[8]The experimental value of $P_K(E_\beta{}^0)$ at the highest energy of $E_\beta{}^0$ is considerably lower than the calculated curve. It is believed that this deviation was caused by the experimental difficulties inherent in our measurements in this energy region; the observed x-ray peak in coincidence with electrons was too small to be estimated accurately.

[9]E. L. Feinberg, Yad. Fiz. 1, 612 (1965) [Sov. J. Nucl. Phys. 1, 438 (1965)].

[10]R. M. Weiner, Phys. Rev. 144, 127 (1966).

Nuclear Level Sequence of Single-Particle Energies in a Momentum-Dependent Potential

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Single-particle energy calculations have been made for nuclei from boron through beyond lead, using a form of momentum-dependent nucleon-nucleus potential. The results are in good agreement with the observed data for energies of single-particle levels.

The motivation for introducing momentum-dependent potentials originally stems from the desirability of representing the short-range repulsive core by a nonsingular potential.[1] From studies based on meson theory, it has been shown that the nucleon-nucleon interaction depends significantly on the relative momentum of the particles involved. Many studies employing momentum-dependent potentials have appeared in the literature.[2-5]

Many investigations over the past few years with nucleon-transfer and knockout reactions have yielded information on the energies of single-particle states. Generally they are fragmented by the residual interactions, but it is still possible to define their energies as the centers of gravity of the fragments, weighted by the spectroscopic factors.

The main purpose of these studies is to provide single-particle energy levels in a momentum-dependent potential for nuclear states covering nearly the entire range of nuclei, all the way from light to heavy nuclei, although schemes producing nuclear systematics are frequently restricted to light, medium, or heavy nuclei.

Under the influence of a nonlocal potential, the energy of a particle at a point $\vec{r}$ depends not only on the wave function at $\vec{r}$ but also on the wave function at other points $\vec{r}\,'$ near $\vec{r}$. Thus the generalized Schrödinger equation can be written as[6]

$$-(\hbar^2/2m)\nabla^2\Psi(\vec{r}) + \int V(\vec{r},\vec{r}\,')\Psi(\vec{r}\,')\,d^3r' = E\Psi(\vec{r}), \tag{1}$$

where m is the reduced mass of the system. The nonlocal potential $V(\vec{r},\vec{r})$ is separated in the variables $\frac{1}{2}(\vec{r}\,'+\vec{r}\,')$ and $(\vec{r}-\vec{r}\,')$, and is given by[7]

$$V(\vec{r},\vec{r}\,') = V(\tfrac{1}{2}(\vec{r}+\vec{r}\,'))\,\delta_b(\vec{r}-\vec{r}\,') + V_r(\tfrac{1}{2}(\vec{r}+\vec{r}\,'))\,\delta(\vec{r}-\vec{r}\,'), \tag{2}$$

where $\delta_b(\vec{r}-\vec{r}\,')$ is a sharply peaked even function of its argument, with range b. The nonlocal potential is transformed into a momentum-dependent form in the effective-mass approximation so that the integral in (1) is given by

$$\int V(\vec{r},\vec{r}\,')\Psi(\vec{r}\,')\,d^3r' = V(\vec{r})\Psi(\vec{r}) + \tfrac{1}{16}b^2\{4V(\vec{r})\nabla^2\Psi(\vec{r}) + 4[\nabla V(\vec{r})]\cdot[\nabla\Psi(\vec{r})] + \Psi(\vec{r})\nabla^2 V(\vec{r}) + V_r(\vec{r})\}\Psi(\vec{r}). \tag{3}$$

In the central portion of the nucleus, where $V(\vec{r})$ is presumably constant, the effect is simply the introduction of effective mass. However, in the surface region, where $V(\vec{r})$ varies, the effect is more complicated; but, for practical purposes, simple potentials can be adjusted to give reasonable single-particle functions. $V_r(\vec{r})$ is the residual potential which may contain several terms characterizing various effects, and b is the range parameter for the momentum-dependent part. A potential of this form can be understood as reflecting the correlations existing in nuclear matter, whereby the presence of a particle at position $\vec{r}$ influences the probability of finding another nucleon at a point $\vec{r}\,'$ in the neighborhood of $\vec{r}$. This in turn affects the energy of the particle at r and leads to a potential energy of the form of the integral in Eq. (1). In these calculations the local static potential and the residual potential

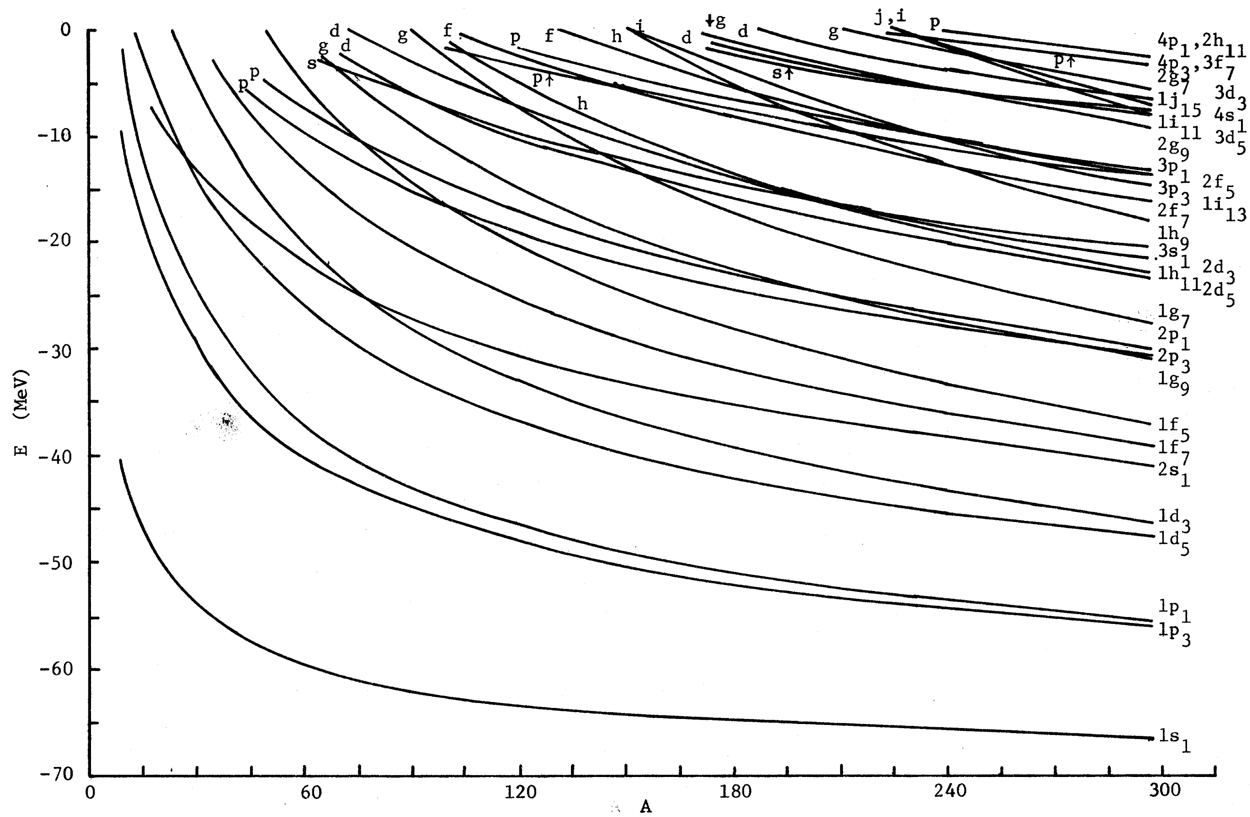

FIG. 1. Single-particle energies for neutrons. The subscript to l values refers to $2j$ values.

are taken to be spherically symmetric so that they can be written as

$$V(r) = V_0/(1+e^{(r-R)/d}), \tag{4}$$

$$V_r(r) = a_{\mathrm{s.o.}}^2(\vec{l}\cdot\vec{s})\frac{1}{r}\frac{\partial V}{\partial r} + a_{\mathrm{sym}}\frac{N-Z}{A}V(r), \tag{5}$$

respectively. The parameters used are as follows: $V_0 = 70$ MeV; $R = r_0A^{1/3}$, with $r_0 = 1.2$ fm; $d = 0.6485$ fm; $a_{\mathrm{s.o.}}^2 = 0.6667$ fm^2; $a_{\mathrm{sym}} = 0.3143$ fm; and $b^2 = 0.6667$ fm^2, the range of nonlocal interaction. Since we are presently concerned with single-particle energies for neutrons only, the Coulomb effect has been ignored in Eq. (4).

In order to generate an effective potential from Eq. (2), the value of E is needed. An initial value of E is used, either from experiment or by guess, to generate the effective potential and thus the eigenvalue, which is characterized as the single-particle energy for a given state in a given nucleus. The procedure is repeated with the eigenvalue thus calculated. The iteration continues until the eigenvalue converges within an error of 10% or 0.4 MeV. In this way the method is self-consistent within the allowed error. Since the errors in observed separation energies are not less than 1 MeV, an accuracy of a few tenths of an MeV is as stringent as necessary for calculated eigenvalues.

Since for the very light nuclei the mode of coupling and internal dynamics are important, they should be dealt with individually. However, we started our calculations with boron and carried them through beyond lead. The systematic variations of the single-particle energies of neutrons are illustrated in Fig. 1. Each value of A in this figure corresponds to a definite value of $Z(N)$, generally that for the most abundant isotopes. The nuclear systematics of various isotopes, isotones, isobars, or exotones (nuclei with constant neutron excess) cannot be illustrated on such a graph. The most apparent observation is the distinctiveness of the single-particle states and the increase in binding energies as A increases, and tends to level off for deeply bound states in heavy nuclei, thus exhibiting the saturation property of nuclear forces. The momentum-dependent potential with a Woods-Saxon–type form factor used for these calculations gives a rather good description of the overall patterns observed in the

TABLE I. Single-particle energies (in MeV); occupied and unoccupied levels are separated by a horizontal line. The experimental values are taken from Ref. 8.

	^{16}O		^{40}Ca		^{208}Pb	
	Calc	Expt	Calc	Expt	Calc	Expt
$1s$	46.75	47.00	57.93		69.04	
$1p_{3/2}$	19.52	22.00	34.78		53.16	
$1p_{1/2}$	12.54	15.70	30.7		52.43	
$1d_{5/2}$	3.68	4.20	19.57	21.30	43.54	
$2s$		3.30	16.78	18.20	36.52	
$1d_{3/2}$		+1.10	12.02	15.80	41.88	
$1f_{7/2}$			6.21	8.30	34.34	
$2p_{3/2}$			5.18	6.20	26.35	
$1f_{5/2}$					31.34	
$2p_{1/2}$			2.84	4.20	25.62	
$1g_{9/2}$					25.35	
$1g_{7/2}$					20.76	
$2d_{5/2}$					18.24	
$2d_{3/2}$					16.47	
$1h_{11/2}$					16.44	
$3s_{1/2}$					16.15	
$2f_{7/2}$					10.59	9.50
$h_{9/2}$					9.75	10.70
$3p_{3/2}$					9.77	8.20
$2f_{5/2}$					7.87	7.80
$3p_{1/2}$					8.92	7.30
$1i_{3/2}$					7.57	8.90
$2g_{9/2}$					3.75	3.94
$3d_{5/2}$					3.51	2.36
$4s_{1/2}$					3.68	1.91
$3d_{3/2}$					2.03	1.40
$2g_{7/2}$					+0.56	1.50

bound-state spectra (see Table I). The momentum-dependent potential as derived from the nonlocal potential can be considered as a kind of phenomenological G matrix which already includes the effect of nucleon correlations. The introduction of the moment-dependent term provides a simple phenomenological representation of many-body effects, and describes the way in which the independent-particle motion of a nucleon within a nucleus is influenced by the presence of other nucleons in its neighborhood.

[1]C. W. Nester, Jr., L. T. R. Davies, S. J. Krieger, and M. Baranger, Nucl. Phys. A113, 14 (1968).
[2]M. H. Johnson and E. Teller, Phys. Rev. 98, 783 (1955).
[3]H. Duerr, Phys. Rev. 103, 469 (1956), and 109, 117 (1958).
[4]Y. C. Tang, R. H. Lemmer, P. J. Wyatt, and A. E. S. Green, Phy. Rev. 116, 402 (1959); A. E. S. Green, G. Dareych, and R. Berezdivin, Phys. Rev. 157, 929 (1967).
[5]M. A. K. Lodhi, Phys. Rev. C 1, 1895 (1970).
[6]M. A. Preston, *Physics of the Nucleus* (Addison-Wesley, Reading, Mass., 1963), p. 192.
[7]W. E. Frahn and R. H. Lemmer, Nuovo Cimento 5, 1564 (1957), and 6, 664 (1957).
[8]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), pp. 325, 328.

Limits on Angular Momentum in Heavy-Ion Compound-Nucleus Reactions

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Experimentally measured fusion excitation functions for the reactions induced on ^{27}Al with ^{16}O and on ^{107}Ag with ^{20}Ne are interpreted in terms of an equilibrium model with fission competition during de-excitation of the compound nucleus. The results of the calculation are in excellent agreement with experimental results, thereby predicting limits to angular momenta for nuclei surviving de-excitation of the compound nucleus.

Data have been published recently for the "fusion cross sections" in heavy-ion–induced compound-nucleus reactions.[1,2] "Fusion" or "complete fusion" cross sections have been defined experimentally as those which involve products that have masses consistent with the formation of a compound nucleus, followed by de-excitation via the emission of some number of nucleons or light