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¹¹In the spirit of the Stoner-Wohlfarth-Slater model, one anticipates exchange splitting of sufficient magnitude to produce the observed saturation magnetic moment. This should approximate the width of the density

of states peak in which the paramagnetic Fermi level falls, ~ 1.0 eV being the band-theory consensus. (See Ref. 10.) One then expects shifts of approximately $\frac{1}{2}$ this amount for structure in the majority spin band as the temperature ranges through the Curie point.

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Compound Particle-Core Coupled States and Direct Nuclear Processes*

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We report a detailed study of the inelastic scattering reaction $^{206}\text{Pb}(p, p')^{206}\text{Pb}^*(2_1^+)$ for incident proton energies in the range from 14.2 to 17.3 MeV. It has been found possible to reproduce the experimental results quantitatively using a theoretical analysis incorporating a number of resonant compounds, as well as direct inelastic, scattering amplitudes. Interference effects play an important role and predictions of shell and weak-coupling models are found to be in excellent agreement with the experimental data.

Recent measurements in the lead region have shown that many of the important structural features may be understood on the basis of a weak-coupling model (WCM)¹ and that study of isobaric-analog states (IAS) provides a powerful probe for study of such structure.

We report herein on a study, both experimental and theoretical, of the $^{206}\text{Pb}(p, p')^{206}\text{Pb}^*(2_1^+)$ inelastic scattering interaction in the incident proton energy range from 14.2 to 17.3 MeV which spans a number of the well-known IAS in this mass region. We have found that, using well-established optical-model parameters for this region and the structural information on the IAS deduced from earlier direct deuteron stripping reaction studies, it has been possible to obtain rather remarkable quantitative reproduction of the experimental data over this entire energy region. Of particular interest has been the dominant role played by interference effects both between different IAS compound-resonance amplitudes and between these and the direct scattering amplitudes.

The spreading of the single-particle strength in ^{207}Pb was initially investigated by Auerbach and Stein.² Their study, based on the coupling of the $2g_{9/2}$, $1i_{11/2}$, $3d_{5/2}$, $4s_{1/2}$, $3d_{3/2}$, and $2g_{7/2}$ single neutrons to the 0_1^+ ground state and the first 2_1^+ excited state of ^{206}Pb , showed that the WCM does predict all the gross features of the experimental

large-angle inelastic scattering excitation functions populating the 2_1^+ state in $^{206}\text{Pb}(p, p')^{206}\text{Pb}^*$ studies. Two prominent effects neglected in this, and numerous other analysis of reactions involving IAS, are the resonance-resonance (RR) interferences resulting from the often observed overlap of the analog states involved and the direct-resonance (DR) interference reflecting the mixing of the analog states with a nonresonant (NR) scattering background. While such neglect has reduced the complexity of the IAS analyses, it constitutes a major limitation in the detailed examination, and careful extraction, of nuclear-structure information.

The theoretical methods³ proposed for removing these limitations have stimulated us to study systematically the strong interference patterns of the $^{206}\text{Pb}(p, p')^{206}\text{Pb}^*(2_1^+)$ angular distributions measured on and off resonance.

Proton beams with incident laboratory energies between 14.2 and 17.3 MeV from the Yale MP tandem were used to bombard a self-supporting isotopically enriched 2-mg/cm²-thick ^{206}Pb target. Two characteristics of the experimental data should be emphasized. The first is the absence of symmetry about 90° which manifests itself in all twelve angular distributions obtained and which is a signature for strong interference effects. The second is the persistence of a common shape for angles between 60° and 100° which

appears to be directly attributable to the nonresonant background.

Theoretical analyses of these data must satisfy several simultaneous stringent conditions and consistency requirements. We discuss briefly the logical connections that exist, within the framework of the data presented, between the WCM and the nuclear-structure information already available.

The first condition concerns the adequacy of the optical-model (OM) parameters to describe direct inelastic scattering in this mass region and the adequacy of available knowledge concerning the degree of deformation (i.e., collectivity) involved. A phenomenological description is required of the smooth energy dependence of the background scattering over the entire experimental energy range. The optimum proton-nucleus OM parameters generated by the Bechetti-Greene's⁴ functional forms gave good fits to the 19.6 and 24.5 off-resonance $^{206}\text{Pb}(p, p')^{206}\text{Pb}^*$ angular distributions of Glashauser *et al.*,⁵ and of Vallois, Saudinos, and Beer,⁶ respectively, and have been adopted. A deformation parameter $\beta = 0.048$ has been deduced from the transition probability $G_2 = 6.2$ obtained by Vallois for the de-excitation of the first 2_1^+ state of ^{206}Pb and provides the necessary collectivity measure.

The second condition concerns the adequacy of the description and parametrization of the IAS resonances. The (p, p') IAS resonant energies and their associated spectroscopic factors are necessarily consistent with the (d, p) results of Moyer, Cohen, and Diehl⁷ and the Coulomb stripping results of Hering, Achterath, and Dost.⁸ Following a remark made by Lenz and Temmer⁹ the excitation function leading to the ^{206}Pb 2_1^+ state was calibrated by assuming that in both $^{207}\text{Bi}^*$ and in $^{209}\text{Bi}^*$ the $g_{9/2}$ single-particle IAS resonates at the same energy. The careful analysis, by Wharton *et al.*,¹⁰ of several $^{208}\text{Pb}^*$ final-state excitation functions with negligible NR background showed that the $g_{9/2}$ single-neutron IAS of ^{209}Pb resonates at a proton laboratory energy of 14.918 MeV. When the above $g_{9/2}$ single-particle reference energy is combined with the corresponding 2.728-MeV excitation energy in ^{207}Pb from the (d, p) experiment, the experimental angular distributions measured at proton energies of 15.80, 16.63, and 16.90 MeV may be associated with the $(2_1^+ \otimes g_{9/2})_{5/2}$, $(0_1^+ \otimes d_{5/2})_{5/2}$, and $(0_1^+ \otimes s_{1/2})_{1/2}$ particle-core configurations, respectively. Furthermore, it is very tempting to associate the structured angular distribution mea-

sured at 15.95 MeV with the $(2_1^+ \otimes g_{9/2})_{9/2}$ configuration of the WCM. The (d, p) experiment of Moyer, Cohen, and Diehl shows a weak state, with an assigned l value of 2 or 3, at 3.725-MeV excitation energy in ^{207}Pb . If this state is assumed to have the sought-after $(2_1^+ \otimes g_{9/2})_{9/2}$ configuration, it becomes possible to deduce an experimental spectroscopic factor of 0.058 based on that quoted for the single-neutron $g_{9/2}$ state at 2.728 MeV. Since the experimentally inferred (0.058) and theoretically predicted (0.06) spectroscopic factors are consistent, we have chosen to associate the WCM $(2_1^+ \otimes g_{9/2})_{9/2}$ configuration with the angular distribution measured at 15.90 MeV.

The third condition concerns the choice of an adequate inelastic scattering reaction formalism which includes both the "compound nuclear" and "direct interaction" processes. The complete S matrix constructed by Weidenmüller and Dietrich³ in their extended theory of nuclear resonance reactions has been adopted. In this theory, each S-matrix element, corresponding to a channel $N = \{lj, l'j'\}$, is resolved into its direct and resonant components $S_N = S_{[D]N} + S_{[R]N}$. The direct terms were generated at each of the experimental energies of interest by the code LISA¹¹ with the OM parameters discussed above as input. The resonant terms were parametrized by a Breit-Wigner shape,

$$S_{[R]N} = -i \sum_r \frac{(\tilde{\Gamma}_{Nr}^p \tilde{\Gamma}_{Nr}^{p'})^{1/2}}{E - E_{Nr} + i \frac{1}{2} \Gamma_{Nr}},$$

where the summation extends over all the compound states associated with channel N . It was further assumed that both the single-particle-model widths Γ_j^{spm} and the total widths Γ_j are those extracted by Wharton *et al.* for shell-model single-neutron states φ_j outside the doubly magic ^{208}Pb core. The Γ_j^{spm} widths were combined with the particle-plus-collective-core wave functions

$$\Psi_{j\alpha}({}^{207}\text{Pb}) = a_{j\alpha} |\psi({}^{206}\text{Pb}(0_1^+)) \otimes \varphi_j\rangle_j \\ + \sum_k b_{j\alpha, k} |\psi({}^{206}\text{Pb}(2_1^+)) \otimes \varphi_{j_k}\rangle_j$$

calculated by Auerbach and Stein² to yield the entrance and exit partial nucleon widths $|\tilde{\Gamma}_{Nr}^p| = a_{j\alpha}^2 \Gamma_j^{\text{spm}}$ and $|\tilde{\Gamma}_{Nr}^{p'}| = b_{j\alpha}^2 \Gamma_j^{\text{spm}}$, respectively.

The predicted inelastic scattering angular distributions were decomposed into direct (D), resonant (R), and DR interference components by means of the program CHRISTINE,¹² a computer code based on a variable-metric minimization algorithm. The program was allowed to search for the "best" phases $\varphi_{Nr} = \arg(\tilde{\Gamma}_{Nr}^p \tilde{\Gamma}_{Nr}^{p'})^{1/2}$ only,

since all remaining parameters had already been chosen to satisfy explicitly the relevant consistency requirements discussed above. Three characteristic types of angular distributions may be distinguished; the first and the second include the angular distributions measured around 15.80 and 16.63 MeV incident proton energy, respectively; the third includes "far-away" resonances such as the single-neutron $g_{9/2}$ state and IAS resonating at energies beyond 17.3 MeV. The contribution of this third group of IAS to the differential cross section can be approximately simulated by forcing reasonably good fits to the shapes and magnitudes of the available off-resonance angular distributions.

In the following, we present a procedure specially tailored to test the degree of accuracy of our knowledge of the reaction mechanism as well as the reliability of the detailed nuclear-structure information used. This involved two steps. In the first an iterative fitting process, in which all the ten IAS of Fig. 1 participated, was used to obtain the "best" simultaneous fits to the on-resonance angular distributions measured at 15.80 and at 16.63 MeV. The resulting fits and their decomposition into direct, resonant, and DR interference components are displayed in Fig. 2. In the second step, we vary *only* the incident proton energy in the calculations, keep the same phases that minimized χ^2 in step 1, and predict all the remaining on- and off-resonance angular distributions. These predictions have been found to yield a remarkably good reproduction of

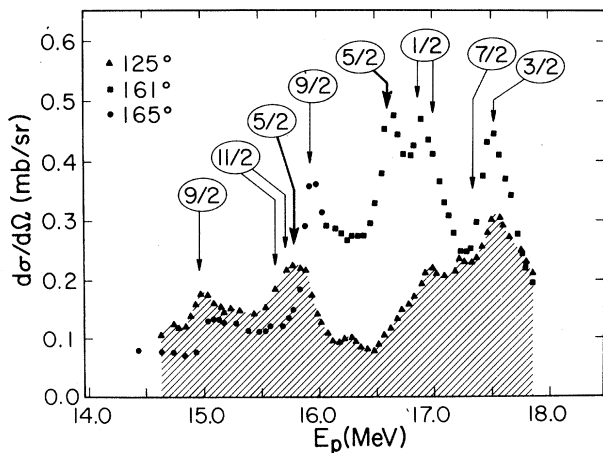


FIG. 1. Excitation functions for the reaction $^{206}\text{Pb}(p, p')^{206}\text{Pb}(2_1^+)$ at laboratory angles of 165° , 161° , and 125° (Ref. 10 and this work). The arrows and spin-parity assignments correspond to the ten strongest IAS identified in the $^{206}\text{Pb}(d, p)$ experiment and included in the calculation.

both the absolute magnitudes and the very dissimilar characteristic shapes of the data measured at 15.85, 15.95, 16.10, 16.30, 16.77, and 16.90 MeV. The quality of the agreement between calculation and experiment shown in Fig. 2 is striking; it provides strong support for the essential reliability of both the detailed reaction-mechanism formalism on which the calculations rest and of the input nuclear-structure information. It should be noted that, apart from trivial penetrability effects, we have implicitly assumed that the complex partial widths of the resonances have no significant energy dependence in the range considered.

Interference effects play a key role in these calculations. The marked changes in the shapes of angular distributions measured at 15.80 and 15.95 MeV is almost entirely a reflection of the energy dependence of the DR interference term. The destruction of the isotropy which would be implied by the $(0_1^+ \otimes s_{1/2})_{1/2}$ configuration of the

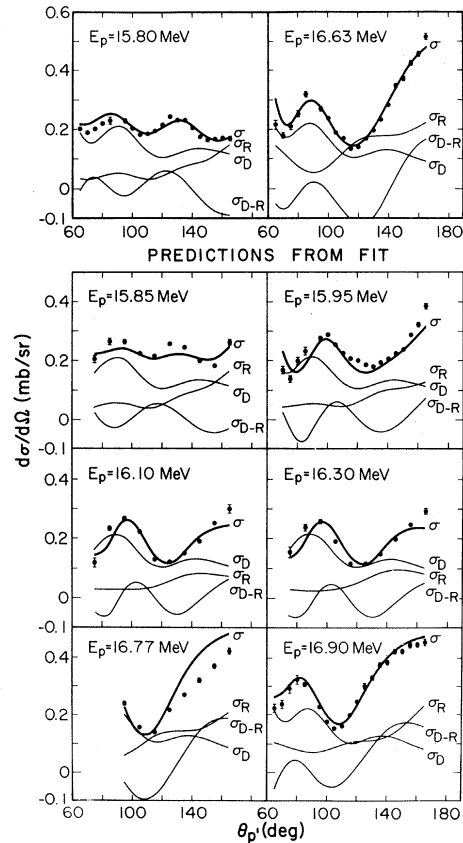


FIG. 2. Best simultaneous on-resonance fits at 15.80 and 16.63 MeV and predictions for other on- and off-resonance angular distributions. The differential cross sections are resolved into direct, resonant, and interference components, $\sigma = \sigma_{[D]} + \sigma_{[R]} + \sigma_{[DR]}$.

angular distribution measured at 16.90 MeV reflects both the DR and the RR interference terms.

We present these results as an illustration of the extent to which the proton inelastic scattering measurements in the lead region can be used as probes both of structure and of interaction mechanisms, and of the degree to which the available information from a variety of reactions can be brought together into a coherent and quantitative picture. The present studies have focused primarily on the dominant components of the wave functions of the states involved. Further prediction of such quantities as polarization and $(p, p'\gamma)$ angular-correlation coefficients will be tested against the corresponding data, as soon as they become available, in order to permit establishment of other than these dominant components.

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Nature of Gravitational Synchrotron Radiation

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Ultrarelativistic geodesic particle orbits in the Schwarzschild geometry produce radiation (GSR) with angular distributions like that of synchrotron radiation. The spectra of high-frequency scalar, electromagnetic, and gravitational GSR are compared to each other and to the spectrum of synchrotron radiation. The differences among the spectra are explained in terms of the shape of the effective potential for GSR and the inapplicability of geometric optics.

Simple arguments¹ based on the local generation of radiation and propagation on null lines predict that a particle accelerated in an ultrarelativistic circular orbit in flat space-time produces radiation (synchrotron radiation) focused in a very narrow cone, and a spectrum characterized by a peak at a frequency much larger than the fundamental frequency, $\omega_0 \equiv d\varphi/dt$, of the particle's motion. Recently Misner *et al.*² have calculated the radiation (GSR³) of scalar waves produced by a particle in an ultrarelativistic ($\gamma \equiv |g_{00}| \times dt/d\tau \gg 1$) circular geodesic orbit in the Schwarz-

schild geometry. They found strong beaming of the radiation (half-angle $\sim 1/\gamma$) and a spectrum peaked at high frequencies,⁴

$$P(\omega) \propto \omega \exp(-2\omega/\omega_{\text{crit}}), \quad (1)$$

$$\omega_{\text{crit}} \equiv 12\pi^{-1}\gamma^2\omega_0,$$

where $P(\omega)$ is the intensity integrated over all angles. We have calculated the high-frequency ($\omega \gg \omega_0$) spectra,⁵ as well as the polarizations,⁶ of electromagnetic and gravitational GSR from ultrarelativistic circular orbits in the Schwarz-