angular distribution measured at 16.90 MeV reflects both the DR and the RR interference terms.

We present these results as an illustration of the extent to which the proton inelastic scattering measurements in the lead region can be used as probes both of structure and of interaction mechanisms, and of the degree to which the available information from a variety of reactions can be brought together into a coherent and quantitative picture. The present studies have focused primarily on the dominant components of the wave functions of the states involved. Further prediction of such quantities as polarization and $(p,p'\gamma)$ angular-correlation coefficients will be tested against the corresponding data, as soon as they become available, in order to permit establishment of other than these dominant components.

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Nature of Gravitational Synchrotron Radiation

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Ultrarelativistic geodesic particle orbits in the Schwarzschild geometry produce radiation (GSR) with angular distributions like that of synchrotron radiation. The spectra of high-frequency scalar, electromagnetic, and gravitational GSR are compared to each other and to the spectrum of synchrotron radiation. The differences among the spectra are explained in terms of the shape of the effective potential for GSR and the inapplicability of geometric optics.

Simple arguments[1] based on the local generation of radiation and propagation on null lines predict that a particle accelerated in an ultrarelativistic circular orbit in flat space-time produces radiation (synchrotron radiation) focused in a very narrow cone, and a spectrum characterized by a peak at a frequency much larger than the fundamental frequency, $\omega_0 \equiv d\varphi/dt$, of the particle's motion. Recently Misner *et al.*[2] have calculated the radiation (GSR[3]) of scalar waves produced by a particle in an ultrarelativistic ($\gamma \equiv |g_{00}| \times dt/d\tau \gg 1$) circular geodesic orbit in the Schwarzschild geometry. They found strong beaming of the radiation (half-angle $\sim 1/\gamma$) and a spectrum peaked at high frequencies,[4]

$$P(\omega) \propto \omega \exp(-2\omega/\omega_{\mathrm{crit}}), \qquad \omega_{\mathrm{crit}} \equiv 12\pi^{-1}\gamma^2\omega_0, \tag{1}$$

where $P(\omega)$ is the intensity intergrated over all angles. We have calculated the high-frequency ($\omega \gg \omega_0$) spectra,[5] as well as the polarizations,[6] of electromagnetic and gravitational GSR from ultrarelativistic circular orbits in the Schwarzschild geometry. The high-frequency spectra for all three types of GSR can be summarized in the following formula for intensity, integrated over all angles:

$$P(\omega) \propto \omega^{(1-s)} \exp(-2\omega/\omega_{\mathrm{crit}}), \tag{2}$$

where s is the spin of the radiation field ($s=0$ for scalar, $s=1$ for electromagnetic, $s=2$ for gravitational waves). The angular distribution in all cases remains highly beamed in the equatorial plane.

Entirely aside from possible astrophysical consequences of the spectra in (2), the absence of high-frequency peaks is interesting for more fundamental reasons. The frequencies we are discussing are very large $[1/\omega \ll (r^3/2M)^{1/2} \approx c^{-1} \times$(radius of curvature of space-time)$]$ so that geometric optics, that is, propagation on null lines, would seem to apply. The high-frequency peak in the synchrotron spectrum is a rather direct consequence of the "headlight effect" of geometric optics in flat space-time. It would not be too surprising if the predictions of *flat* space-time geometric optics did not apply to GSR since the null lines in the Schwarzschild geometry, near the source, differ greatly from the null lines of gravitation-free Minkowski space-time. The gravitational-lens effect might be expected to spread "headlight" beams. The crucial point, however, is that the same null lines apply to waves of different spin, so that the bending of the null geodesics should affect the spectra, angular distributions, etc. of all the waves in the same way. We are forced to conclude, in view of (2), that the qualitative conclusions of geometric optics fail to apply to GSR. An explanation of this failure and of the difference in the spectra is important to the understanding of the generation of radiation in curved space-time.

The details of the calculations of the properties of the high-frequency radiation, which will be published elsewhere, are rather intricate. Spin-weighted spherical harmonics[7] and the Newman-Penrose null-tetrad formalism[8] have proved to be very useful in simplifying the calculations and more so in giving insight into the nature of the physical processes. These techniques, as applied to perturbation fields, are outlined in a recent paper.[9] The insight that arises from these calculations can be separated from the mathematical details and an understanding of the nature of GSR generation can be gained by examining the wave equations and the source terms governing scalar, electromagnetic, and gravitational waves.

The equation governing the generation and propagation of scalar waves of a particular frequency, in a particular (l,m) multipole mode, is

$$\begin{gathered} -\frac{d^2 u_{lm}{}^{\mathrm{sc}}}{dr^{*2}} + [V(r^*) - \omega^2] u_{lm}{}^{\mathrm{sc}} = S(l,m), \\ r^* \equiv r - 3M + 2M \ln(r/M - 2), \quad \omega = m\omega_0, \end{gathered} \tag{3}$$

where $S(l,m)$ is the source term, M is the mass (in units $c=G=1$) giving rise to the Schwarzschild geometry, and the scalar field is

$$\Phi = \sum_{l,m} u_{lm}{}^{\mathrm{sc}}(r^*) \exp(-im\omega_0 t) Y_l{}^m(\theta,\varphi). \tag{4}$$

In the case of ordinary synchrotron radiation, we we can ignore gravitational effects, so that $r^* = r$ and the effective potential $V(r)$ is just the centrifugal potential $l(l+1)/r^2$. In the case of GSR the curvature of space-time induces an effective potential barrier which is the same as the centrifugal potential for $r \gg M$ but which has a peak near $r = 3M$, and which vanishes exponentially in r^* as $r^* \to -\infty$ (i.e., $r \to 2M$).

For significant production of GSR, the circular orbits must be near $r=3M$, and hence near the peak of the effective potential. Misner *et al.* solve (3) by viewing it as a problem in barrier penetration near a potential peak. The usual WKB technique is inapplicable for such a calculation. Rather, they approximate the potential near its peak by a parabola so that the homogeneous solutions of (3), from which the Green's function is constructed, are parabolic cylinder functions of the variable $(l/27)^{1/2} r^*/M$ near the potential peak. The requirement that radiation must not be appreciably damped inside the potential means that $\omega^2 \approx V_{\mathrm{peak}}$ and leads to the important features of the radiation: $l \approx m = \omega/\omega_0$ and significant intensity for m as large as several times γ. A similar approach can be used for ordinary synchrotron radiation, with the spherical Bessel functions of the argument (ωr) used as the homogeneous solutions of (3). The requirement that there is small damping of the radiation inside the potential means that $\omega^2 \approx V(r_{\mathrm{source}})$ and again leads to the significant features of the radiation.

With the guidance of the null-tetrad formalism, equations like (3) can be derived for electromagnetic and gravitational waves. These equations are characterized by similar potential barriers, independent of the type of waves, and by wave functions u^{em} and u^{gw} (see Table I) related to the "real" physical field rather than a potential (e.g., for electromagnetism u^{em} is related to the elec-

TABLE I. Comparison of scalar, electromagnetic, and gravitational wave descriptions.

Type of Radiation	Variable in Wave Equation[a]	Origin of Wave Equation	Relation of Power Flux to the Magnitude of the Wave Variable at Large	Source Term in Wave Equation
Scalar	$u^{SC} \propto r\Phi$ Φ = Scalar Field	Usual scalar wave equation $\Phi_{;\nu}{}^{;\nu} = 4\pi\rho$	$P_{SC}(\omega) \propto \omega^2 \lvert u^{SC}\rvert^2$	Density of scalar charge $\propto (\omega)^0$ for GSR $\propto (\omega)^0$ for synchrotron radiation
Electromagnetic	$u^{EM} \propto r^2\Phi_{-1}$ [$\Phi_{-1} \equiv$ spin-weight -1 projection[b] of the electromagnetic field tensor $F^{\mu\nu}$.] $u^{EM} \propto r^2[E_{\hat\theta} - iE_{\hat\phi}]$ for $r \gg m$ and $r \gg 1/\omega$	Combinations of the second order differential equations that result from differentiating Maxwell's equations	$P_{EM}(\omega) \propto \lvert \frac{u^{EM}}{r}\rvert^2$	Derivatives of the current J^μ. $\propto \omega^{\frac{1}{2}}$ for GSR $\propto \omega$ for synchrotron radiation
Gravitational	$u^{GW} = r^3\,\Psi_{-2}$ [Ψ_{-2} is the spin-weight -2 projection[b] of the Weyl tensor.] $u^{GW} \propto r^3[R_{\hat 0\hat\theta\hat 0\hat\theta} + iR_{\hat 0\hat\theta\hat 0\hat\phi}]$	Combinations of the second order equations that result from differentiating the Bianchi identities	$P_{GW}(\omega) \propto \omega^2 \lvert \frac{u^{GW}}{r^2}\rvert^2$	Second derivatives of the stress-energy $T^{\mu\nu}$ $\propto \omega$ for GSR $\propto \omega^2$ for synchrotron radiation

[a]Carets denote components on an orthonormal basis. [b]See Ref. 9.

tromagnetic field tensor $F^{\mu\nu}$ rather than the vector potential A^μ).

The mathematical structure of the sources in these equations differ significantly. In the scalar wave equation, the source is the density of scalar charge. Since the radiation arises from a scalar charged particle in a circular orbit at r_0^* the source term in (3) goes as $\delta(r^* - r_0^*)$. For electromagnetic radiation, the wave equation is constructed by differentiating and combining Maxwell's equations. The source therefore consists of derivatives of the current density and is characterized by $d\delta(r^* - r_0^*)/dr^*$. The gravitational wave equation is constructed by differentiating and combining the Bianchi identities so that the source consists of second derivatives of the Ricci tensor, or the stress energy; the source is characterized then by $d^2\delta(r^* - r_0^*)/dr^{*2}$.

Misner *et al.*[2] show that the intensity of scalar GSR is of the form

$$P(\omega) \propto \omega \exp(-2\omega/\omega_{\rm crit}). \tag{5}$$

Since all the radiation fields satisfy the same type of wave equation and are characterized by the same effective potentials, the high-frequency ($\omega \gg \omega_0$) GSR spectra for scalar and electromagnetic waves are known as soon as the "strength" of the sources in ω is known. Since the source terms are all δ functions in r^* or their derivatives, the strength of each source is evaluated by operating with these δ functions on the homogeneous solutions (i.e., by integrating the source over the Green's function). The strengths thus evaluated are found to go as $\omega^{1/2s}$ for GSR. With this the power spectrum is easily calculated. The strength of the electromagnetic source, for instance, is $\omega^{1/2}$ so that $|u^{\rm em}|^2$ is larger than $|u^{\rm sc}|^2$ by ω at large ω. If we combine this with the fact that $P_{\rm em} \approx |u^{\rm em}|^2$ (this follows from $P_{\rm em} \propto E^2$) and $P_{\rm sc} \propto \omega^2 |u^{\rm sc}|$, then a comparison with (3) shows that $P_{\rm em} \propto \exp(-2\omega/\omega_{\rm crit})$. This conclusion and the analogous one for gravitational waves are summarized in (2).

For ordinary synchrotron radiation from accelarated particles, the strengths must again be evaluated. They are now found to go as ω^s so that, by arguments similar to those above, and by comparison with the known result for electromagnetism

$$P(\omega) \propto \omega \exp(-2\omega/\omega_{\rm crit}) \tag{6}$$

for all three types of ordinary synchrotron radiation. For ordinary synchrotron radiation, then, the prediction of geometric optics is verified, all the spectra are independent of the spin of the waves and have high-frequency peaks. We now consider why the prediction is not valid for GSR.

Mathematically the difference between GSR spectra and the synchrotron radiation spectra lies in the different strengths of the sources when evaluated on the homogeneous solutions for GSR [parabolic cylinder functions of $(l/27)^{1/2}r^*M$] and the homogeneous solutions for synchrotron radiation (spherical Bessel functions of ωr). The homogeneous solutions have a different mathematical form and different forms of the argument due to the different shape of the potential in the region of the source. The sources are of different strength because differentiating the GSR Green's function by r^* brings out a factor of order $\omega^{1/2}$ while differentiating the synchrotron radiation Green's function brings out a factor of order ω. Thus the difference between the strengths of the sources for GSR and for synchrotron radiation lies in the shapes of the effective potential at the radial location of the source: For synchrotron radiation the effective potential is sharply changing while for GSR the potential is near its peak.

These mathematical insights lead very naturally to a physical interpretation. In (3) we can view $V-\omega^2$, the difference between the effective potential and the effective energy of the waves, as the reciprocal of the square of the effective wavelength. In GSR the slow variation of the effective potential near the source, where $V-\omega^2 \approx 0$ means the wavelength remains very large near the source. In ordinary synchrotron radiation the quick variation of $V(r^*)$ near the source, where again $V(r^*)-\omega^2\approx 0$, means that the wavelength decreases quickly. The geometrical-optics approximation requires a short wavelength, to be valid. Just as the WKB approximation can be used for functions $V(r^*)-\omega^2$ with first-order zeros, it is not surprising that the predictions of geometrical optics are valid for synchrotron radiation. For the barrier peak problem of GSR neither the WKB method nor geometrical optics can be expected to give correct conclusions.

Heuristically, near the source the homogeneous solutions for ordinary synchrotron radiation have radial derivatives much smaller than time derivatives and therefore do not propagate on null lines, and are not within the scope of geometric optics.

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[1]See, for example, J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1963), Sec. 14.4.

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[3]We use GSR, an abbreviation for geodesic sychrotron radiation, to mean radiation of *any* type (scalar, etc.) produced by *geodesic* motion of an ultrarelativistic particle.

[4]The formula for $P(\omega)$ is only accurate for $\omega \gg \omega_{\mathrm{crit}}$. Our comparison of the three types of GSR is, however, good for $\omega \gg \omega_0$.

[5]These spectra have also recently been numerically computed. See R. A. Breuer and C. V. Vishveshwara (to be published) for the electromagnetic GSR spectrum and M. Davis *et al.*, Phys. Rev. Lett. 28, 1352 (1972), for the gravitational-wave GSR spectrum. These numerical results confirm Eq. (2), and show it to be qualitatively correct for low frequencies also.

[6]We find, for both electromagnetic and gravitational radiation that the polarization *near* the equatorial plane is predominantly the same as that precisely *in* the equatorial plane (~ 90% the same at the intensity half-angle). A discussion of polarizations is not central to the issues of this paper and will not be presented here.

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