ward means of normalization. We calculated the $K_L \to \gamma\gamma$ branching ratio by using the unlikely normalization to our 120 $K_L \to \pi^0\pi^0$ events. The result is

$$B(K_L \to \gamma\gamma) = (4.32 \pm 0.55) \times 10^{-4} \times |\eta_{00}/\eta_{+-}|^2 .$$

We have explicitly removed η_{00}/η_{+-} in order to separate the error due to the uncertainty in $B(K_L \to \pi^0\pi^0)$. With our measurement,[4] $|\eta_{00}/\eta_{+-}|^2 = 1.05 \pm 0.14$, we find $B(K_L \to \gamma\gamma) = (4.54 \pm 0.84) \times 10^{-4}$. The previous world average for $B(K_L \to \gamma\gamma)$ was $(4.91 \pm 0.37) \times 10^{-4}$.[6,9,10] Complete details on both the $K_L \to \gamma\gamma$ and $K_S \to \gamma\gamma$ results will be presented in a forthcoming article.

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K^--Nucleus Scattering Lengths

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We extract K^--nucleus scattering lengths from all K^--mesonic atom data currently available. The scattering lengths obtained are shown to be model independent as long as the K^--nucleus optical potential is local.

Using recently reported x-ray transitions in K^--mesonic atoms,[1-3] there have been made some phenomenological studies[3-6] of the K^--nucleus optical potential. The results of the studies may be summarized as follows: (1) The potential appears to be local and proportional to the nuclear density.[3] (2) Because of a large uncertainty in the strength parameter (ξ) in the potential, finding detailed structure in the nuclear density is very difficult.[5] Of technical importance, perturbation theory is found to be inadequate because of the large absorption effect.[4,5]

In this Letter we report K^--nucleus scattering lengths extracted from all K^--mesonic atom data currently available. In contrast to the K^--nucleus *potential*, the scattering length will be demonstrated to be practically free from the uncertainty mentioned in (2) above.

We first determine the best-fit ξ for the observed energy level and spread ΔE by numerically integrating the Klein-Gordon equation for Coulomb and strong-interaction potentials. The lat-

TABLE I. Measured level shifts and half-widths ΔE, best-fit potential parameters ξ, and calculated scattering lengths a. a depends negligibly upon the nuclear density parameters (third column), as discussed in the text. The measured values are from Ref. 3, except those noted otherwise.

	Transition	(α) or (c,t) and the rms radius (F)	E (keV)	ξ (F)	a (F)
B^{10}	3d → 2p	(1) 2.32	0.208 ± 0.35 -i(0.405 ± 0.05)	- 1.14 + 0.18 - 0.61 -i(0.73 + 0.13 - 0.14)	2.96 + 0.04 - 0.25 -i(1.15 + 0.00 - 0.06)
B^{11}	3d → 2p	(1) 2.28	0.167 ± 0.035 -i(0.350 ± 0.04)	- 0.49 + 0.80 - 0.05 -i(0.50 + 0.19 - 0.01)	2.36 + 0.45 - 0.01 -i(1.07 + 0.08 - 0.03)
C^{12}	3d → 2p	(4/3) 2.34	0.59 ± 0.08	- 1.10 + 0.17 - 0.26 -i(0.70 + 0.21 - 0.11)	2.88 + 0.09 - 0.18 -i(1.18 + 0.02 - 0.08)
P^{31}	4f → 3p	(2.75, 2.45) 2.96	0.33 ± 0.08 -i(0.72 ± 0.06)	-0.80 ± 0.22 -i(1.33 ± 0.29)	3.89 ± 0.07 -i(1.21 ± 0.08)
S^{32}	4f → 3p	(2.68, 2.60) 3.02	0.55 ± 0.06 -i(1.165 ± 0.03)	-0.74 ± 0.07 -i(1.32 ± 0.13)	3.97 ± 0.04 -i(1.27 ± 0.04)
			0.37 ± 0.25 -i(1.10 ± 0.30)[a]	-0.69 ± 0.52 -i(0.94 ± 0.43)	3.83 + 0.18 - 0.21 -i(1.34 + 0.20 - 0.18)
Cl^{35}	4f → 3p	(3.07, 2.60) 3.24	0.77 ± 0.40 -i(1.9 ± 0.5)	-0.70 ± 0.52 -i(0.86 ± 0.39)	4.06 + 0.16 - 0.15 -i(1.38 + 0.18 - 0.19)
			0.94 ± 0.40 -i(1.96 ± 0.50)[b]	- 0.76 + 0.86 - 0.59 -i(1.06 + 0.44 - 0.52)	4.14 + 0.14 - 0.19 -i(1.34 + 0.12 - 0.19)

[a]From Ref. 1.

[b]From Ref. 2.

ter is written as a local optical potential,

$$V(r) = (4\pi\hbar^2/2\mu)(1+m_K/m_N)\bar{\xi}\rho(r). \quad (1)$$

$\rho(r)$ is the nuclear density function, normalized to the nuclear mass number, of the form

$$\rho(r) = \rho_0\left(1+\frac{\kappa\alpha r^2}{r_0^2}\right)\exp\left(\frac{-\kappa r^2}{r_0^2}\right) \text{ for } Z<8,$$
$$\rho(r) = \rho_0\left[1+\exp\left(\frac{r-c}{t}\,4\ln 3\right)\right]^{-1} \text{ otherwise,} \quad (2)$$

where $\kappa = 3(2+5\alpha)/2(2+3\alpha)$ and r_0 is the rms nuclear radius. The density parameters c and r_0 are determined so that the square of the rms nuclear radius is less than that of the rms charge radius[7] by (0.8 F)² = (the nucleon rms charge radius)². The values of t and α are taken to be the same as those of the charge distributions[7] because these parameters for charge and matter distributions are expected to differ little. The detailed description of the optical potential and of determination of ξ are given elsewhere.[5]

The result is shown in Table I. The errors in ξ, coming from the experimental errors in ΔE, are sometimes asymmetric, reflecting a nonlinear relation between ξ and ΔE.[4,5,8] It may be of some interest to examine the isospin dependence of ξ: We find that the best fit is for

$$\mathrm{Re}\,\xi = -(0.79\pm0.06) + (2.2\pm3.7)(N-Z)/A \text{ F} \quad (\chi^2 = 5.4),$$
$$\mathrm{Im}\,\xi = -(0.99\pm0.13) + (5.1\pm2.3)(N-Z)/A \text{ F} \quad (\chi^2 = 16.5), \quad (3a)$$

by assigning the averaged error to each ξ and

$$\mathrm{Re}\,\xi = -(0.78\pm0.05) - (0.71\pm2.92)(N-Z)/A \text{ F} \quad (\chi^2 = 3.3),$$
$$\mathrm{Im}\,\xi = -(0.99\pm0.12) + (4.2\pm2.2)(N-Z)/A \text{ F} \quad (\chi^2 = 15.0), \quad (3b)$$

by shifting each central value of ξ so that it falls in the middle of the error. Both results show that the isospin shift in Reξ is not recognizable because of the large uncertainty and that, because of the bad fitting, the isospin shift in Imξ is statistically not acceptable. The magnitude of our ξ is larger than that in Ref. 6. This is because the nuclear charge distribution is directly used for $\rho(r)$ in Ref. 6.

Using ξ thus obtained, we then calculate the S-wave K^--nucleus scattering lengths a. The Schrödinger equation without the Coulomb potential is numerically solved at the threshold. The result is listed in the last column of Table I. The nature of a thus calculated is displayed probably best in the best-fit nuclear mass-number (A) dependence (in fermis):

$$a = (1.33 \pm 0.01)A^{0.316\pm0.002} \quad (\chi^2 = 7.26), \qquad (4a)$$
$$= -i(0.907 \pm 0.015)A^{0.097\pm0.006} \quad (\chi^2 = 3.49),$$

by assigning the average error to each a, and

$$a = (1.29 \pm 0.01)A^{0.323\pm0.002} \quad (\chi^2 = 2.86), \qquad (4b)$$
$$= -i(0.850 \pm 0.014)A^{0.117\pm0.006} \quad (\chi^2 = 1.56),$$

by shifting each central value of a so that it falls in the middle of the error. The approximate $A^{1/3}$ dependence in Rea shows the strongly absorptive nature of the K^--nucleus interaction, which corresponds to the hard-core–type behavior as discussed in Ref. 5. We emphasize that the positive Rea and negative Reξ do not imply an existence of a bound state by the attractive K^--nucleus interaction. In contrast to Rea, a simple picture is hard to apply to Ima which has a small magnitude, slowly varying over A. The limiting case $A = 1$ in Eq. (4) would be of some interest: It may be compared with the average of the known K^--n and K^--p scattering lengths $0.50 - i\,0.59$ F.[9] The significant difference between them shows that the K^--nucleon and K^--nucleus interactions near the threshold are dominated by different physical mechanisms. In particular, the role of the $\Lambda(1405)$ would be quite different in two cases, as has been discussed.[10] This significant difference also shows us a wild deviation from any prediction using partial conservation of axial-vector currents for the K^--nucleus scattering length, in contrast to the case of the π-nucleus interaction.[11]

The scattering lengths obtained would be of little significance if they should depend strongly on the potential parameters. In the following we show that the scattering lengths are practically independent of the choice of these values, using a model calculation in the K^--S^{32} atom.

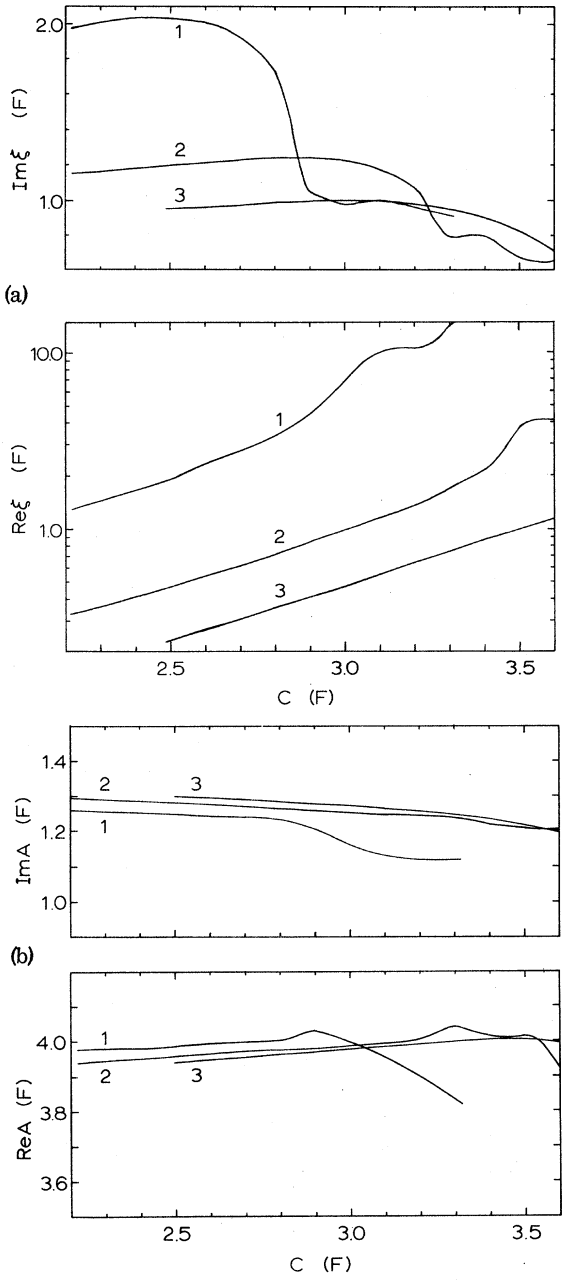

FIG. 1. Model calculation in the K^--S^{32} atom. The variations of the potential strength ξ and of the S-wave scattering length a are shown in (a) and (b), respectively, as functions of the nuclear half-density c. Curves 1, 2, and 3 correspond to rms nuclear radii of 2.76, 3.07, and 3.22 F, respectively.

We start with finding three groups of (c,t) values to yield rms radii 3.07 F, 3.22 F (= 3.07 $\times$1.05), and 2.76 F (= 3.07$\times$0.90). The value of c is varied systematically from 3.2 to 3.6 F. This

variation causes decreases of the t values from about 3.0 F to about 1.2 F, the precise values of which depend on the rms radii. Next, we calculate ξ numerically for each group of (c,t) values in such a way that each potential produces the same measured ΔE value, $0.55-i1.165$ keV. The result is displayed in Fig. 1(a). We observe large variations, 0.2–15 F in $-\mathrm{Re}\xi$ and 0.6–2.0 F in $-\mathrm{Im}\xi$. In the figure we also observe somewhat oscillatory variations. This seems to correspond to oscillations observed previously in the ΔE-ξ relation.[4,5,8] Using these ξ's, we then calculate, again numerically, the S-wave scattering lengths a. The result is shown in Fig. 1(b). We see that the variation in a is at most 0.2 F, and practically less than 0.1 F. Comparing with the very large variation in ξ, we conclude that the value of a is determined almost uniquely from the observed ΔE, independent of the potential shape.

We emphasize that in this analysis an extrapolation is made to determine the S-wave scattering lengths from ΔE of the higher-angular-momentum states. If the optical potential should be a nonlocal one, the extrapolation would not be valid. Recently, it has been demonstrated that the potential has little nonlocality and little $\rho^2(r)$ dependence.[3] We can safely conclude that a in Table I is rather reliable, particularly in view of the fact that the optical potential plays simply an intermediary role from ΔE to a in our analysis.

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