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Absence of a Quadratic Term in the ^{4}He Excitation Spectrum*

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By directly measuring the difference in velocity of 30 and 90 MHz sound waves in liquid ^{4}He below 100 mK, we have determined that the elementary excitation spectrum cannot be described by an expression $\epsilon(k)=c\hbar k(1+\alpha_1 k+\alpha_2 k^2+\cdots)$ which has a value for α_1 that is greater than 0.01 Å.

For many years it has been assumed that the long-wavelength portion of the ^{4}He elementary excitation spectrum could be described by

$$\epsilon(k)=c\hbar k(1+\alpha_2 k^2+\cdots), \quad (1)$$

where ϵ is the energy of the excitation, k is its wave number, and c is the sound velocity; α_2 is traditionally taken to be negative. Recently, however, considerable experimental and theoretical evidence has accumulated which supports different forms for the spectrum. Phillips, Waterfield, and Hoffer[1] concluded from their measurements of the heat capacity that α_2 was *positive* at low pressures. Barucchi, Ponzano, and Regge[2] predicted that the excitation spectrum could be described by

$$\epsilon(k)=c\hbar k(1+\alpha_1 k+\alpha_2 k^2+\cdots), \quad (2)$$

and Molinari and Regge[3] suggested that an improved fit to the inelastic neutron scattering data[4] for the excitation spectrum could be achieved with this expression; α_1 was positive in the resultant fit. Molinari and Regge also showed that their expression was consistent with the heat-capacity data of Phillips, Waterfield, and Hoffer.[1] In order to explain the shoulder which appeared at high pressures in our ultrasonic attenuation data,[5] Jäckle and Kehr[6] needed to assume that the excitation spectrum initially curved *upward*. Recently, Maris[7] successfully explained the unusual frequency ordering that we found in the temperature dependence of the ultrasonic velocity.[8] This theory also used a *positive* value of α_2 determined from the heat-capacity data of Phillips, Waterfield, and Hoffer.[1]

Since the frequency-dependent sound (phase) velocity according to Eq. (2) is just $c(1+\alpha_1 k+\alpha_2 k^2+\cdots)$, then a measurement of the frequency dispersion of the velocity is a direct indication of the higher-order terms in the excitation spectrum. Anderson and Sabisky[9] recently reported just such sound-velocity measurements obtained on helium films at frequencies between 20 and 60 GHz. At their experimental temperature of 1.38 K they find a dispersion whose leading term is linear (i.e., a quadratic term in the excitation spectrum) and is of the same sign and magnitude as suggested by Molinari and Regge.[3]

In an attempt to measure the dispersion in a way that is independent of temperature effects and of possible peculiarities of He films, we have measured the dispersion in bulk ^{4}He below 100 mK to an accuracy sufficient to allow the observation of a linear term if it is as large as predicted. Our method is to send simultaneous sound waves of frequencies f and $3f$ through the helium and to measure the relative phase delay of the two received signals as the sound path length is changed. Essential to this technique are (1) the ability to obtain phase-locked rf sources of frequencies f and $3f$, and (2) the ability to continuously change the sound path length at low temperatures without changing the alignment necessary for detecting the sound signal. A block diagram of the electronics for the experiment is shown in Fig. 1. We used two oscillators working at 30 and 90 MHz, which were phase locked to each other by a Hewlett-Packard syn-

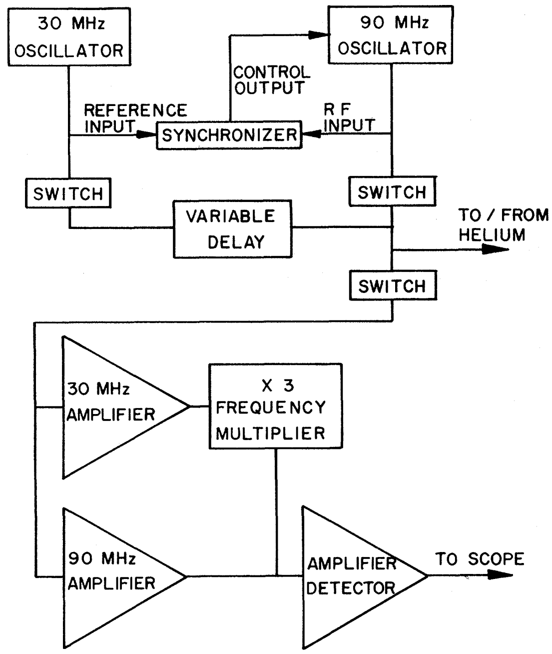

FIG. 1. Simplified block diagram of the electronics of the experiment.

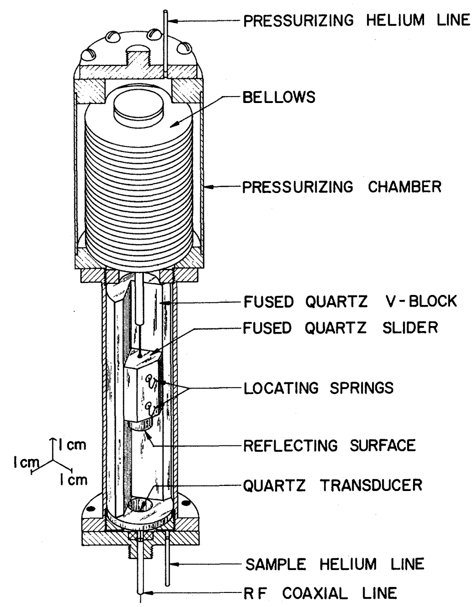

FIG. 2. Moving reflector sound cell.

chronizer.[10] The switches at the oscillator outputs gated rf pulses into the sound cell containing a 30-MHz quartz piezoelectric transducer which could also be driven at 90 MHz. The variable delay in the 30-MHz branch compensated for the difference in transit time of the two sound signals. The switch at the amplifier inputs kept the original rf pulses from disturbing the amplifiers and only passed the signals returning from the sound cell after transit through the helium. The 30-MHz signal was then frequency tripled by a nonlinear element and its phase and amplitude adjusted to exactly cancel the 90-MHz signal at the input to the final amplifier detector. Then as the sound path in the helium was changed (by moving a reflector in the helium) any difference in the velocities of the two signals would show up as a phase delay of one signal with respect to the other; the delay would be exactly proportional to the change in the sound path length. This differential delay was compensated and measured by adjusting the variable delay in the 30-MHz input branch.

Figure 2 shows the sound cell used in the experiment. The main feature of the cell is the fused-quartz V block and slider, which were lapped to optical accuracy in order to allow the reflecting surface to be moved up and down above the transducer while maintaining exact orientation. Precise alignment is required to insure that the reflected wave front is accurately parallel to the transducer. The slider is moved up and down by a shaft which is attached to the top of the metal bellows. A differential pressure of liquid helium between the inside and the outside of the bellows causes it to collapse or expand by several centimeters, thus moving the slider by this amount. The slider is lightly pressed against the V block by springs which contact the inside of the lower chamber. The cell was attached to the copper mixing chamber of a ^{3}He-^{4}He dilution refrigerator. This allowed us to perform the experiment at temperatures below 60 mK where there is essentially no temperature dependence to either the sound velocity or attenuation.

According to Molinari and Regge[3] the value of α_1 in Eq. (2) is 0.275 Å. This implies that the fractional difference in velocity between 30- and 90-MHz sound waves is 4.35×10^{-5} and that for every centimeter of travel the 90-MHz wave will gain approximately 1.8 nsec over the 30-MHz wave. Since we can change our round-trip path length by nearly 8 cm and since our resolution is about 0.1 nsec, we should have no difficulty observing the predicted effect if it exists. Figure 3 shows the results of our measurements of the differential delay versus total path length. The solid line is the expected result according to the prediction of Molinari and Regge[3] and the dashed line is the expected result from the measurements of Anderson and Sabisky.[9] Clearly, we

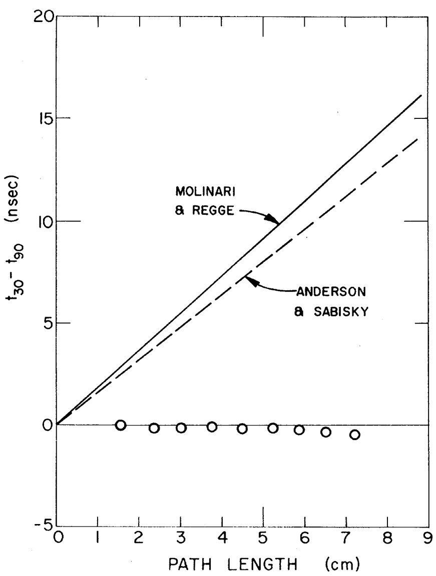

FIG. 3. Differential delay of 30- and 90-MHz sound waves versus round-trip path length (circles). Solid line, result expected for the prediction of Molinari and Regge (Ref. 3); dashed line, result according to the measurements of Anderson and Sabisky (Ref. 9).

see nothing approaching the expected magnitude for α_1. At the frequencies of our experiment any reasonable value for α_2 would lead to an effect that is completely unobservable within our resolution. The advantage of this experiment is that the only variable is the path length; as long as the amplitude of the reflected sound signal remains constant as the reflector is moved, then such undesirable effects as attenuation in the liquid, beam spreading, or tilting of the reflector must be absent. The very small amount of delay that we do occasionally observe does not tend to be reproducible or proportional to path length and is usually associated with some such amplitude effect.

The large amount of dispersion found by Anderson and Sabisky[9] is almost certainly due to the relatively high temperature at which they performed their experiment. At their temperature of 1.38 K the velocity can differ from its zero-temperature value by about 1%. It is not surprising, therefore, that the difference in the temperature dependence of the velocity for the different frequencies used by Anderson and Sabisky could account for the effect they observed.

The present experiment thus establishes that the magnitude of the coefficient of a quadratic term in the energy spectrum of ^{4}He must be less than 0.01 Å.

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Ion Heating via Turbulent Ion Acoustic Waves*

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Three-dimensional ion acoustic turbulence is excited by injecting a cold ion beam into a nonmagnetized plasma with $T_e/T_i > 5$. A rapid, approximately exponential heating of the ion beam is observed along the beam path. The heating rates level off when the ion temperature approaches $0.2T_e$. The turbulent ion acoustic wave energy density does not exceed $(1-3)\times 10^{-3}T_e$. Wave saturation, which occurs before significant dissipation of ion beam energy, is consistent with orbit-diffusion strong-turbulence theory.

Ion heating from electron-current–driven ion acoustic turbulence has been investigated in both linear and toroidal devices.[1-3] The turbulent-heating experiment reported here differs from previous experiments in that there is no magnetic field and the ion acoustic turbulence is excited by