TABLE I. Single-particle energies (in MeV); occupied and unoccupied levels are separated by a horizontal line. The experimental values are taken from Ref. 8.

	^{16}O		^{40}Ca		^{208}Pb	
	Calc	Expt	Calc	Expt	Calc	Expt
$1s$	46.75	47.00	57.93		69.04	
$1p_{3/2}$	19.52	22.00	34.78		53.16	
$1p_{1/2}$	12.54	15.70	30.7		52.43	
$1d_{5/2}$	3.68	4.20	19.57	21.30	43.54	
$2s$		3.30	16.78	18.20	36.52	
$1d_{3/2}$		+1.10	12.02	15.80	41.88	
$1f_{7/2}$			6.21	8.30	34.34	
$2p_{3/2}$			5.18	6.20	26.35	
$1f_{5/2}$					31.34	
$2p_{1/2}$			2.84	4.20	25.62	
$1g_{9/2}$					25.35	
$1g_{7/2}$					20.76	
$2d_{5/2}$					18.24	
$2d_{3/2}$					16.47	
$1h_{11/2}$					16.44	
$3s_{1/2}$					16.15	
$2f_{7/2}$					10.59	9.50
$h_{9/2}$					9.75	10.70
$3p_{3/2}$					9.77	8.20
$2f_{5/2}$					7.87	7.80
$3p_{1/2}$					8.92	7.30
$1i_{3/2}$					7.57	8.90
$2g_{9/2}$					3.75	3.94
$3d_{5/2}$					3.51	2.36
$4s_{1/2}$					3.68	1.91
$3d_{3/2}$					2.03	1.40
$2g_{7/2}$					+0.56	1.50

bound-state spectra (see Table I). The momentum-dependent potential as derived from the nonlocal potential can be considered as a kind of phenomenological G matrix which already includes the effect of nucleon correlations. The introduction of the moment-dependent term provides a simple phenomenological representation of many-body effects, and describes the way in which the independent-particle motion of a nucleon within a nucleus is influenced by the presence of other nucleons in its neighborhood.

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Limits on Angular Momentum in Heavy-Ion Compound-Nucleus Reactions

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Experimentally measured fusion excitation functions for the reactions induced on ^{27}Al with ^{16}O and on ^{107}Ag with ^{20}Ne are interpreted in terms of an equilibrium model with fission competition during de-excitation of the compound nucleus. The results of the calculation are in excellent agreement with experimental results, thereby predicting limits to angular momenta for nuclei surviving de-excitation of the compound nucleus.

Data have been published recently for the "fusion cross sections" in heavy-ion–induced compound-nucleus reactions.[1,2] "Fusion" or "complete fusion" cross sections have been defined experimentally as those which involve products that have masses consistent with the formation of a compound nucleus, followed by de-excitation via the emission of some number of nucleons or light clusters by an evaporation mechanism.

One very interesting observation from this work is that the ratio of fusion cross section ($\sigma_{\rm fus}$) to reaction cross section ($\sigma_{\rm reac}$) decreased from nearly unity near the Coulomb barrier to a fraction at bombarding energies several times the barrier energy. Such an effect was predicted qualitatively by Kalinkin[3] in a model for the fusion reaction involving arbitrary shapes for the fusing system. Other suggestions have also been made, e.g., that the limit is determined by the unavailability of higher spin states in the compound nucleus, or by the predicted disappearance of the fission barrier at higher angular momenta.[4] Swiatecki has emphasized[5] that even before the total disappearance of the barrier at a sufficiently high angular momentum (J), significant fission competition with particle emission during the de-excitation of the compound nucleus is expected to take place at lower values of J. It is this fission competition in an equilibrium approach which we have investigated in this work. Results of our calculations will be compared with some of the other assumptions listed above. This general area of investigation has been cited as one of the prime reasons for building new heavy-ion accelerator facilities; it is therefore important that possible reaction models be fully explored.[6]

It will be assumed that for all impact parameters which contribute to the total reaction cross section in any given heavy-ion reaction, a compound nucleus is formed, and that fission may compete with particle emission in the de-excitation process. The fission barrier is a function of angular momentum (and hence of the impact parameter) and was obtained from the liquid-drop model as described below. The assumption of compound-nucleus formation at all impact parameters is required for the consideration of fission and particle emission competition in the equilibrium model. We wish, however, to point to the following qualification: In the case of a large impact parameter when the fission barrier is small, the system may not actually form a compound nucleus. To determine whether or not a compound nucleus is formed in such a case would probably require a more sophisticated dynamical calculation. In this work, when the fission barrier is small, the nucleus de-excites by fission and thus, whether or not a compound nucleus is formed, it does not in either case contribute to $\sigma_{\rm fus}$. As the fission barrier tends to zero, therefore, our assumption of compound-nucleus formation may not be valid, but calculated $\sigma_{\rm fus}/\sigma_{\rm reac}$ results remain unaffected.

The calculations presented here are based on the calculations of Plasil and Swiatecki[7] for uniformly charged rotating drops in which the disruptive rotational and Coulomb energies are counteracted by the cohesive nuclear binding energy, approximated by surface tension. These liquid-drop calculations are static (or more precisely gyrostatic) and give ground-state and saddle-point equilibrium shapes. From Ref. 7 we have obtained values of $E_{\rm rot}(J)$, the minimum energy of a rotating drop with angular momentum J at equilibrium deformation [$E_{\rm rot}(J)$ is the liquid-drop yrast energy], and of $E_{\rm sp}(J)$, the energy of the saddle-point shape. The fission barrier of the drop, $B_{\rm fis}(J)$, is the difference between $E_{\rm sp}(J)$ and $E_{\rm rot}(J)$. In relating liquid drops to nuclei, the nuclear constants of Myers and Swiatecki were used.[8] These rotating ground-state and saddle-point energies were then used with the Bohr-Wheeler[9,10] expression for neutron and fission widths modified in the following ways:

(1) Proton and α-particle emission widths were also included in calculating the fission to total width ratios, $\Gamma_{\rm fis}/\Gamma_{\rm tot}$, by substituting the appropriate inverse cross sections, binding energies, etc., into the Bohr-Wheeler expression.

(2) The calculation was performed for each partial wave populating the compound nucleus. Partial-wave cross sections for the incident heavy ions were computed using the parabolic-potential approximation due to Thomas[11] with parameters of Ref. 11.

(3) Calculations were performed for multiple n, p, and α emission over all paths appropriately weighted over spectra of residual excitations. These evaporation calculations were governed by the Weisskopf-Ewing formula,[12] using the s-wave approximation, as described below. The calculations were performed using a modified version of a computer program previously described.[13]

(4) Lacking experimental information on the ratio of average single-particle level densities for particle emission to those for fission in the mass region considered in this work, a value of 1 was assumed, i.e., $a_{\rm fis} = a_{n,p,\alpha} = A/8$, where A is the nucleus mass number.

The resulting expression for $\Gamma_{\rm fis}/\Gamma_{\rm tot}$ at angular momentum J may be represented as

$$\frac{\Gamma_{\rm fis}(J)}{\Gamma_{\rm tot}(J)} = N\Big[N + 2\sum_{\nu=n,p,\alpha}\int_0^{E'} g_\nu \sigma_\nu(\epsilon)\mu\epsilon \times \rho(E - E_{\rm rot}(J) - B_\nu - \epsilon)\, d\epsilon\Big]^{-1}, \quad (1)$$

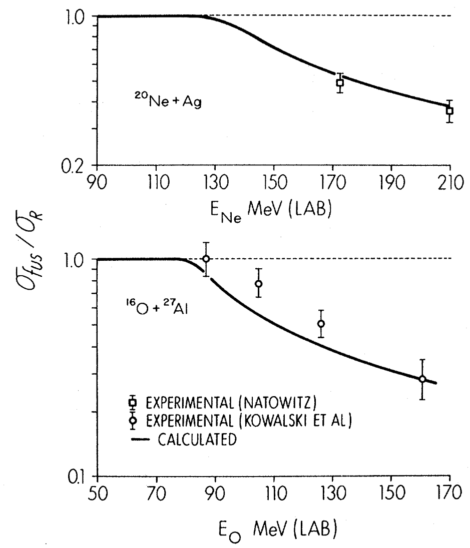

FIG. 1. Experimental and calculated fusion excitation functions. The ordinate is the ratio of fusion cross section (as defined in the text) to total reaction cross section (as calculated with the parabolic approximation of Ref. 11). Experimental results of Refs. 1 and 2 are shown as points with error bars. Calculated results are shown as solid lines. The ratio of 1.0 on the ordinate (complete fusion limit) has been extended by a dashed line as a visual guide.

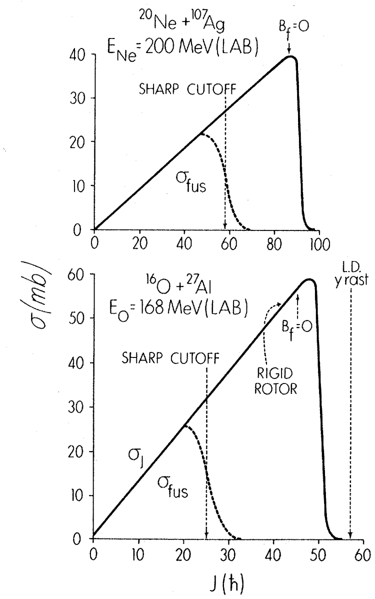

FIG. 2. Calculated cross section surviving fission as a function of angular momentum for two systems. The solid curve represents the partial reaction cross sections versus angular momentum. The dashed curve represents the cutoff imposed by fission as given by the calculations of this work. Also indicated are the sharp-cutoff J value (which gives the same fusion cross section as the calculated result), and the J values for which the fission barrier becomes zero, for which the rotational energy of a rigid rotor equals the excitation energy, and for which the rotational energy of a rotating drop at equilibrium deformation equals the excitation energy. The latter two values are not shown for the ^{20}Ne + ^{107}Ag system since both values are at J values higher than those populated in the reaction.

where $E' = E - E_{rot}(J) - B_\nu$ and

$$N = \pi\hbar^2 \int_0^{E''} \rho(E - E_{sp}(J) - \epsilon)\, d\epsilon ,$$

with $E'' = E - E_{sp}(J)$. In (1), E is the compound-nucleus excitation energy, ρ represents the level density at the appropriate excitation, ϵ the channel energy, μ the reduced mass, g_γ the statistical factor for particle type γ, $\sigma_\gamma(\epsilon)$ the inverse-reaction cross section, and B_γ the binding energy of the evaporated particle. The integrals over particle emission were replaced by a sum over a 1-MeV mesh size in the computer code used; the fission width integrals were replaced with sums with a 100-keV mesh size.

Calculations were performed for the systems ^{16}O + ^{27}Al and ^{20}Ne + ^{107}Ag, for which fusion cross sections have been measured using solid-state track detectors.[1,2] Results are shown in Fig. 1 for the fusion excitation functions. The experimental and calculated cross sections have both been divided by the calculated total reaction cross sections of this work and are shown as fraction of total reaction cross section versus bombarding energy. Results seem to be in agreement nearly to within experimental uncertainties for these systems.

The predicted partial-wave cross sections surviving fission are shown as a function of angular momentum J in Fig. 2 for 168-MeV ^{16}O + ^{27}Al and for 200-MeV ^{20}Ne + ^{107}Ag. The initial distribution of partial-wave cross sections is indicated as well. The angular momentum at which the fission barrier goes to zero is indicated; it may be seen that the fission competition during de-excitation of the compound nucleus places a much lower limit on the maximum angular momentum surviving de-excitation than the value at which the barrier disappears. The J value for which the rotational energy of a rigid sphere equals the total excitation energy is also shown in Fig. 2. This value of J is one of the suggested limits to the maximum angular momentum for formation of a compound nucleus. Also indicated is the J value for which a rotating drop with equilibrium deformation has rotational energy equal to the excitation energy. This more realistic limit on J lies higher than that deduced from a rigid sphere. However, as indicated by the results of this work, a much lower limit is obtained from the J-dependent fission competition in the de-excitation process. We have also indicated the value of the angular momentum J in the sharp-cutoff approximation which gives the same fusion cross section as that given by the calculation described in this work. It may be seen that the maximum calculatted J values exceed the sharp-cutoff limit by $(5-8)\hbar$.

Several observations can be made from the results of this work. Detailed supporting evidence for our conclusions will be given in a later paper. The main conclusions are as follows:

(A) The experimentally observed limit on the fusion cross sections for the heavy-ion bombardments studied here has at least a consistent interpretation in terms of angular-momentum–enhanced fission competition during the de-excitation process.

(B) This fission competition has significant contributions from many nuclides in the de-excitation cascade, not just from the compound nucleus on a "first-chance fission" basis.

(C) Because of binding-energy considerations, proton and α emission can have greater widths than neutron emission in these reactions and must therefore be considered in the de-excitation cascade in competition with fission.

(D) Maximum angular-momentum values in excess of the sharp-cutoff values are predicted in these calculations, but these values are far less than those for which the fission barrier goes to zero or for which no states are thought to exist in the compound nucleus. At lower bombarding energies the fission cross sections are predicted to go to zero, consistent with experimental observations that $\sigma_{\rm fus}/\sigma_{\rm reac}$ goes to 1 at lower bombarding energy.

(E) For a given target-projectile system, the maximum angular momentum surviving de-excitation was found to increase with increasing bombarding energy to some maximum value, but then was found to decrease slowly as the projectile energy is further increased.

(F) For a given compound nucleus, the maximum angular momentum surviving de-excitation will result from the target projectile system giving the minimum excitation energy at a given J, as given by the energy dependence of Eq. (1).

It remains to be seen if our interpretation of the observed behavior of $\sigma_{\rm fus}/\sigma_{\rm reac}$ with increasing heavy-ion bomarding energy is the correct one. Nonetheless, it seems to give very good agreement with experimental results with no parameter adjustment. An equilibrium model should be the simplest one with which to attempt an interpretation of complete fusion phenomena, and we conclude that this model should be investigated further since it suggests that equilibrium potential-energy surfaces give valuable estimates even in the absence of dynamical calculations.

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