Measurement of an Optical Frequency and the Speed of Light

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(Received 12 May 1972)

We report the measurement of the frequency of the 633-nm red laser line. This is the first measurement of an optical frequency in the visible range without reference to the speed of light or to a measured wavelength. Combination of the optical frequency with the known wavelength yields c to an accuracy higher than previously known. This method demonstrates the practicability of a single-standard time-length measurement system unified via a defined value of the speed of light.

Progress in an experiment whose principle was described previously[1-3] has led recently to the determination of the optical frequency of the red (632.9-nm) He-Ne laser line and a redetermination of the speed of light. The method is based on the idea that if the difference and ratio of two optical frequencies are known, then the optical frequencies themselves are known.

By electro-optic modulation of the laser light (frequency ν) at the microwave frequency ω, the sideband frequencies $\nu \pm \omega$ are generated. The two sidebands are introduced into an evacuated Fabry-Perot cavity. The length L of the cavity and ω are simultaneously adjusted so that both sidebands pass the cavity with maximum intensity. When this is accomplished, the ratio of the two frequencies is the ratio of the two order numbers (N_+ and N_-, respectively) in the cavity. Then the optical frequency can be expressed as

$$\nu = [(N_+ + N_-)/(N_+ - N_-)]\,\omega \tag{1}$$

or

$$\nu = (N/n)2\omega, \tag{2}$$

where

$$N = (N_+ + N_-)/2 \tag{3}$$

is the order number of ν, and

$$n = N_+ - N_- \tag{4}$$

is the order number belonging to the beat note between the two sideband frequencies, 2ω. The interferometer thus establishes a relationship between the optical and the microwave frequencies.

Thus, the optical frequency is determined by measuring a microwave frequency, which is directly related to the primary frequency standard, and by determining the two order numbers N and n. Note that the measurement of ν in terms of ω makes reference only to the unit of time. It does not depend on any definition of a unit of length. It does not require knowledge of a wavelength, of the speed of light or of the length of the cavity. It merely depends upon tuning the cavity simultaneously to the two sidebands.

The condition of maximum transparency for both sidebands is produced by two servo loops as shown in Fig. 1. After passing the interferometer the sidebands are directed to two etalons, each of which passes only one sideband. The length of the interferometer is modulated at a frequency of approximately 40 kHz. The first-harmonic signals from the photomultipliers are added and phase detected in one circuit, subtracted and phase detected in another. The sum of the two first harmonics is used to servo L, via a piezoelectric transducer, to minimize that sum. The difference of the two first harmonics servos ω via a voltage-controlled oscillator to minimize that difference. (The frequency of this oscillator is added to a high-order multiple of the in-house 100-kHz frequency, derived from a frequency standard.) Clearly, if the sum and the difference of the two first harmonics are zero, the first-harmonic signals themselves are zero; i.e., the interferometer is tuned to both sidebands simultaneously and the conditions for Eqs. (1) and (2) are satisfied.

The precision of locking L to ν is similar in this experiment to that in other experiments in which a high-finesse interferometer is locked to a laser. The finesse of our interferometer is about 500, and a precision within 1% of the width of the transparency curve is easily achieved.

There are several favorable features of this experiment which allow the setting of ω to L with high precision. (i) Since the sidebands are generated by the microwave modulation of ν, the measurement of their separation, 2ω, on the frequency scale is limited only by the stability of the frequency standard. (ii) The two first har-
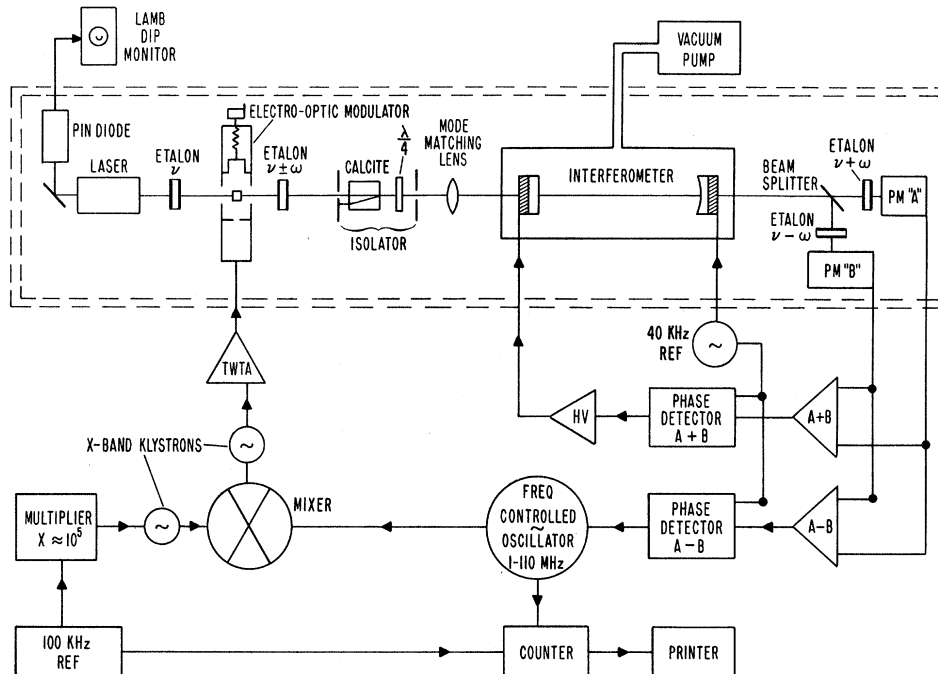
FIG. 1. Modulation interferometer for determining optical frequencies. A stable optical frequency ν is generated by the laser. Microwave frequencies $\pm\omega$ are added to ν in an electro-optic crystal mounted in the microwave cavity. The etalons shown are tuned to reject frequencies other than those indicated. The length L of the interferometer and frequency ω are adjusted, via servos, to achieve maximum transmission simultaneously at $\nu+\omega$ and $\nu-\omega$. The frequency ν is then related simply to ω (see text). Portions of the experiment included inside the dotted lines are contained in a mechanically and thermally isolated box.

monics belonging to the sidebands are strongly correlated. Thus their difference, which is used to drive ω, is insensitive to short-term fluctuations in ν, in L, or in the optics and the air path between the laser and the interferometer. (iii) To diminish uncorrelated noise between the sideband signals (e.g., shot noise) long time constants can be used in the servoing of ω. The "effective" time constant is finally given by the time of counting of ω. Successive counts, each taken for a counting time of 100 sec, fluctuated only by a few hundred hertz. This corresponds to about 10^{-4} of the bandwidth of the interferometer, or to a few parts in 10^8 for $\omega \sim 10^{10}$ Hz for the present experimental configuration.

Counts extended over 10–20 min showed slow variations in excess of noise fluctuations, amounting to a few kilohertz. We attribute these to optical feed-back effects and expect improvements by further refinements in the isolation between the Fabry-Perot and the laser. Observations showed these slow variations to be of random character. We decided, therefore, to use the equipment in its present state to produce, concurrently with other types of experiments being

performed in the infrared, a preliminary measurement of ν (and hence of c) in the visible spectrum.

In the evaluation of the data, one has to take into account that N and n in Eq. (2) are not integers, even for maximum transmission of both sidebands by the interferometer. Their deviations from integral numbers, caused by the reflection phase shifts of the mirrors and by diffraction phase shifts, are treated by Bay and Luther,[3] and by Bay.[4]

To eliminate the reflection phase shifts, the interferometer is set successively to two different lengths, L_1 and L_2, and the corresponding ω_1 and ω_2 measured. Then[3,4]

$$\nu = \frac{N_2 - N_1 + \Phi_2 - \Phi_1}{n_2/2\omega_2 - n_1/2\omega_1}, \tag{5}$$

where $N_2 - N_1$, n_2, and n_1 are now integral numbers, and Φ_2 and Φ_1 (the diffraction phase shifts) are calculated to better than 10^{-3} by cavity theory.[5] Diffraction phase shifts do not appear in the denominator of Eq. (5) because cavity theory shows that those phase shifts depend only on the geometry of the cavity and not on wavelength.

TABLE I. Estimated systematic errors.

100-kHz in-house frequency	$<10^{-9}$
Diffraction phase-shift correction	$<10^{-9}$
Phase uncertainties from mirror irregularities	$<10^{-9}$
Zeroing of electronic servoing	$<10^{-9}$
Wavelength of Lamb-dip laser (relevant for c measurement only)	$<1.5\times10^{-8}$

Consequently, the diffraction phase shifts for the two sideband waves are identical. The small errors due to irregularities of the mirror surfaces and slightly different mode diameters of the two sideband waves have been treated in Ref. 4. Their estimated values for the present experiment are included in Table I.

In the experiments $L_1 \sim 12$ cm, $L_2 \sim 25$ cm, $\omega_1 \sim \omega_2 \sim 1.034\times10^{10}$ Hz were used. This results in the values $n_1 = 17$, $n_2 = 35$.

For the determination of $N_2 - N_1$ in Eq. (5) a trial number for ν is used. Since $N_2 \sim 8\times10^5$, $N_1 \sim 4\times10^5$, and $N_2 - N_1 \sim 4\times10^5$, a trial number for ν with an uncertainty of less than 10^{-6} can be used to exclude the possibility of an error of one unit in N_2 and N_1. Such a number is available, since c is known to within 3×10^{-7},[6] and our λ is known to be $(632.99147 \pm 1)\times10^{-5}$ nm by measurements of C. Sidener at the National Bureau of Standards. It should be noted that the use of such a trial number does not in any way bias our measurement of ν or c. The measurements replace the trial number by the measured value for ν.

In the course of taking data, the length of the interferometer was alternated between the positions corresponding to $n_1 = 17$ and $n_2 = 35$. In each position, several counts of ω were taken, corresponding to different order numbers N_1 and N_2, respectively. Their departure from integral numbers, $\mathscr{E}_1$ and $\mathscr{E}_2$, respectively, were calculated and checked for statistical randomness. From 37 determinations of $\mathscr{E}_1$ and 36 determinations of $\mathscr{E}_2$, we obtained a correction to the trial number ν of -1.3×10^{-7} with the statistical standard deviation of the mean of $\pm6\times10^{-8}$. All known systematic errors, for which estimates are given in Table I, contribute substantially less than this value. This leads to the following conclusions:

(1) The frequency of the red laser line used is

$$\nu = 473\,612\,166 \pm 29 \text{ MHz.}$$

This is the first absolute determination of an optical frequency in the visible spectrum. This frequency is about 5 times higher than the highest infrared frequency yet measured.[7] It should be noted that this method of measurement is applicable to any laser line, and that it relates the optical to the microwave frequency in one step, without the use of intermediary (infrared) frequencies. The method can be applied conveniently to tunable lasers.

(2) From the above measurement of ν and the measured wavelength, the speed of light is calculated to be

$$c = 299\,792.462 \pm 0.018 \text{ km/sec.}$$

The accepted value of c $(299\,792.50 \pm 0.1$ km/sec$)$[8] is the same as measured by Froome.[6] Our error bar is smaller by a factor of about 5 than that reported by Froome.

(3) These experiments are a prototype of length measurements which are based on c and the measurement of microwave frequencies.[4] Indeed, the denominator in Eq. (5), multiplied by c, gives $2(L_2 - L_1)$. Thus, lengths are determined to optical precision without the need to know an optical wavelength or an optical order number, if c is known to the desired accuracy or if the value of c is defined. Alternatively, the method can be used to make a most direct measurement of c in the present units of time and length by simultaneously determining $L_2 - L_1$ in terms of the Kr^{86} wavelength.

(4) These experiments demonstrate the feasibility of determining optical order numbers via modulation techniques.[4] Indeed, the product $\nu n/2\omega$ determines immediately the integral part of the optical order number. These techniques can replace fringe counting and optical multiplication methods which are especially difficult and inconvenient for long interferometers.

Future improvements in the accuracy of this method can be achieved by (a) diminishing the effects of optical feed back, (b) increasing the length of the interferometer, and (c) increasing the microwave frequency. Since (b) and (c) increase the precision linearly, there can be little doubt that, still confined to a table top, the experiment can yield two more orders in the precision.

These improvements can lead to a measurement of the speed of light limited in accuracy only by that of the Kr^{86} wavelength standard. For such a c measurement in the visible spectrum the 633-nm He-Ne laser locked to the Lamb dip in the present experiments can be replaced by one locked to the saturated absorption of the I_2

molecule.[9]

The ultimate accuracy of optical-frequency measurements and that of length measurements by this method is expected to be limited only by imperfections of mirror surfaces.[4] This technological limitation is applicable also to wavelength comparisons and length measurements based on any wavelength standards, irrespective of their possible better quality. Thus, since they are applicable throughout the entire spectrum, these experiments demonstrate the possibility and practicability of a unified time-length measurement system[2,3,4,10,11] based on a frequency standard and on a defined value of the speed of light, compatible with the present meter but otherwise arbitrary.

We wish to acknowledge the contributions of several people at the National Bureau of Standards, especially R. G. Carpenter, C. Sidener, R. O. Stone, B. N. Taylor, and K. W. Yee.

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