maximum. This might be related to the weak shoulder at $|t| = 0.04$ (GeV/c)^2 noted by Miller *et al.*[1] Let us now turn to polarization effects. It is easy to check that Eq. (1) yields zero polarization. This is in fair agreement with the data[13] for $0 < |t| < 0.005$ (GeV/c)^2.

Finally, it is interesting to observe that in our model the *value* of the forward peak is provided for by the second (constant) "background" term in Eq. (3) due to u-channel exchange, whereas the strong decrease with $|t|$ is provided for by the last term in (3), i.e., by the destructive interference between the background term and the t-channel term. The t-channel term itself [i.e., the first term on the right-hand side of Eq. (3)] has little influence on the $|t|$ dependence of the forward peak. This brings us back to the model suggested by Phillips[3] where we now possess a natural explanation for the background term and for its *destructive* interference with the OPE amplitude.

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Lepton Symmetry

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Assuming a weak interaction invariant under $SU(2)_M \otimes SU(2)_I$, we show that M-spin conservation leads to the rule $A(\nu_e e^- | \nu_e e^-) - A(\nu_\mu e^- | \nu_\mu e^-) = A(\nu_e \mu^- | \nu_\mu e^-)$ and to other sum rules for leptonic, semileptonic, and trident production processes. The additional assumption of total isospin conservation yields simple relations for lepton-hadron scattering. We also show that μ decay is forbidden in lowest order for various symmetry schemes such as the SU(3) and O(3) theories based upon the Konopinski-Mahmoud assignment of lepton number.

Lepton symmetries based upon the observed spectrum of leptons lead to simple relations among leptonic and semileptonic processes that are becoming amenable to experimental observation. We consider the consequences of assuming lepton-number conservation, a $V-A$ form for the four-fermion interaction, and an invariance under an $SU(2)_M \otimes SU(2)_I$ symmetry. M spin describes the μ content of a system of leptons, and I is the weak isospin[1]; the classification of the known leptons is given in Table I. The $SU(2)_M \otimes SU(2)_I$ symmetry is of interest both for its di-

TABLE I. Lepton quantum numbers.

Lepton	M_z	I_z	Lepton No.
ν_e	1/2	1/2	1
e^-	1/2	−1/2	1
ν_μ	−1/2	1/2	1
μ^-	−1/2	−1/2	1
$\bar{\nu}_e$	−1/2	−1/2	−1
e^+	−1/2	1/2	−1
$\bar{\nu}_\mu$	1/2	−1/2	−1
μ^+	1/2	1/2	−1

rect consequences, and also as a possible basis for a renormalizable gauge theory of weak and electromagnetic interactions.

Muon-number conservation in leptonic and semileptonic processes requires $\Delta M_z = 0$. If total M spin is also conserved, then purely leptonic processes of the form $a+b \to c+d$ depend on two complex independent amplitudes, $M(0)$ and $M(1)$. An immediate consequence is the relation

$$A(\nu_e e^-|\nu_e e^-) - A(\nu_\mu e^-|\nu_\mu e^-) = A(\nu_e \mu^-|\nu_\mu e^-), \tag{1}$$

where

$$A(\nu_e e^-|\nu_e e^-) = M(1), \tag{2a}$$

$$A(\nu_\mu e^-|\nu_\mu e^-) = \tfrac{1}{2}[M(1) + M(0)], \tag{2b}$$

$$A(\nu_e \mu^-|\nu_\mu e^-) = \tfrac{1}{2}[M(1) - M(0)]. \tag{2c}$$

The right-hand side of (1) is related to the μ-decay constant. Equation (1) leads to the inequality

$$|[\bar{\sigma}(\nu_e e^-|\nu_e e^-)]^{1/2} - [\bar{\sigma}(\nu_\mu e^-|\nu_\mu e^-)]^{1/2}| \leq |[\bar{\sigma}(\nu_e \mu^-|\nu_\mu e^-)]^{1/2}|, \tag{3a}$$

or, equivalently,

$$|[\bar{\sigma}(\nu_e e^-|\nu_e e^-)]^{1/2}| \geq |[\bar{\sigma}(\nu_e \mu^-|\nu_\mu e^-)]^{1/2} - [\bar{\sigma}(\nu_\mu e^-|\nu_\mu e^-)]^{1/2}|; \tag{3b}$$

$\bar{\sigma}$ is the experimental cross section divided by the phase-space factor for that process. Equations (3) are consistent with the present rough limits on the electron neutrino scattering cross section.[2]

The production of charged-lepton pairs by energetic ν_μ's incident on heavy nuclei Z (trident production) provides a further test of M-spin conservation. Since all hadrons are assumed to have zero M spin, Eqs. (2b) and (2c) and the relation

$$A(\nu_\mu \mu^-|\nu_\mu \mu^-) = M(1) \tag{4}$$

lead to

$$A(\nu_\mu + Z \to \mu^- \mu^+ \nu_\mu + Z) - A(\nu_\mu + Z \to e^+ e^- \nu_\mu + Z) = A(\nu_\mu + Z \to \mu^- e^+ \nu_e + Z). \tag{5}$$

A search for such processes is currently underway.[3]

Immediate consequences of M-spin conservation are also found in elastic and quasielastic lepton-hadron scattering. Since hadrons are assumed to have zero M-spin, we obtain simple relations between amplitudes for electron-type leptons and muon-type leptons by a reflection in M space, i.e., $M_z = \frac{1}{2} \to M_z = -\frac{1}{2}$ and $M_z = -\frac{1}{2} \to M_z = +\frac{1}{2}$:

$$A(\nu_e p \to \nu_e p) = A(\nu_\mu p \to \nu_\mu p), \tag{6a}$$

$$A_{\rm weak}(e^- p \to e^- p) = A_{\rm weak}(\mu^- p \to \mu^- p), \tag{6b}$$

$$A(\nu_e n \to e^- p) = A(\nu_\mu n \to \mu^- p). \tag{6c}$$

Thus *μ-e universality* as typified by Eq. (6c) *is generated by an M-spin reflection.*

The quantum numbers in Table I are such that the charge of a lepton is related to its lepton number L and isospin component $I_z^{(l)}$ by the formula

$$Q^l = I_z^{(l)} - \tfrac{1}{2}L. \tag{7}$$

As a result the sum $I_z^{(l)} + I_z^{(h)}$ of the z components of leptonic and hadronic isospins is conserved in strangeness-conserving semileptonic processes such as neutron β decay, Σ-Λ β decay, and the neutrino reactions of Eq. (6). We may therefore speculate that *total* isospin is conserved in these processes,[4] i.e.,

$$\Delta I \equiv \Delta(I^{(l)} + I^{(h)}) = 0. \tag{8}$$

Of necessity this assumption requires the existence of neutral currents and of processes like

$$\nu_\mu + p \to \nu_\mu + p + \pi^0, \tag{9a}$$

$$\nu_\mu + p \to \nu_\mu + n + \pi^+, \tag{9b}$$

$$\nu_\mu + n \to \nu_\mu + n + \pi^0. \tag{9c}$$

If the pion-nucleon final state is dominated by the $\Delta(1236)$[5] and isospin is conserved, then the processes of Eqs. (9a), (9b), and (9c) are governed by a single amplitude $g_{3/2}$, and the cross-section ratios

$$R_2 = \sigma(\nu_\mu p \to \nu_\mu n \pi^+)/\sigma(\nu_\mu p \to \mu^- p \pi^+), \tag{10a}$$

$$R = \frac{\sigma(\nu_\mu p \to \nu_\mu p \pi^0) + \sigma(\nu_\mu n \to \nu_\mu n \pi^0)}{2\sigma(\nu_\mu n \to \mu^- p \pi^0)} \tag{10b}$$

are predicted to be

$$R_2=\tfrac{1}{9}, \quad R=1. \tag{11}$$

The present experimental limits,[6,7]

$$R_2<0.16, \quad R<0.14, \tag{12}$$

are consistent with the prediction for R_2, but differ substantially from that for R.

To reconcile the limit on R with total isospin conservation, we must include an $I=\frac{1}{2}$ component in the pion-nucleon final state.[8] This gives us two more amplitudes, h_0 and h_1 (the subscripts 0 and 1 indicate the total t-channel isospin of the hadrons), and we can fit Eq. (12) with ratios like $h_0/g_{3/2}\approx 0.3$, and $h_1/g_{3/2}\approx 0.6$. We then find that the branching ratio for the process in Eq. (9c) should be

$$R_3=\sigma(\nu_\mu n\to\nu_\mu n\pi^0)/\sigma(\nu_\mu n\to\mu^- p\pi^0)\simeq 0.03. \tag{13}$$

This value is well within the experimental upper limit of 0.2.[6]

The use of I_z reflections ($I_z\to -I_z$ and $-I_z\to I_z$) and the $\Delta I=0$ rule yields two relations for lepton-hadron scattering:

$$A_{\text{weak}}(\nu_\mu p\to\nu_\mu p)=A_{\text{weak}}(\mu^- n\to\mu^- n), \tag{14a}$$

$$A_{\text{weak}}(\nu_e p\to\nu_e p)=A_{\text{weak}}(e^- n\to e^- n). \tag{14b}$$

In addition, because these processes depend upon only two independent I-spin amplitudes, we obtain the sum rules

$$A(\nu_\mu p\to\nu_\mu p)-A(\nu_\mu n\to\nu_\mu n)=A(\nu_\mu n\to\mu^- p), \tag{15a}$$

$$A(\mu^- n\to\mu^- n)-A(\mu^- p\to\mu^- p)=A(\mu^- p\to\nu_\mu n), \tag{15b}$$

and their M_z reflection partners,

$$A(\nu_e p\to\nu_e p)-A(\nu_e n\to\nu_e n)=A(\nu_e n\to e^- p), \tag{16a}$$

$$A(e^- n\to e^- n)-A(e^- p\to e^- p)=A(e^- p\to\nu_e n). \tag{16b}$$

The present experimental limit[9] on the ratio $R_1=\sigma(\nu_\mu p\to\nu_\mu p)/\sigma(\nu_\mu n\to\mu^- p)$,

$$R_1<0.24, \tag{17}$$

implies, via Eq. (15a), that

$$R_4=\sigma(\nu_\mu n\to\nu_\mu n)/\sigma(\nu_\mu n\to\mu^- p) \tag{18}$$

has the limit $0.24\le R_4\le 2.22$.

For more complicated situations such as inclusive reactions, we obtain restrictions upon the isospin of X in the final state $\mu^- + X^{++}$ of neutrino-hadron scattering, and we can relate its amplitudes to the corresponding ones for the $\nu_\mu + X^+$ final state.[10] As an example, consider

$$\nu_\mu + d\to\mu^- + X^{++} \tag{19a}$$

$$\to\nu_\mu + X^+. \tag{19b}$$

Since the deuteron has zero isospin, X can have either $I=0$ or $I=1$. In the case $I_X=0$, the conservation of isospin implies that the amplitude $A(\nu_\mu d\to\mu^-(X^{++})_{I=0})$ must vanish, and in the case of $I_X=1$ it requires

$$A(\nu_\mu d\to\mu^-(X^{++})_{I=1})=-\sqrt{2}A(\nu_\mu d\to\nu_\mu(X^+)_{I=1}). \tag{20}$$

Having surveyed some of the consequences of the $SU(2)_M\otimes SU(2)_I$ lepton symmetry scheme, we now give our reasons for choosing it rather than some other groups. Since there are four lepton states, the most obvious alternative to our scheme is an SU(4) in which the leptons $\psi\equiv(\nu_e, e^-, \nu_\mu, \mu^-)$ are assigned to the representation $\underline{4}$ and their antiparticles $\bar\psi$ to a $\underline{4}^*$. No SU(4) symmetric interaction constructed from the product $(\bar\psi\times\psi)\times(\bar\psi\times\psi)$ contains the terms $(\bar\nu_\mu\mu^-)\times(\overline{e^-}\nu_e)$, and hence μ-meson decay is not allowed. It may only occur by breaking the SU(4) symmetry. Since we prefer to have a scheme in which the process $\mu^-\to e^- + \bar\nu_e + \nu_\mu$ is engendered by a symmetric interaction in lowest order, we reject SU(4) as a possible lepton symmetry.

An alternative to SU(4) is the symplectic group Sp(4) which contains as a subgroup $SU(2)_I\otimes O(2)_M$, and which can accommodate ψ in its four-dimensional representation. In this scheme the symmetric four-fermion interactions *do* contain terms giving rise to μ decay, but they are always of the form

$$(\bar{\nu}_\mu \mu^-)(\overline{e^-}\nu_e) - (\bar{\nu}_\mu \nu_e)(\overline{e^-}\mu^-). \tag{21}$$

Because we use $V-A$ for the space-time interaction, the two terms in Eq. (21) cancel one another by virtue of the symmetry of $V-A$ under Fierz transformations. Once again, we find that μ decay must break the symmetry.

Various authors have considered schemes with the Konopinski-Mahmoud[11] $L=1$ triplet $\varphi=(e^-,\nu,\mu^+)$ as their basis. If we assign them to the $\underline{3}$ representation of an SU(3), and their antiparticles to a $\underline{3}^*$, then we find that exactly as in the case of SU(4), SU(3) symmetric interactions of the type $(\bar{\varphi}\times\varphi)\times(\bar{\varphi}\times\varphi)$ do not engender μ decay.[12] This, in fact, is a general result: If leptons behave as the quarks of an SU(n) symmetry, μ decay is forbidden in lowest order and only takes place by breaking that symmetry. As an alternative to SU(3), we could assign φ to the three-dimensional representation of an O(3).[13] In this case the symmetric interaction does give rise to μ decay, but because e^- and μ^+ are in the same multiplet, it cannot give rise to a pure $V-A$ interaction. Thus we must again reject this alternative.

To summarize, our principal conclusions are:

(1) The assumption of an invariance under the $SU(2)_M \otimes SU(2)_I$ symmetry for leptonic and semileptonic interactions leads to simple sum rules for lepton-lepton scattering and for trident production. These relations, based on invariance under M spin, are analogous to U-spin scattering amplitude relations in strong interactions.

(2) The additional assumption of *total isospin conservation* provides us with additional relations for lepton-hadron scattering which may soon be subject to experimental test.

(3) A simple criterion for ruling out various symmetries is whether or not μ decay breaks the symmetry. Weinberg-type theories based on the Konopinski-Mahmoud lepton-number assignment, and other SU(4), Sp(4), and SU(3) theories in which the leptons are in the fundamental representation, are ruled out.

Finally, we note that our theory does not *require* the existence of heavy leptons. If they are found, however, they will have to form complete multiplets with respect to $SU(2)_M \otimes SU(2)_I$.

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