rise to the apparent "α-particle removal" seen here is quite sensitive to such effects as (a) the height of the Coulomb barrier, (b) the extent of a possible neutron tail, which may extend significantly further beyond the protons in Br and Ag than in the Ni region, and (c) the binding energy per nucleon, which has its maximum around $A \approx 56$.

Very little other experimental information on nuclear γ rays from exotic atoms is available for comparison with the present results. Poelz *et al.*[6] reported some nuclear γ rays following pion capture, but since their spectrometer covered only a limited range of γ-ray energies (100–650 keV), many lines would have been missed—in particular, those in even-even nuclei. Recently published work by Wiegand, Gallup, and Godfrey[7] on γ rays from kaon capture by S and Cl is even more restricted in the energy interval observed. For the Cl target, they report a weak 78-keV line (~5% per stopped kaon) which they assign to a transition in ^{32}P; and for the S target an even weaker (1%) line at 197 keV was attributed to ^{19}F.

To summarize, the nuclear γ-ray spectra following K^- capture in Ni and Cu show a pattern that implies that the removal of one, two, or three α particles is the favored mode. This conclusion is not inconsistent with any previous work on the interaction between stopped kaons and complex nuclei. However, far more than the present fragmentary data will be required before a mechanism can be definitely established. We would like to thank the ZGS staff for their help and assistance in the course of this experiment.

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Magnetic Moments of ^{3}He and ^{3}H†

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The electromagnetic structure of a nucleon in a nucleus is different from that of a free nucleon. This difference is examined for the two-pion part of the form factor by considering interactions with other nucleons in the course of pion emission and absorption, but without exchanging charge. For ^{3}He and ^{3}H, the isovector part of the magnetic moment increases by an amount which is comparable to the outstanding discrepancy between the experimental and theoretical values.

The purpose of this note is to show that the magnetic moment of an individual nucleon in the trinucleon systems ^{3}He and ^{3}H is modified by a mechanism different from the conventional exchange-current effect, to the extent that the bulk of the outstanding discrepancy between the theoretical and experimental values of the magnetic moments of these nuclei can be explained.

To begin with, let us summarize the background of the problem. The static magnetic moments of ^{3}He and ^{3}H are known very accurately. For a theoretical analysis it is convenient to introduce the isovector and isoscalar parts of the magnetic moment, defined by

$$\mu_v = \tfrac{1}{2}[\mu(^3\mathrm{He}) - \mu(^3\mathrm{H})], \tag{1}$$

$$\mu_s = \tfrac{1}{2}[\mu(^3\mathrm{He}) + \mu(^3\mathrm{H})]. \tag{2}$$

TABLE I. Expectation values of the one-body moment operators and their deviation from corresponding experimental values. The experimental values are $\mu_v^{\text{exp}}=2.553$ nm and $\mu_s^{\text{exp}}=0.426$ nm. Cases A and B are for the Hamada-Johnston potential (Ref. 1) and Reid's soft-core potential (Ref. 3), respectively.

Case	P_S (%)	$P_{S'}$ (%)	P_D (%)	$\mu_v^{(1)}$ (nm)	$\mu_v^{\text{exp}}-\mu_v^{(1)}$ (nm)	$\mu_s^{(1)}$ (nm)	$\mu_s^{\text{exp}}-\mu_s^{(1)}$ (nm)
A	89	2	9	2.134	0.419 (16%)	0.406	0.020 (5%)
B	90.56	0.52	8.92	2.182	0.371 (15%)	0.406	0.020 (5%)

Their experimental values are $\mu_v^{\text{exp}}=2.553$ nm and $\mu_s^{\text{exp}}=0.426$ nm.

The theoretical interpretation starts with taking the expectation value of the one-body moment operator. For a wave function which consists of S, S', and D components, one obtains

$$\mu_v^{(1)}=\tfrac{1}{2}(\mu_p-\mu_n)(P_S-\tfrac{1}{3}P_{S'}+\tfrac{1}{3}P_D)-\tfrac{1}{6}P_D\,, \quad (3)$$

$$\mu_s^{(1)}=\tfrac{1}{2}(\mu_p+\mu_n)(P_S+P_{S'}-P_D)+\tfrac{1}{2}P_D\,. \quad (4)$$

Here $\mu_p=2.793$ nm and $\mu_n=-1.913$ nm are the magnetic moments of proton and neutron, respectively, and the P's are the probabilities for the component states indicated by the suffix. Extensive calculations for the trinucleon bound states have been done using realistic nucleon-nucleon interactions such as the Hamada-Johnston potential[1,2] and Reid's soft-core potential.[3] The wave functions obtained by those calculations are characterized by a large D-state probability, $P_D\approx 9\%$.[1,3] The values of $\mu_{v,s}^{(1)}$ together with the P's used are shown in Table I. They are both clearly smaller than the corresponding experimental values.

This situation resembles that of the deuteron magnetic moment μ_d. There the one-body operator also yields too small a value for μ_d if one takes the D-state probability of about 7% which is predicted by the Hamada-Johnston and Reid's potentials.[4] Adler and Drell[5] showed that this discrepancy can be explained as due to the effect of exchange currents. Blankenbecler and Gunion[6] considered a vector-meson correction to the deuteron form factor, which is analogous to the double-scattering process in the Glauber approximation.

For the trinucleon systems, Chemtob and Rho[7] examined exchange-current contributions, $\mu_v^{(2)}$ and $\mu_s^{(2)}$, in great detail and found that

$$\mu_v^{(2)}=0.193\pm 0.041 \text{ nm}\,, \quad (5)$$

$$\mu_s^{(2)}=0.0093\,^{+0.0077}_{-0.0053} \text{ nm}\,, \quad (6)$$

which reduce the discrepancy between μ^{exp} and $\mu^{(1)}$ as follows:

$$\mu_v^{\text{exp}}-(\mu_v^{(1)}+\mu_v^{(2)}) = \begin{cases} 0.226\ (9\%) \text{ for case } A \\ 0.178\ (7\%) \text{ for case } B, \end{cases} \quad (7)$$

$$\mu_s^{\text{exp}}-(\mu_s^{(1)}+\mu_s^{(2)}) =0.011\ (3\%) \text{ for cases } A \text{ and } B. \quad (8)$$

But the discrepancy still remains, particularly for μ_v. If one requires that $\mu_{v,s}^{\text{exp}}=\mu_{v,s}^{(1)}+\mu_{v,s}^{(2)}$, one is led to $P_{S'}\approx 0$ and $P_D\lesssim 6\%$.[7]

Green and Schucan[8] examined effects of an admixture of a state containing $\Delta(1236)$ and found that, for a reasonable range of admixture, μ_v can be enhanced by as much as 1% while the effect on μ_s is negligible. Arenhövel and Danos[9] pointed out that an admixture of $\Delta(1470)$ will increase μ_v, but the amount of this admixture is unknown. Contributions from other components of the wave function, namely, the P state and the $I=\frac{3}{2}$ states are negligible. In summary, we are still left with a discrepancy of 6 to 8% for μ_v, and a smaller discrepancy for μ_s.

In Eqs. (3) and (4), the magnetic moments of the *free* nucleons, μ_p and μ_n, have been used. However, the magnetic moment of a nucleon bound in a nucleus will be different from that of a free nucleon because the nucleon will be dis-

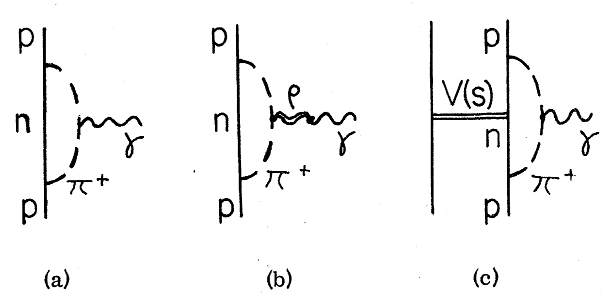

FIG. 1. Diagrams (a) and (b) are for the electromagnetic structure of a free proton. Diagram (c) depicts a process in which the neutron in the intermediate state interacts with another nucleon through an effective interaction $V(s)$.

turbed by other nucleons. To be more explicit let us consider the "two-pion contribution" to the electromagnetic structure of the proton as depicted in Fig. 1(a). The neutron in the intermediate state can interact with other nucleons, while π^+ interacts with the electromagnetic field, as in Fig. 1(c). As we shall see, μ_p and μ_n are modified by this process in such a way that μ_v is enhanced by about 5% or more. On the other hand, because of G-parity conservation, the two-pion diagram does not contribute to μ_s.[10] There are other similar processes, but the energies involved in intermediate states are larger than that for the two-pion diagram and, hence, they will be less affected by interactions with other nucleons. Note also that the process we are considering is different from the conventional exchange-current effect in which charge is exchanged between nucleons.

We examine the pion effect by perturbation theory in the static approximation which will be sufficiently accurate for our purposes. First, let us quote relevant results for a free nucleon. Although our primary interest is in the magnetic moment, we also examine the charge distribution because it will give us some more insight into the problem. The charge and current densities in the nucleon which is at rest at the origin are given by[11,12]

$$\rho_c(r) = \frac{4(f/m)^2}{(2\pi)^5}\tau_3 \int d^3k\, d^3k'\, \frac{vv'\vec{k}\cdot\vec{k}'}{\omega\omega'(\omega+\omega')} \times e^{i(\vec{k}-\vec{k}')\cdot\vec{r}}, \tag{9}$$

$$\vec{j}(\vec{r}) = \frac{-4i(f/m)^2}{(2\pi)^5}\tau_3 \int d^3k\, d^3k'\, \frac{vv'\vec{k}\vec{\sigma}\cdot(\vec{k}\times\vec{k}')}{(\omega\omega')^2} \times e^{i(\vec{k}-\vec{k}')\cdot\vec{r}}, \tag{10}$$

where $f^2 = 0.08$ is the πN coupling constant, m is the pion mass, and the units are such that $c = \hbar = 1$. k and k' are the pion momenta, $v = v(k)$ and $v' = v(k')$ are cutoff factors for the πN interaction, and $\omega = (k^2+m^2)^{1/2}$ and $\omega' = (k'^2+m^2)^{1/2}$. For the current it is convenient to introduce a magnetization density ρ_m defined by

$$\vec{j}(\vec{r}) = -(\vec{\sigma}\times\nabla)\rho_m(r). \tag{11}$$

Then ρ_c and ρ_m can be interpreted as the Fourier transforms of the standard electromagnetic form factors $F_1(q^2)$ and $F_2(q^2)$ in the nonrelativistic limit, respectively. The charge and magnetic moment carried by π^+ in $p \to n + \pi^+$ are given by

$$Q = \int \rho_c(r)\, d^3r = (2f^2/\pi m^2)\int_0^\infty dk\, v^2k^4/\omega^3, \tag{12}$$

$$\mu = \int \rho_m(r)\, d^3r = (4f^2/3\pi m^2)\int_0^\infty dk\, v^2k^4/\omega^4. \tag{13}$$

The r^2 moments of the charge and magnetization distributions can also be computed easily.[11,12]

Now let us turn to the process depicted in Fig. 1(c). We assume that the two nucleons are interacting via an effective potential $V(s)$, where s is the distance between the two. Also, with an application to the trinucleon systems in mind, we assume that $V(s)$ is independent of spin and isospin. The charge distribution due to this process is denoted by $\Delta\rho_c(r)$, which also depends on s through $V(s)$. We treat $V(s)$ in lowest-order perturbation theory; therefore, $\Delta\rho_c$ is simply proportional to $V(s)$, and $\Delta\rho_c$ is spherically symmetric with respect to r. We then take the expectation value of $\Delta\rho_c$ with respect to the nuclear wave function, replacing $V(s)$ in $\Delta\rho_c$ by its expectation value $\langle V\rangle$. The current $\Delta\vec{j}$ can be treated similarly.

An expression for $\Delta\rho_c$ can be obtained by putting the extra factor

$$-\langle V\rangle(\omega+\omega')/\omega\omega' \tag{14}$$

into the integrand of Eq. (9). In terms of time-ordered diagrams, diagram (c) has one additional intermediate state as compared with diagram (a). The energy denominator of that state is either ω or ω', hence the factor $\omega^{-1}+\omega'^{-1}$ in Eq. (14). It is known that ρ_c is somewhat overestimated in the static approximation.[12] However, because of the additional factor in $\Delta\rho_c$ the low-momentum part of the integration becomes relatively more important for $\Delta\rho_c$ than for ρ_c, and the static approximation will give a better estimate for $\Delta\rho_c$. Similarly, $\Delta\vec{j}$ is obtained from Eq. (10) by introducing the following extra factors into the integrand:

$$-\langle V\rangle(\omega^2+\omega\omega'+\omega'^2)/\omega\omega'(\omega+\omega'). \tag{15}$$

The additional charge and magnetic moment due to $\Delta\rho_c$ and $\Delta\vec{j}$ are given as follows:

$$\Delta Q = \int \Delta\rho_c(r)\, d^3r = -5\mu\langle V\rangle, \tag{16}$$

$$\Delta\mu = \int \Delta\rho_m(r)\, d^3r = -(2f^2\langle V\rangle/\pi m^2)\int_0^\infty dk\, v^2k^4/\omega^5. \tag{17}$$

The change in the r^2 moments can be similarly obtained.

For the expectation value of $-V(s)$ in ^{3}He and ^{3}H, we take $\frac{2}{3}$ of the total potential energy E_p, because two out of the three bonds are contributing to it. According to the variational calculations by Ohmura, Morita, and Yamada,[13] E_p is in the range of $55\sim 80$ MeV. The potential without hard core gave 55 MeV, while the one with a hard core of radius 0.6 F gave 80 MeV. Our effective interaction should be regarded as a K matrix which shows no singular behavior like the realistic nucleon-nucleon potential. Therefore, we should take E_p which is obtained from a potential without a hard core. On the other hand, Law and Bhaduri[14] showed that, for the binding-energy calculation for the triton, it is a good approximation to take only the long-range part of the nucleon-nucleon potential in the spirit of Moszkowski and Scott's separation method,[15] and to include its second-order perturbation term. They obtained $E_p \approx 37$ MeV for the triton (see their Table II). There is a large gap between the two values, $E_p = 37\sim 55$ MeV. In view of the fact that the triton binding energy has been underestimated by Law and Bhaduri and overestimated by Ohmura, Morita, and Yamada, we take an in-between value, $E_p \approx 45$ MeV, and hence

$$-\langle V\rangle = \tfrac{2}{3}E_p \approx 30 \text{ MeV}. \qquad (18)$$

This value is smaller than the one taken by Ohtsubo, Fujita, and Takeda[16] for the same quantity which appeared in a different context.

The quantities Q, ΔQ, μ, $\Delta\mu$, and also various r^2 moments have been evaluated using a cutoff function $v(k) = (\Lambda^2 - m^2)/(k^2+\Lambda^2)$ for two cases, $\Lambda = 5m$ and $\Lambda = 6m$. The coupling constant has been taken as $f^2 = 0.08$ and the pion mass as $m = 139.6$ MeV. The results are summarized in Table II. For the magnetic moment we obtain $\Delta\mu = 0.10 \sim 0.12$ nm or $\Delta\mu/\mu = 9\sim 10\%$.

There are many other processes contributing to the electromagnetic form factor. The effect due to the process $p \to \Delta^0(1236) + \pi^+$ will increase $\Delta\mu$ by about 10%,[12] provided that the N-Δ interaction is similar to the N-N interaction. The most important one is probably the ρ-2π process shown in Fig. 1(b). This effect can be estimated, with some ambiguity, but one can show at least that the correction is to increase the contribution. However, because of the larger energy involved in the intermediate state, this ρ-2π contribution will be modified to an extent less than the simple 2π diagram of Fig. 1(a). We conclude that the isovector part of the nucleon magnetic moment is enhanced in the trinucleon system by

$$\Delta\mu \equiv \tfrac{1}{2}(\Delta\mu_p - \Delta\mu_n) \gtrsim 0.12 \text{ nm}. \qquad (19)$$

Here, the inequality sign means that other less important contributions will add to the 2π contribution.

TABLE II. Q, ΔQ, μ, $\Delta\mu$, and r^2 moments. For example, the pair of numbers in the upper left-hand corner mean that $Q = \int\rho_c(r)\,d^3r = 0.507$ and $\Delta Q = \int\Delta\rho_c(r)\,d^3r = 0.046$, when the cutoff function $v(k) = (\Lambda^2 - m^2)/(k^2+\Lambda^2)$ with $\Lambda = 5m$ is used. The charge is in units of e, the magnetic moment in nanometers and length in fermis.

Cutoff \ Integrand	ρ_c ($\Delta\rho_c$)	$r^2\rho_c$ ($r^2\Delta\rho_c$)	ρ_m ($\Delta\rho_m$)	$r^2\rho_m$ ($r^2\Delta\rho_m$)
$\Lambda = 5m$	0.507	0.258	0.957	0.638
	(0.046)	(0.051)	(0.095)	(0.115)
$\Lambda = 6m$	0.772	0.303	1.281	0.703
	(0.061)	(0.056)	(0.116)	(0.123)

For μ_v of the trinucleon system, in addition to the conventional $\mu_v^{(1)}$ and $\mu_v^{(2)}$, we now have the contribution

$$\mu_v'^{(2)} \equiv \Delta\mu\,(P_S - \tfrac{1}{3}P_{S'} + \tfrac{1}{3}P_D), \qquad (20)$$

where $\Delta\mu$ is given by Eq. (19). With the values of P's in Table I, we find that $\mu_v'^{(2)} \approx \Delta\mu \gtrsim 0.12$ nm. Therefore, the effect we have considered can explain the bulk of the discrepancy shown in Eq. (7).

Finally, let us point out that the changes in the r^2 moments of charge and magnetic-moment distributions are quite substantial, as shown in Table II. This should be taken into account in the analysis of the electromagnetic form factors of ^{3}He and ^{3}H.

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Experimental Study of the Decays $K_{S,L}\to\gamma\gamma$†*

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In an optical spark-chamber experiment, we measured the branching ratio $(K_S\to\gamma\gamma)/(K_S\to\text{all})$ to be $(-1.9\pm2.4)\times10^{-4}$. The result assumes no interference between $K_S\to\gamma\gamma$ and $K_L\to\gamma\gamma$. We also measured the branching ratio $(K_L\to\gamma\gamma)/(K_L\to\text{all})$ to be $(4.32\pm0.55)\times10^{-4}\times|\eta_{00}/\eta_{+-}|^2$.

Some recent attempts to explain the anomalously small $K_L\to\mu^+\mu^-$ branching ration have relied upon destructive interference between the $K_2\to\mu^+\mu^-$ amplitude and an unexpectedly large $K_1\to\mu^+\mu^-$ amplitude.[1-3] It is possible that the mechanism which produces such a $K_1\to\mu^+\mu^-$ enhancement would also produce a large $K_S\to\gamma\gamma$ decay rate. We have studied the decays $K_L\to\gamma\gamma$ and $K_S\to\gamma\gamma$ in an experiment performed in the neutral 4.7° beam at the Brookhaven alternating-gradient synchrotron. These data were collected simultaneously with data on the decays $K_{S,L}\to\pi^0\pi^0$. The $\pi^0\pi^0$ results have already been published,[4] and the apparatus shown in Fig. 1 is also described in Ref. 4. We were able to achieve a high sensitivity to $K_S\to\gamma\gamma$ decayse because the pair spectrometer had a γ-ray transverse-momentum resolution of 5 MeV/c. This enabled us to separate $\gamma\gamma$ decays from most of the dominant $2\pi^0$ background, since the γ-ray transverse-momentum spectrum from $2\pi^0$ decays cuts off at 229 MeV/c while the spectrum from the $\gamma\gamma$ mode extends up to 249 MeV/c. The K_S's were produced by regeneration from a K_L beam in an 8-in. uranium block. The $K_L\to\gamma\gamma$ decays which occurred downstream of the regenerator were used for normalization.

The vector momentum of one γ ray was measured in the pair spectrometer with an rms error of 2.5% in energy and 3 mrad in production angle. The decay vertex was established by the intersection of the spectrometer γ-ray trajectory with the beam which was a 0.8-in. wide by 8-in. high verti-

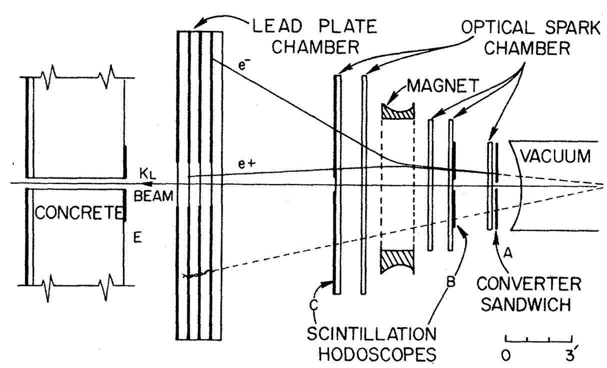

FIG. 1. Schematic view of the apparatus.