = (S^2 + \tfrac{1}{2}D^2)^{1/2}, \quad S > D \gg \omega. \qquad (6)$$

In summary, the results of the present paper indicate that a suitable mixing of static and dynamic fields can produce a satellite structure substantially different from the one associated with either the conventional Blokhintsev theory

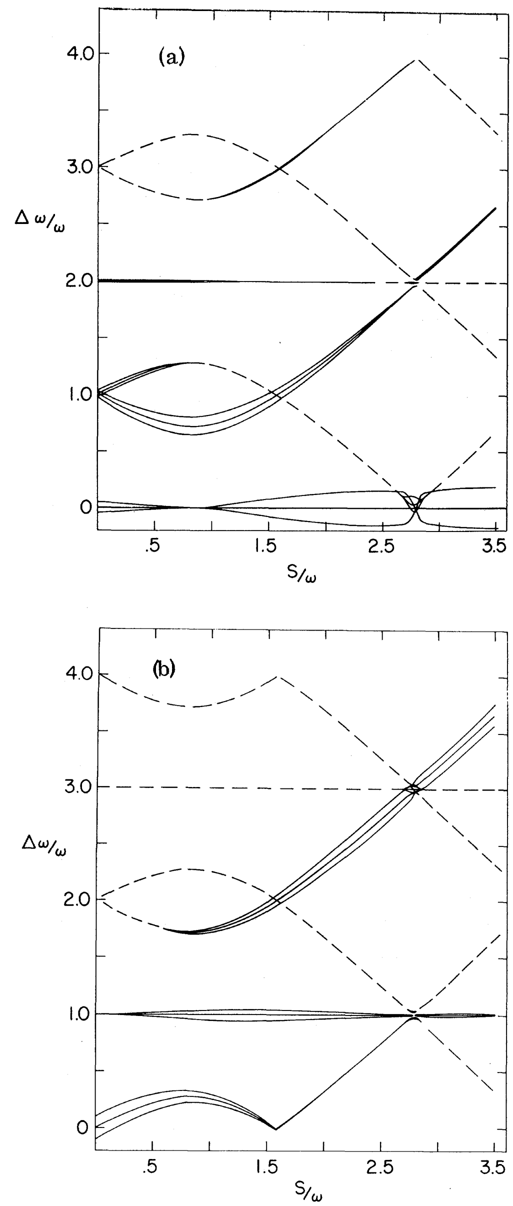

FIG. 2. Harmonic intensity and location versus S/ω for $D/\omega = 1.5$. The center of each (solid) line shows the harmonic location, the width of each is proportional to the intensity (as in Ref. 7); each part should be complemented by a mirror image below its axis. Dashed lines are for < 1% intensity. (a) $|2p_z\rangle$; (b) $|2p_x\rangle$.

or with a Baranger-Mozer–type perturbation spectrum. In particular, certain harmonics can be greatly emphasized without leading to high intensities elsewhere in the spectrum. In the recent experiments on the H_β line by Gallagher and Levine,[10] the observed satellite structure did not appear to conform to the simple Blokhintsev theory, and exhibited a much stronger second-harmonic intensity than predicted by the latter. Therefore, it seems to be warranted to seek an explanation of the Gallagher-Levine results along the lines of the present theory. In an actual plasma the fluctuating dynamic and quasistatic fields can be at arbitrary angles with respect to each other and their intensities are also distributed statistically. Quantitative comparison with experimental results, without taking into account these effects, would not be realistic. Detailed consideration of these aspects, and a more detailed theoretical analysis will be presented in forthcoming papers.

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Observation of a Broad Resonance in the 2^3S Excitation of Helium by Electron Impact*

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We have observed a broad P-wave resonance at about 50 eV primary energy in the angular distribution of electrons scattered after excitation of the 2^3S state of helium. The resonance has a width of about 15 eV and affects the differential excitation cross section by more than 3 orders of magnitude. It is suggested that this resonance is associated with the temporary formation of triply excited states of the negative helium ion.

Narrow resonances in the 2^3S excitation cross section of helium just above threshold have previously been reported by Schulz and Philbrick[1] at an angle of 72°, and by Ehrhardt and Willmann[2] at angles between 7° and 110°. These resonances were also observed in the forward scattered electron current after excitation of the 2^3S state by Chamberlain and Heideman[3] and are associated with the formation of resonant states of doubly excited He^-.

Structure in the transmitted electron current at 57.1 and 58.2 eV, reported by Kuyatt, Simpson, and Mielszarek,[4] was interpreted by Fano and Cooper[5] as associated with the temporary formation of the triply excited states of He^-, $(2s^2 2p)^2P$ and $(2s2p^2)^2D$. Simpson, Menendez, and Mielczarek[6] investigated the angular dependence of narrow resonances in the 2^3S excitation due to these states.

All of the above resonances are extremely narrow, less than 100 meV, and give rise to small changes in the excitation cross section. In the present work a resonance has been observed with a width of about 15 eV, due to which the excitation cross section varies by more than 3 orders of magnitude.

The present apparatus is described in detail elsewhere.[7] Briefly, an electron gun produced a 1–5 μA beam of electrons with full width at half-maximum of 150 meV that interacted with the static target gas which was at a pressure of 1 mTorr. Electrons ejected at an angle θ with respect to the primary beam, with energy loss corresponding to excitation of the 2^3S state of