Si single crystal cleaved in air immediately before mounting. The result is shown in Fig. 3 and is compared with the absorption measurements performed by Brown and Rustgi[20] on a polycrystalline sample. Small differences could be mainly attributed to a lower resolution in our case. The main features, however, are present in both spectra. The rise of the yield at the high-energy end of the spectrum is due to the first peak in the spectrum[21] of SiO_2 at 106 eV since our sample was not completely free from oxide. (This was even more pronounced when using a polished single crystal.) When measurements under ultrahigh-vacuum conditions become possible, the investigation of single crystals by yield spectroscopy will certainly be of primary interest.

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Magnetic Field Effect on Plasma-Wave Dispersion in a Dielectric Layer

R. E. De Wames and W. F. Hall
North American Rockwell Science Center, Thousand Oaks, California 91360
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We present dispersion curves and field amplitudes for a plasma layer in a magnetic field. Various effects such as the nonreciprocal character of the dispersion, the pileup of bulk states at ω_∞, and the existence of surface states above ω_∞ should be observable in optical experiments.

The recent measurements of the surface plasma dispersion relation in InSb by Marschall, Fischer, and Queisser,[1] are in good agreement with predictions derived by treating the semiconductor as a dielectric half-space.[2] However, the dispersion relation in this case is a single, structureless curve asymptotic at small frequency to the light-line ($\omega = kc$) and at large ω to the surface plasma frequency $\omega_s = \omega_p[\kappa/\kappa+1)]^{1/2}$, κ being the high-frequency dielectric constant of the lattice. A more exacting test of the dielectric model can be made if the system is perturbed by the introduction of a magnetic field permeating the sample, and a finite thickness is considered for the dielectric.

The effect of a magnetic field on the optical constants deduced from bulk properties has been discussed by Palik and Wright.[3] The modes discussed in this paper, usually referred to as nonradiative solutions,[4] do not couple to a plane-wave field incident on an infinite film. However, in real samples anomalies in the optical and electron spectra have been identified with the excitation of these modes. Much theoretical and experimental work has recently been carried out elucidating the coupling mechanism, which has its origin in surface roughness.[5]

This Letter reports the results of a calculation of the plasma-wave dispersion relation for the simplest arrangement of applied field and propagation vector relative to the surface normal, namely, a dc magnetic field lying in the plane of the surface and a propagation vector parallel to the surface but perpendicular to the applied field. The case of infinite thickness has recently been discussed by Brion, Wallis, and Burstein,[6] while the dispersion relation for a plasma layer ($\kappa = 1$) was derived by Kondratyev[7] in 1964. Because we are concerned with the magnetoplasma effects on semiconductors (κ of order 10), the case $\kappa = 1$ will not be treated.

In this paper we treat the free carriers in a semiconductor as a plasma of negative charges moving together in a uniform background of compensating positive charge. We also take the effective mass m^* to be isotropic. The bulk properties of this model are discussed in Ref. 3.

Using Maxwell's equations and the equation of motion of a particle subjected to external electric and magnetic fields, one obtains for the electric field $\vec{E}$ an equation of the form

$$-\nabla^2\vec{E} + \nabla(\nabla\cdot\vec{E}) - (\omega/c)^2\epsilon\cdot\vec{E} = 0 \tag{1}$$

inside the medium, together with the usual requirements that tangential E and H be continuous across the vacuum boundary. The effect of a dc magnetic field (taken to lie in the z direction) on the free-charge carriers in an isotropic material is to alter the dielectric tensor ϵ from a multiple of the identity to a tensor of the form

$$\epsilon_{ij} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ -\epsilon_{xy} & \epsilon_{xx} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix}, \tag{2}$$

where for free carriers having plasma frequency ω_p and cyclotron frequency ω_c, the components of this tensor are

$$\epsilon_{xx} = \kappa\left(1 + \frac{\omega_p^2}{\omega_c^2 - \omega^2}\right), \quad \epsilon_{xy} = -i\kappa\frac{\omega_c\omega_p^2}{\omega(\omega_c^2 - \omega^2)}, \quad \epsilon_{zz} = \kappa\left(1 - \frac{\omega_p^2}{\omega^2}\right); \quad \omega_p = \left(\frac{Ne^2}{m^*\epsilon_0\kappa}\right)^{1/2}, \quad \omega_c = e\mu_0 H/m^*. \tag{3}$$

For a plasma layer with boundaries at $x = 0$ and $x = t$, whose dielectric tensor has the form (2), there exist solutions of (1) which decay exponentially outside the layer with distance from the boundaries. These nonradiative or "bound" solutions, for the special case of propagation in the y direction {dependence $\exp[i(\omega t - k_y y)]$}, have nonvanishing values only for E_x, E_y, and H_z. A direct application of (1) to a wave of this form, together with the boundary conditions on E_y and H_z, gives the equation

$$\tanh(k_y^2 - q^2)^{1/2}t = \frac{-2k^2q^2[(k_y^2 - q^2)(k_y^2 - k^2)]^{1/2}}{k^4(k_y^2 - q^2) + q^4(k_y^2 - k^2) + (\epsilon_{xy}/\epsilon_{xx})^2 k^4 k_y^2}, \tag{4}$$

where $k^2 = \omega^2/c^2$ and $q^2 = k^2(\epsilon_{xx}^2 + \epsilon_{xy}^2)/\epsilon_{xx}$. The dispersion relation[7] determined by (4) is illustrated in Fig. 2 for the dielectric function (3) with $\kappa = 20$ and $\omega_c/\omega_p = 0.8$. Figure 1 shows the dispersion relation in the absence of a magnetic field, while Fig. 3 illustrates the behavior of the electric fields inside the sample for these two cases. In these figures k_y is plotted in units of $k_p = \omega_p/c$; x_p denotes ω/ω_p.

In the absence of a magnetic field, the effect of introducing a finite thickness for the dielectric is to split the surface-wave dispersion relation into two branches which merge at the light-line and at infinity, corresponding to solutions for E_y which are odd or even with respect to reflection about the plane $x = t/2$.[2,4] Also, bulk solutions, which for infinite thickness fill the region $\omega > [\omega_p^2 + (k_y c)^2/\kappa]^{1/2}$, are restricted to begin at the light-line with an integral number of half-wavelengths across the layer $[(k_y^2 - q^2)^{1/2} = in\pi/t]$.

In Figs. 1 and 2 the lines starting at the left-hand side of the light-line demark regions where $(k_y^2 - q^2)^{1/2} = k_x$ in the medium is real or imaginary, leading to surface or bulk solutions, respectively. In the left-hand side of Fig. 3 we have schematically drawn the behavior of the fields inside the medium for specific points denoted by numbered dots on the dispersion curves in Fig. 1. The even and odd properties of the solution about $x = t/2$, the variations of the fields along the bulk branches, and the derivative and phase relationship of E_x to E_y are some of the important properties illustrated in Fig. 3.

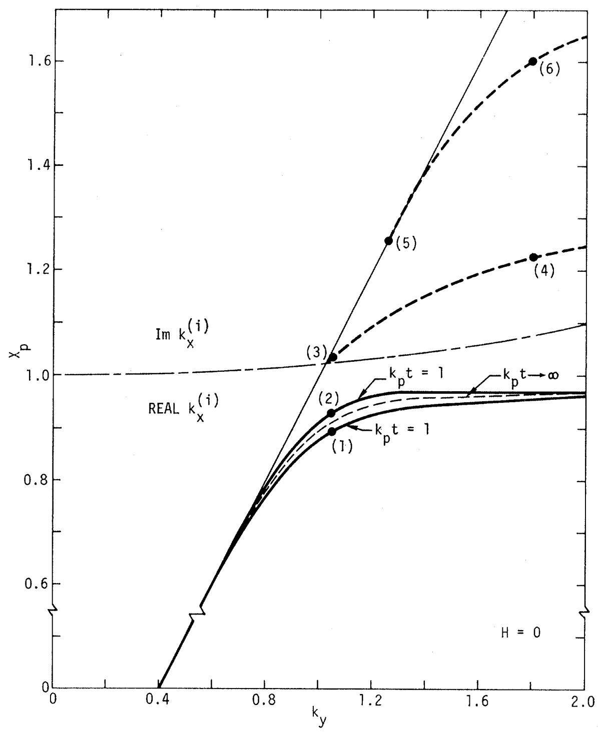

FIG. 1. Dispersion curves for a dielectric layer with $\epsilon = \kappa(1 - \omega_p^2/\omega^2)$ and $\kappa = 20$. Light dashed curve, infinite thickness. Heavy solid curves, surface-wave branches for $k_p t = 1$. Dot-dash curve, boundary between surface and bulk solutions. Heavy dashed curves, first two bulk solutions for $k_p t = 1$. Note that all solutions begin at the light-line $k_y = \omega/c$, (light solid line).

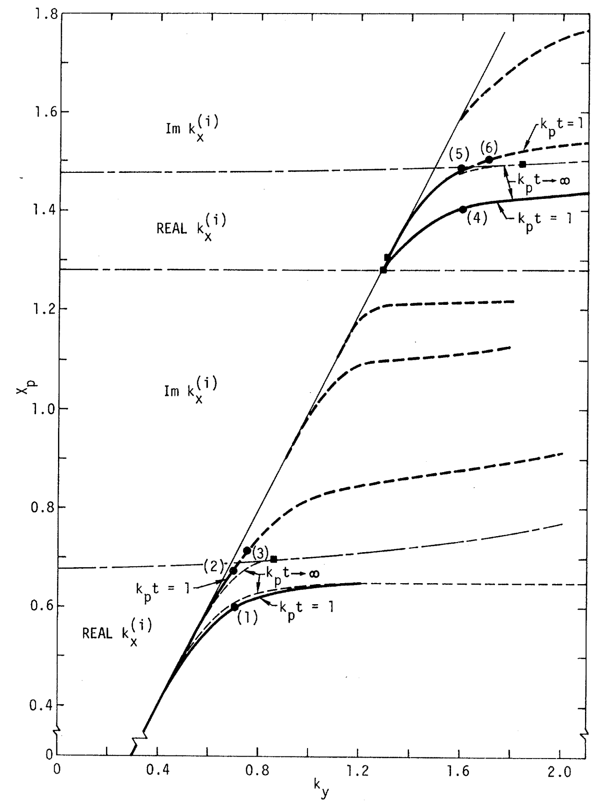

FIG. 2. Dispersion curves for a dielectric layer in a magnetic field with $\omega_c/\omega_p = 0.8$ and $\kappa = 20$. Symbolism is the same as for Fig. 1. The three dot-dashed curves are the boundaries between bulk and surface solutions, with the middle curve being the straight line $\omega = \omega_\infty = (\omega_c^2 + \omega_p^2)^{1/2}$.

Applying a magnetic field perpendicular to the propagation direction and parallel to the surface introduces a great deal of structure in the dispersion curves, primarily because q^2, the square of the bulk propagation vector, now has a singularity at $\omega^2 = \omega_c^2 + \omega_p^2 \equiv \omega_\infty^2$. For a vanishingly small applied field, one observable effect is a difference in the asymptotic values $\omega_{s\pm}$ of the two surface wave branches of the dispersion relation

$$\omega_{s\pm} = (\omega_s^2 + \omega_c^2/4)^{1/2} \pm \omega_c/2. \tag{5}$$

In addition, the singularity of q^2 becomes an accumulation point for bulk solutions for frequencies less than ω_∞, and no solutions exist in the region between ω_∞ and the higher frequency at which $q^2 = k^2$.

When the applied field becomes sufficiently strong that $\kappa\omega_c > \omega_\infty$, a condition which is not attainable for $\kappa = 1$, the various branches of the dispersion relation take on the appearance illustrated in Fig. 2. Two new surface branches appear[8] in the region above ω_∞, one beginning at the point

$$k_y = \kappa\omega_c\omega_\infty/c(\kappa^2\omega_c^2 - \omega_\infty^2)^{1/2}, \quad \omega = \omega_\infty, \tag{6}$$

and having asymptote ω_{s+} at large k_y, and the other beginning at the light-line, where $\omega = [\omega_\infty^2 + \omega_p^2(\kappa - 1)^{-1}]^{1/2}$. This latter branch crosses into the region where $q^2 > k^2$, becoming a bulk solution. This pattern is repeated by the original surface branches at frequencies below ω_∞: The lower branch tends asymptotically to ω_{s-}, while the upper branch makes the transition to bulk. For large k_y these bulk solutions tend toward the line $\omega = \omega_\infty$ for $\omega < \omega_\infty$ and toward the line $k_y = \sqrt{\kappa}\,\omega/c$ for $\omega > \omega_\infty$.

The electric fields associated with these surface branches, as illustrated on the right-hand side of Fig. 3, exhibit several interesting features. First, the lack of symmetry between propagation in the $\pm y$ directions for the semi-infinite medium[6] shows up here as a localization of the fields on the surface $x = 0$ for the lowest and highest surface branches [points (1) and (5)], and a

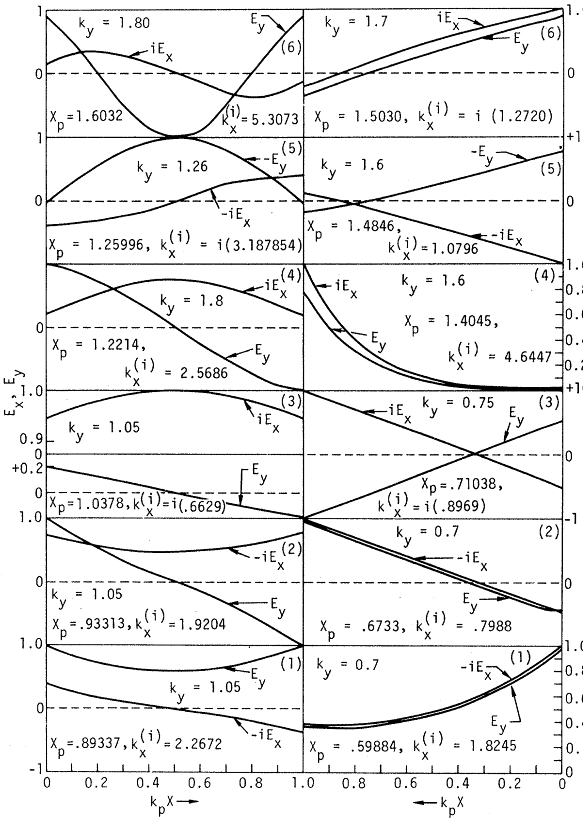

FIG. 3. Electric-field eigenvectors for various frequencies from Figs. 1 and 2 for the case $k_p t = 1$. Left-hand plots show symmetric and antisymmetric behavior of the fields about $x = t/2$, corresponding to the points on the dispersion curves of Fig. 1 labeled by the numbered dots. Right-hand plots show localization of fields near $x = 0$ and $x = t$ for the corresponding points on Fig. 2.

localization on the surface $x = t$ for the two middle surface branches [points (2) and (4)]. [As the thickness tends toward infinity, this localization becomes complete, so that a wave propagating in the $+y$ direction, which is localized on the back surface ($x = t$) can, by translation and rotation of the coordinate system, be regarded as a wave propagating in the $-y$ direction on the front surface.] The transition between surface and bulk modes has little effect on the position dependence of the fields, as can be seen by comparing E_x at points (2) and (3). For all surface modes, E_x and E_y differ in phase by $\pi/2$, and there is, of course, an arbitrary normalization. At point (3) this normalization has been chosen to reflect the plotted values of E_y about zero relative to E_x to remove the two lines from near coincidence; a similar choice was made at point (5).

The primary effect of finite thickness for the dielectric layer is the substitution of discrete branches for the bulk continuum. The "band edge" at $\omega = \omega_\infty$, where an infinite number of bulk solutions pile up, should be easily observable in the reflectance spectrum of such a layer. A further point is that by applying a magnetic field, the dispersion curve observed by Marschall, Fishcher, and Queisser,[1] should split into the two states described in this Letter. The manner in which the surface branch is continuous across the various regions of k_x is clearly illustrated in this calculation.

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